1.1(a) $z = j10 \qquad z = Ae^{j\theta} \qquad A = \sqrt{0^2 + 10^2} = 10$

$z = 0 + j10 \qquad \theta = \tan^{-1}\left[\frac{Im(z)}{Re(z)}\right] = \tan^{-1}\left(\frac{10}{0}\right) = \frac{\pi}{2}$

$z = 10e^{j\pi/2}$

1.1(b) $z = -10 = -10 + j0 = 10e^{j\pi}$

1.1(c) $z = (-10, -10)$ (an ordered pair)

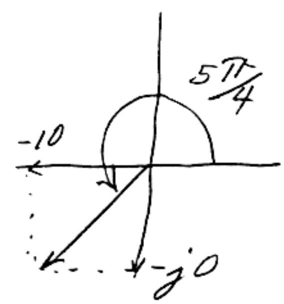
$z = -10 - j10$

$z = Ae^{j\theta} \qquad A = \sqrt{10^2 + 10^2} = 10\sqrt{2}$

$\theta = \tan^{-1}\frac{10}{10} = \frac{\pi}{4}$ <u>BUT</u> this is the principal.

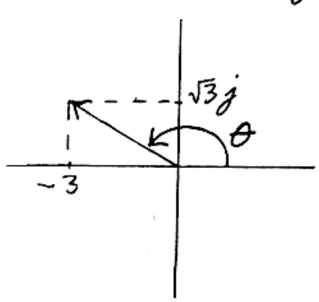
The actual value $= \frac{\pi}{4} + \pi = \frac{5\pi}{4}$

$z = 10\sqrt{2}\, e^{j\frac{5\pi}{4}}$



1.1(d) $z = -2 + j2 = 2\sqrt{2}\, e^{j\frac{3\pi}{4}}$

1.1(e) $z = -3 + j\sqrt{3} \qquad A = \sqrt{3^2 + \sqrt{3}^2} = \sqrt{12} = 2\sqrt{3}$



$\tan^{-1}\frac{\sqrt{3}}{-3} = -0.52\text{ rad.} = -\frac{\pi}{6}$

Obviously $\theta = -\pi/6 + \pi = \frac{5\pi}{6}$

$z = 2\sqrt{3}\, e^{j\frac{5\pi}{6}}$

1.1(f) $z = 20 = 20e^{j0}$

1.2(a) $z = 3\sqrt{2}\, e^{-j\frac{3\pi}{4}} = 3\sqrt{2}\left(\cos\left(-\frac{3\pi}{4}\right) + j\sin\left(-\frac{3\pi}{4}\right)\right)$

$z = 3\sqrt{2}\left(\frac{-\sqrt{2}}{2} + j\left(-\frac{\sqrt{2}}{2}\right)\right) = -3 - j3$

1.2(b) $z = 5e^{j\pi/2} = 0 + j5$

1.2(c) $z = 4\angle\pi/3 = 4\left(\cos\frac{\pi}{3} + j\sin\pi/3\right)$

$z = 4\left(0.5 + j\,\sqrt{3}/2\right) = 2 + j2\sqrt{3}$

1.2(d) $z = 5\angle-61\pi = 5\left(\cos(-61\pi) + j\sin(-61\pi)\right)$

$-61\pi = \pi$

$z = -5 + j0$

1.3(a) $z_1 = -2 - j2 \qquad z_1^* = -2 - (-j2) = -2 + j2$

1.3(b) $jz_2 = j\left(3e^{-j\frac{3\pi}{4}}\right) = e^{j\pi/2} \cdot 3e^{-j\frac{3\pi}{4}} = 3e^{j\left(\frac{\pi}{2} - \frac{3\pi}{4}\right)}$

$= 3e^{-j\frac{\pi}{4}}$ or $2.12 - j2.12$

1.3(c) $\frac{z_2}{z_1} = \frac{3e^{-j\frac{3\pi}{4}}}{-2-j2} = \frac{3e^{-j\frac{3\pi}{4}}}{2\sqrt{2}\,e^{j\frac{5\pi}{4}}} = 1.06e^{-j\left(\frac{3\pi}{4} + \frac{5\pi}{4}\right)}$

$= 1.06e^{-j\frac{8\pi}{4}} = 1.06e^{-j2\pi} = 1.06e^{j0} = 1.06 + j0$

1.3(d) $z_2^2 = \left(3e^{-j3\pi/4}\right)^2 = 9e^{-j\frac{3\pi}{2}} = 0 - 9j$

1.3(e) $z_1^{-1} = \dfrac{1}{-2-j2} = \dfrac{-2+j2}{(-2-j2)(-2+j2)} = \dfrac{-2+j2}{4+4}$

