ECE-2025 Fall-00

LECTURE OBJECTIVES
for ALL 26 Lectures

COURSE OBJECTIVE

- Students will be able to:
- Understand **mathematical** descriptions of signal processing **algorithms** and express those algorithms as computer **implementations** (MATLAB)

LECTURE #1

- Write general formula for a "sinusoidal" waveform, or signal
- From the formula, plot the sinusoid versus time
- What's a **signal**?
  - It's a **function** of time, x(t)
  - in the mathematical sense

LECTURE #2

- Define Sinusoid from a plot
- Relate TIME-SHIFT to PHASE
- Introduce an **ABSTRACTION**:
  - Complex Numbers **represent** Sinusoids
  - Complex Exponential Signal

$$z(t) = Xe^{j\omega t}$$

LECTURE #3

- Phasors = Complex Amplitude
  - Complex Numbers **represent** Sinusoids

$$z(t) = Xe^{j\omega t} = (Ae^{j\varphi})e^{j\omega t}$$

- Develop the ABSTRACTION:
  - Adding Sinusoids = Complex Addition
  - **PHASOR ADDITION THEOREM**

LECTURE #4

- Sinusoids with DIFFERENT Frequencies
  - SYNTHESIZE by Adding Sinusoids

$$x(t) = \sum_{k=1}^{N} A_k \cos(2\pi f_k t + \varphi_k)$$

- SPECTRUM Representation
  - Graphical Form shows **DIFFERENT** Freqs

LECTURE #5

- Signals with HARMONIC Frequencies
  - Add Sinusoids with $f_k = kf_0$

$$x(t) = A_0 + \sum_{k=1}^{N} A_k \cos(2\pi k f_0 t + \varphi_k)$$

- FREQUENCY can change vs. TIME
  - Chirps: $x(t) = \cos(\alpha t^2)$
  - Introduce Spectrogram Visualization
    (specgram.m) **(plotspec.m)**

LECTURE #6

- Work with the Fourier Series Integral

$$a_k = \frac{1}{T_0}\int_0^{T_0} x(t)e^{-j(2\pi k/T_0)t}\,dt$$

- **ANALYSIS** via Fourier Series
  - For PERIODIC signals: $x(t+T) = x(t)$
  - **SPECTRUM from the Fourier Series**

LECTURE #7

- SAMPLING can cause ALIASING
  - Sampling Theorem
  - Sampling Rate > 2(Highest Frequency)
- Spectrum for digital signals, x[n]
  - Normalized Frequency

$$\hat{\omega} = \omega T_s = \frac{2\pi f}{f_s} + 2\pi\ell$$

ALIASING

LECTURE #8

- FOLDING: a type of ALIASING
- DIGITAL-to-ANALOG CONVERSION is
  - Reconstruction from samples
    - SAMPLING THEOREM applies
  - Smooth Interpolation
- Mathematical Model of D-to-A
  - SUM of SHIFTED PULSES
    - Linear Interpolation example

LECTURE #9

- INTRODUCE FILTERING IDEA
  - Weighted Average
  - Running Average
- FINITE IMPULSE RESPONSE FILTERS
  - **FIR** Filters
  - Show how to compute the output y[n] from the input signal, x[n]

LECTURE #10

- BLOCK DIAGRAM REPRESENTATION
  - Components for Hardware
  - Connect Simple Filters Together to Build More Complicated Systems
- GENERAL PROPERTIES of FILTERS
  - LINEARITY
  - TIME-INVARIANCE
  - ==> CONVOLUTION

LTI SYSTEMS

LECTURE #11

- SINUSOIDAL INPUT SIGNAL
  - DETERMINE the FIR FILTER OUTPUT
- FREQUENCY RESPONSE of FIR
  - PLOTTING vs. Frequency
  - MAGNITUDE vs. Freq
  - PHASE vs. Freq

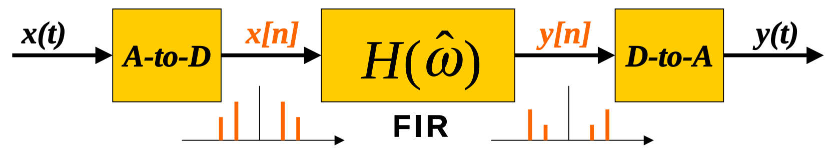
MAG

PHASE

$$H(\hat{\omega}) = |H(\hat{\omega})| e^{j\varphi(\hat{\omega})}$$

LECTURE #12

- Two Domains: Time & Frequency
- Track the spectrum of x[n] thru an FIR Filter.
- **UNIFICATION**: How does Frequency Response affect x(t) to produce y(t) ?



