GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Fall 2000
Problem Set #2

Assigned: 1-Sept-00
Due Date: Week of 11-Sept-00

Reading: In *DSP First*, all of Chapter 2 on *Sinusoids*; and start reading in Chapter 3: *Spectrum Representation,* especially pp. 48–61.

$\Longrightarrow$ Please check the "Bulletin Board" often. All official course announcements are posted there.

ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web. CD-ROM.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.

PROBLEM 2.1:

Each of the following signals may be simplified, and expressed as a single sinusoid of the form: $A\cos(\omega t + \phi)$. For each signal, draw a vector diagram of the complex amplitudes (phasors), and use vector addition to estimate the amplitude A and phase ϕ of the sinusoid. Then use the phasor addition theorem to find the exact values for A and ϕ.

(a) $x_a(t) = 3\cos(388\pi t - 4\pi/3) + \cos(388\pi t + 3\pi/4)$

(b) $x_b(t) = \sqrt{2}\cos(12.6\pi t + 11\pi) + 2\cos(12.6\pi t - 12.5\pi) + \sqrt{3}\cos(12.6\pi t + 38\pi)$

(c) $x_c(t) = \cos(60\pi t + 3\pi/4) + \cos(60\pi t + 5\pi/4) + 2\cos(60\pi t - \pi/4) + 2\cos(60\pi t + \pi/4)$

PROBLEM 2.2*:

Define $x(t)$ as

$$x(t) = \sqrt{3}\cos(\omega_0 t + \pi/3) + \sin(\omega_0 t + \pi/2)$$

(a) Find a complex-valued signal $z(t)$ such that $x(t) = \Re e\{z(t)\}$. Simplify $z(t)$ as much as possible, so that you can identify its complex amplitude. Hint: Be careful to note that the second term in $x(t)$ is a sine rather than a cosine.

(b) Assume that $\omega_0 = 0.1\pi$ rad/sec. Make a plot of $\Re e\{(1-j)e^{j\omega_0 t}\}$ over the range $-10 \le t \le 10$ secs. How many periods are included in the plot?

PROBLEM 2.3*:

Define $x(t)$ as

$$x(t) = 5\sqrt{2}\cos(20\pi t + \pi/4) + A\cos(20\pi t + \phi) \tag{1}$$

where A is a *positive* number. In addition, assume that $x(t)$ has a phase of zero, so that it may be written as

$$x(t) = B\cos(20\pi t), \tag{2}$$

where B is a *positive* number.

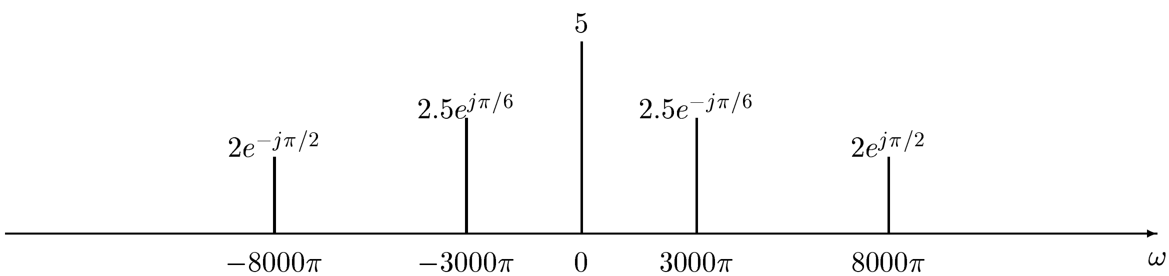
(a) What relationship must exist between A and ϕ in order for $x(t)$ to have zero phase as indicated in Eq. 2?

(b) If $B = 10$, what are the values for A and ϕ?

(c) Now assume that B is unspecified. Find the values for A, B, and ϕ so that the value of A is *minimized.* Draw a plot of the complex amplitudes to prove using a geometrical argument that you have found the minimum for A. *Hint: Recall the geometrical "theorem" that tells you how to find the shortest distance between a line and a point that is not on the line (have you heard the term "projection"?).*

PROBLEM 2.4*:

A real signal $x(t)$ has the following two-sided spectrum:



(a) Write an equation for $x(t)$ as a sum of cosines.

(b) Plot the spectrum of the signal $y(t) = 2x(t) - 3\cos(5000\pi(t - 0.002))$.

PROBLEM 2.5*:

In AM radio, the transmitted signal (voice or music) is modulated by a sinusoid at the assigned broadcast frequency of the AM station. For example, WSB in Atlanta has a *carrier frequency* of 750 kHz. Therefore, if $x(t)$ is the voice/music signal, then the transmitted signal would be:

$$y(t) = [x(t) + A]\cos(2\pi(750 \times 10^3)t)$$

where A is a constant. (A is introduced to make the AM receiver design easier, in which case A must be chosen to be larger than the maximum value of $x(t)$.) Suppose that the signal that is to be transmitted is

$$x(t) = 3\cos(2000\pi t + \pi/4) + \cos(4000\pi t + \pi/2)$$

Draw the spectrum for $y(t)$ assuming a carrier at 750 kHz with $A = 5$. *Hint: Substitute for $x(t)$ and expand $y(t)$ into a sum of cosines.*

PROBLEM 2.6*:

Complex exponentials obey the expected rules of algebra when doing integrals and derivatives. Consider the complex signal $z(t) = Ze^{j\pi t/2}$ where $Z = e^{-j\pi/3}$.

(a) Show that the first derivative of $z(t)$ with respect to time can be represented as a new complex exponential $Qe^{j\pi t/2}$, i.e., $\frac{d}{dt}z(t) = Qe^{j\pi t/2}$. Determine the value for the complex amplitude Q. How much greater (or smaller) is the angle of Q than the angle of Z.

(b) Evaluate the definite integral of $z(t)$ over the range $0 \le t \le 1$: $\int_0^1 z(t)dt = ?$

Note that integrating a complex quantity follows the expected rules of algebra: you could integrate the real and imaginary parts separately, but you can also *use the integration formula for an exponential* directly on $z(t)$.

(c) Evaluate the integral of the magnitude squared $|z(t)|^2$ over the range $-1 \le t \le 1$:

$$\int_{-1}^{1} |z(t)|^2 dt = ?$$