$= -\frac{1}{4} + j\frac{1}{4} = \frac{\sqrt{2}}{4}e^{j\frac{3\pi}{4}}$

1.3(f) $z_1 z_2 = (-2-j2)3e^{-j\frac{3\pi}{4}} = 2\sqrt{2}e^{j\frac{5\pi}{4}} \cdot 3e^{-j\frac{3\pi}{4}}$

$= 6\sqrt{2}\,e^{j\frac{2\pi}{4}} = 6\sqrt{2}\,e^{j\frac{\pi}{2}} = 0 + j6\sqrt{2}$

1.3(g) $z_1 + z_2^* = -2+j2 + 3e^{+j\frac{3\pi}{4}} =$

$= -2+j2 + 3\left(\cos\frac{3\pi}{4} + j\sin\frac{3\pi}{4}\right)$

$= -2+j2 + 3\left(-\frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}\right) = -2+j2 - \frac{3}{2}\sqrt{2} + j\frac{3\sqrt{2}}{2}$

$= -4.12 + j4.12 = \sqrt{2}(4.12)e^{j\frac{3\pi}{4}}$

1.3(h) $|z_2^2| = z_2 z_2^* = 3e^{-j\frac{3\pi}{4}} \cdot 3e^{+j\frac{3\pi}{4}} = 9$

1.3(i) $z_2 + z_2^* = 2\mathcal{Re}[z_2] = 2\left(3\cos\frac{3\pi}{4}\right)$

$= 6\frac{\sqrt{2}}{2} = 3\sqrt{2}$

1.4(a) $z = Ae^{-j\pi/3} = A\left(\cos\left(\frac{-\pi}{3}\right) + j\sin\left(\frac{-\pi}{3}\right)\right)$

$$z^* = A\left(\cos\left(-\frac{\pi}{3}\right) - j\sin\left(-\frac{\pi}{3}\right)\right)$$

$$= A\left(\cos\left(-\frac{\pi}{3}\right) + j\sin\left(\frac{\pi}{3}\right)\right)$$

$$\mathrm{Im}[z^*] = A\sin\left(\frac{\pi}{3}\right) = A\frac{\sqrt{3}}{2}$$

1.4(b) $z = Ae^{-j\frac{\pi}{3}}$

$$z + z^* = Ae^{-j\pi/3} + Ae^{+j\frac{\pi}{3}}$$

but $\cos\theta = \dfrac{e^{j\theta} + e^{-j\theta}}{2}$

so $z + z^* = A\left(e^{j\pi/3} + e^{-j\pi/3}\right) = 2A\cos\pi/3$

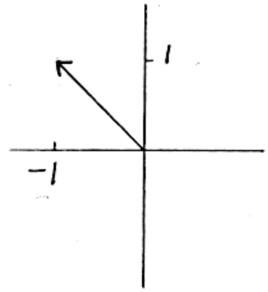
$$= 2A\left(\tfrac{1}{2}\right) = A$$

1.4(c) $z = 10e^{j\phi} = 10(\cos\phi + j\sin\phi)$

$$\mathrm{Re}[jz] = -10\sin\phi$$

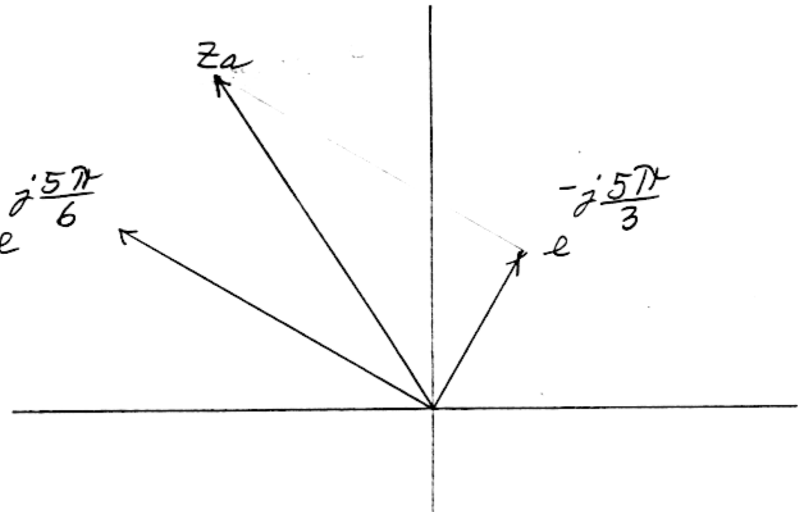
1.4(d) $z = -\alpha + j\alpha = \alpha(-1 + j)$