LECTURE #13

- INTRODUCE the Z-TRANSFORM
  - Give Mathematical Definition
  - Show how the H(z) POLYNOMIAL simplifies analysis
    - **CONVOLUTION** EXAMPLE
- Z-Transform can be applied to
  - FIR Filter: h[n] --> H(z)
  - Signals: x[n] --> X(z)

$$H(z) = \sum_n h[n] z^{-n}$$

LECTURE #14

- ZEROS and POLES
- Relate H(z) to FREQUENCY RESPONSE

$$H(\hat{\omega}) = H(z)\Big|_{z=e^{j\hat{\omega}}}$$

- THREE DOMAINS:
  - Show Relationship for FIR:

$$h[n] \leftrightarrow H(z) \leftrightarrow H(e^{j\hat{\omega}})$$

LECTURE #15 (Review)

- ZEROS and POLES
- Relate H(z) to FREQUENCY RESPONSE

$$H(\hat{\omega}) = H(z)\Big|_{z=e^{j\hat{\omega}}}$$

- THREE DOMAINS:
  - Show Relationship for FIR:

$$h[n] \leftrightarrow H(z) \leftrightarrow H(e^{j\hat{\omega}})$$

LECTURE #16

- Bye bye to D-T Systems for a while
- The UNIT IMPULSE signal
  - Definition
  - Properties
- Continuous-time signals and systems
  - Example systems
  - Review: Linearity and Time-Invariance
  - Convolution integral: impulse response

LECTURE #17

- Review of C-T LTI systems
- Evaluating convolutions
  - Examples
  - Impulses
- LTI Systems
  - Cascade and parallel connections
  - Stability and causality

LECTURE #18

- Review of convolution
  - **THE** operation for **LTI** Systems
- Complex exponential input signals
  - Frequency Response
  - Cosine signals
    - Real part of complex exponential
- Fourier Series thru $H(j\omega)$
  - These are Analog Filters

LECTURE #19

- Review
  - Frequency Response
  - Fourier Series
- Definition of Fourier transform

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

Relation to Fourier Series

- Examples of Fourier transform pairs

LECTURE #20

- The Fourier transform

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

- More examples of Fourier transform pairs
- Basic properties of Fourier transforms
  - Convolution property
  - Multiplication property

LECTURE #21

- Review of FT properties
  - Convolution <--> multiplication
  - Frequency shifting
- Sinewave Amplitude Modulation
  - AM radio
- Frequency-division multiplexing
  - FDM

LECTURE #22

- Sampling Theorem Revisited
  - GENERAL: in the FREQUENCY DOMAIN
  - Fourier transform of sampled signal
  - Reconstruction from samples
- Review of FT properties
  - Convolution <--> multiplication
  - Frequency shifting
  - Review of AM

LECTURE #23

- INFINITE IMPULSE RESPONSE FILTERS
  - Define **IIR** DIGITAL Filters
  - Have **FEEDBACK**: use PREVIOUS OUTPUTS

$$y[n] = \sum_{\ell=1}^{N} a_\ell y[n-\ell] + \sum_{k=0}^{M} b_k x[n-k]$$

  - Show how to compute the output y[n]
    - FIRST-ORDER CASE (N=1)
    - Z-transform: Impulse Response h[n] <--> H(z)

LECTURE #24

- SYSTEM FUNCTION: H(z)
- H(z) has **POLES** and ZEROS
- FREQUENCY RESPONSE of IIR
  - Get H(z) first

$$H(e^{j\hat{\omega}}) = H(z)\big|_{z=e^{j\hat{\omega}}}$$

- THREE-DOMAIN APPROACH

$$h[n] \leftrightarrow H(z) \leftrightarrow H(e^{j\hat{\omega}})$$

LECTURE #25

- Discrete-Time Filtering of Continuous-Time Signals
  - Basic Configuration
    - CT Input -> A/D -> DT System -> D/A -> CT Output
- **EFFECTIVE** FREQUENCY RESPONSE
  - For Bandlimited Input Signals
  - Relies on the General Version of the Sampling Theorem

LECTURE #26

- THREE-DOMAIN APPROACH
  - EXHIBIT BANDPASS FILTERS
- RE-UNIFICATION:
  - How does Frequency Response affect x(t) to produce y(t) ?