$$z = \sqrt{2}\,\alpha\, e^{j\frac{3\pi}{4}}$$

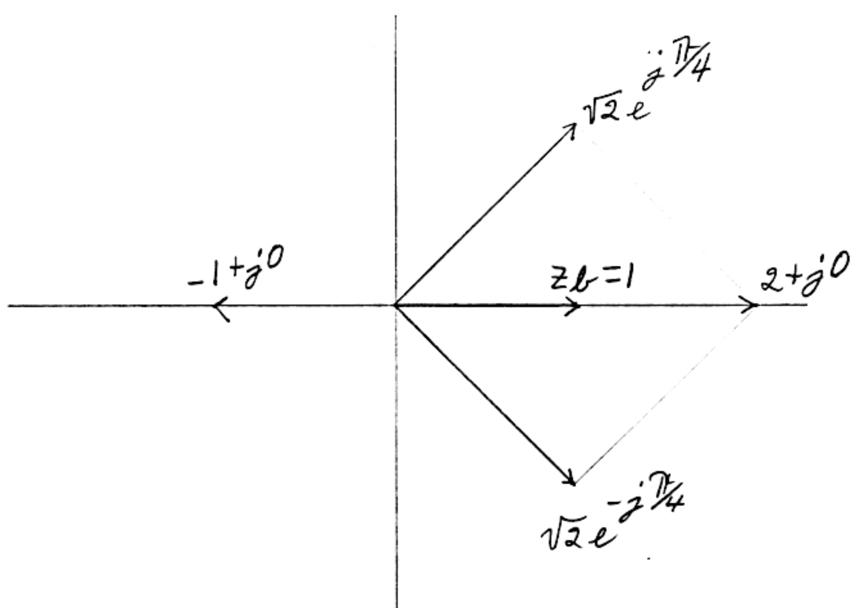


1.5(a) $z_a = e^{-j\frac{5\pi}{3}} + 2e^{j\frac{5\pi}{6}}$

$$z_a = 0.5 + j\sqrt{3}/2 + 2\left(-\sqrt{3}/2 + j0.5\right)$$
$$= 0.5 + j\sqrt{3}/2 - \sqrt{3} + j1 = -1.23 + j1.87$$
$$= 2.24e^{j2.15}$$



1.5(b) $z_b = \sqrt{2}e^{j\pi/4} + \sqrt{2}e^{-j\pi/4} - 1 = 1 + j1 + 1 - j1 - 1 = 1$



1.6 By inspection of the graph

$A = 50 \qquad f_0 = \frac{1}{T} = \frac{1}{4 \cdot 10^{-3}} = 250\,Hz$

$\omega_0 = 2\pi f_0 = 500\pi$

$t_m = -10^{-3}\,sec \qquad \phi = -\omega_0 T = -500\pi(-10^{-3})$

$\phi = 0.5\pi$

1.7 The amplitude can be determined from the complex amplitude, Z.

$Z = \sqrt{2}(1-j) \qquad A = \sqrt{2} \cdot |1-j| = \sqrt{2} \cdot \sqrt{2} = 2$

The period $T = \frac{1}{F_0} = \frac{1}{8} = 0.125$

The phase is determined from the complex amplitude $Z = \sqrt{2}(1-j)$. Expressed in polar form,

$Z = 2e^{-j\pi/4} \quad \therefore \phi = -\pi/4$

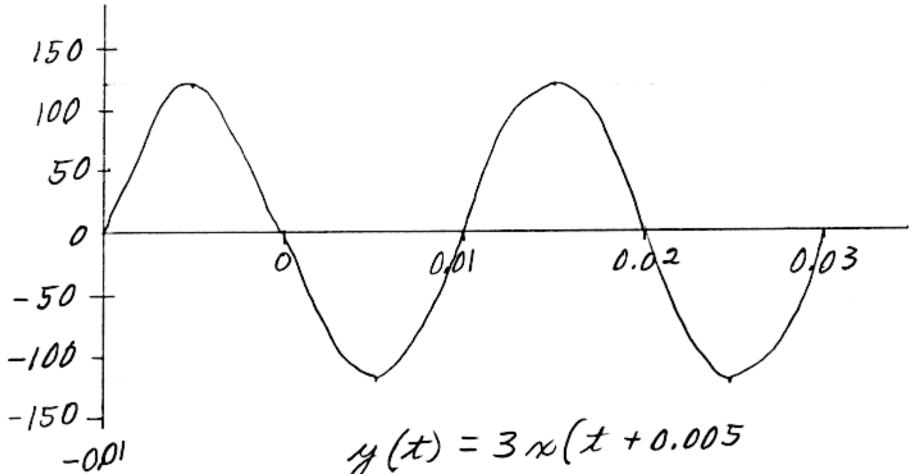
1.8(a) $x(t) = A\cos(\omega_0 t + \phi)$

$A = 40 \qquad T = 0.02 \quad f = \frac{1}{T} = 50 \quad \omega_0 = 2\pi f = 100\pi$

$\phi = 0$

1.8(b) $x(t) = 40\cos(100\pi t) \qquad Z = 40e^{j0}$

1.8(c)



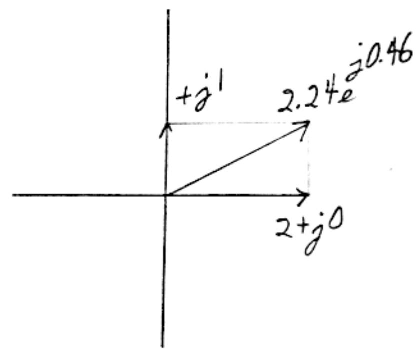
$y(t) = 3x(t + 0.005$

1.9(a) $x_a(t) = 2\cos(333\pi t) - \sin(333\pi t)$
$= 2\cos(333\pi t) - \cos(333\pi t - \pi/2)$

Expressed as complex phasors:

$2e^{j0} - e^{-j\pi/2} = 2 + j0 - (0 - j1) = 2 + j1 = 2.24e^{j0.46}$

$\therefore x_a(t) = 2.24\cos(333\pi t + 0.46)$



1.9(b) $x_b(t) = 7\cos(245t + 3\pi/4)$
$+ 7\cos(245t + \pi/2)$

As phasors

$X_b = 7e^{j3\pi/4} + 7e^{j\pi/2}$

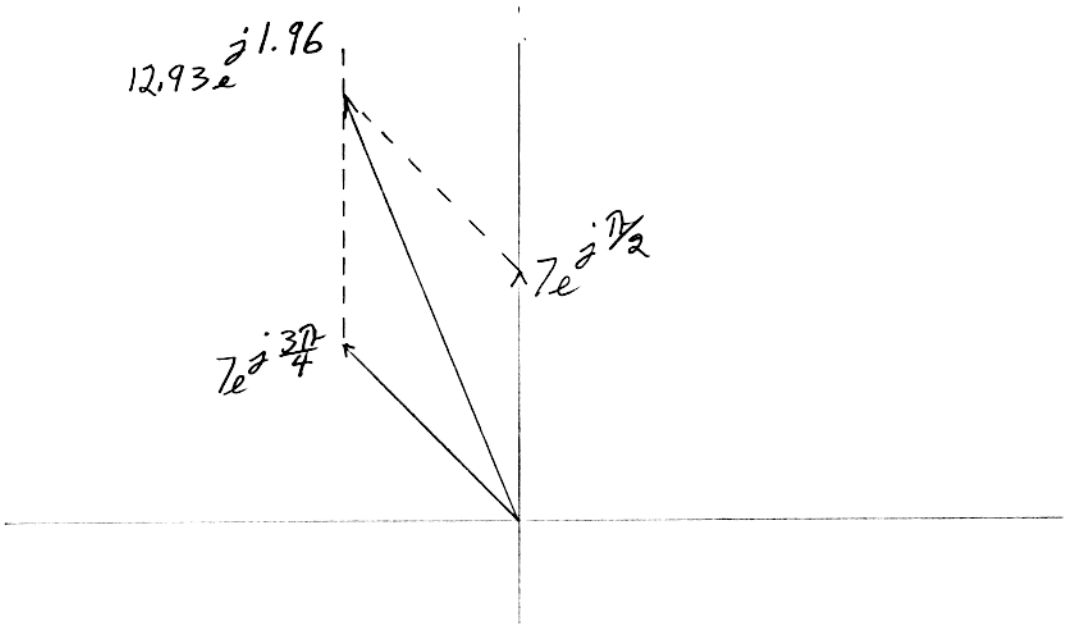
and in rectangular form

$X_b = -7\frac{\sqrt{2}}{2} + j\frac{7\sqrt{2}}{2} + 7j = -4.95 + j11.95$

1.9(b) continued

in polar form, $X_b = 12.93e^{j1.96}$

$x_b(t) = 12.93\cos(245t + 1.96)$



1.9(c) $x_c(t) = \cos(41t + 17\pi) + \sqrt{2}\cos(41t + \pi/4) + \sqrt{2}\cos(41t - \pi/4)$

$17\pi = \pi$ (17 is an odd number)

$X_c = 1e^{j\pi} + \sqrt{2}e^{j\pi/4} + \sqrt{2}e^{-j\pi/4}$

Note that $e^{j\pi} = -1$ and that the last two numbers are conjugates, so they add to $2\cdot\mathcal{Re}\left[\sqrt{2}e^{-j\pi/4}\right] = 2\cdot 1 = 2$

$X_c = -1 + 2 = 1$