# GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

**ECE 2025 Fall 2000**
**Problem Set #3**

Assigned: 9-Sept-00
Due Date: Week of 18-Sept-00

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**Quiz #1 will be held in lecture on Monday 18-Sept-00.** It will cover material from Chapters 2 and 3, as represented in Problem Sets #1 and #2.
**Closed book, calculators permitted, and one hand-written formula sheet ($8\frac{1}{2}'' \times 11''$, both sides)**

Reading: In *DSP First*, all of Chapter 3 on *Spectrum Representation,* especially pp. 48–73.

$\Longrightarrow$ **Please check the "Bulletin Board" often. All official course announcements are posted there.**

**ALL** of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web. Some problems have solutions similar to those found on the CD-ROM.

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**Your homework is due in recitation at the beginning of class.** After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.

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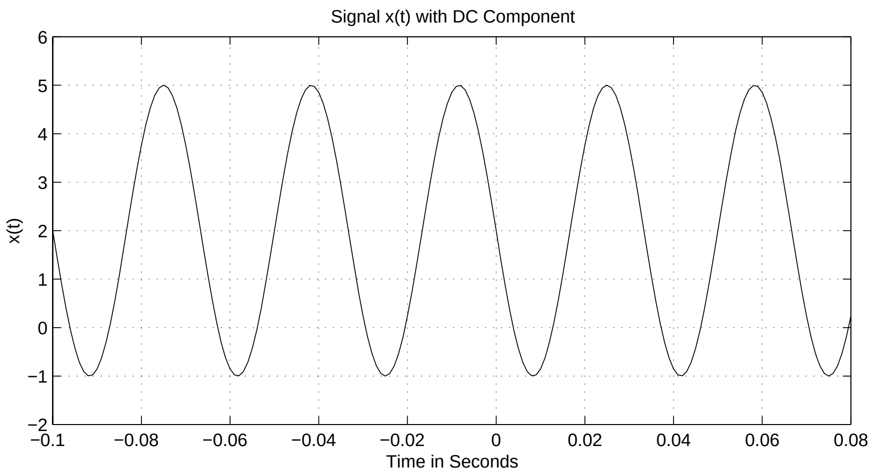
## PROBLEM 3.1*:




The above signal $x(t)$ consists of a DC component plus a cosine signal. The terminology *DC component* means a component that is constant versus time.

(a) What is the frequency of the DC component? What is the frequency of the cosine component?

(b) Write an equation for the signal $x(t)$. You should be able to determine numerical values for all the amplitudes, frequencies, and phases in your equation by inspection of the above graph.

(c) Expand the equation obtained in the previous part into a sum of positive and negative frequency complex exponential signals.

(d) Plot the two-sided spectrum of the signal $x(t)$. Show the complex amplitudes for each positive and negative frequency contained in $x(t)$.

**PROBLEM 3.2***:
A signal composed of sinusoids is given by the equation

$$x(t) = 3\cos(50\pi t - \pi/8) - 5\cos(150\pi t + \pi/6)$$

(a) Sketch the spectrum of this signal indicating the complex amplitude of each frequency component. You do not have to make separate plots for real/imaginary parts or magnitude/phase. Just indicate the complex amplitude value at the appropriate frequency.

(b) Is $x(t)$ periodic? If so, what is the period? Which harmonics are present?

(c) Now consider a new signal $y(t) = x(t) + 7\cos(160\pi t - \pi/3)$. How is the spectrum changed? Is $y(t)$ periodic? If so, what is the period?

(d) Finally, consider another new signal $w(t) = x(t) + \cos(5\sqrt{2}\,\pi t + \pi/3)$. How is the spectrum changed? Is $w(t)$ periodic? If so, what is the period? If not, why not?

**PROBLEM 3.3***:
A periodic signal $x(t)$ with a period $T_0 = 4$ is described *over one period,* $0 \le t \le 4$, by the equation

$$x(t) = \begin{cases} 2 & 0 \le t \le 2 \\ 0 & 2 < t \le 4 \end{cases}$$

(a) Sketch the periodic function $x(t)$ for $-4 < t < 8$.

(b) Determine the D.C. coefficient of the Fourier Series, $a_0$.

(c) Use the Fourier $\boxed{\textit{analysis}}$ integral (for $k \neq 0$)

$$a_k = \frac{1}{T_0}\int_0^{T_0} x(t)e^{-jk\omega_0 t}dt$$

to find the <u>first</u> Fourier series coefficient, $a_1$. Note: $\omega_0 = 2\pi/T_0$.

(d) Does the value of $a_1$ change if we add a constant value of one to $x(t)$, i.e., if we replace $x(t)$ with

$$x(t) = \begin{cases} 3 & 0 \le t \le 2 \\ 1 & 2 < t \le 4 \end{cases}$$

Explain why or why not. (Note: You should not have to evaluate $a_1$ explicitly to answer this question.)

## PROBLEM 3.4*:

(Similar to *DSP First*, Chapter 3, Problem 8, page 80.)
We have seen that musical tones can be modeled mathematically by sinusoidal signals. If you read music or play the piano you are aware of the fact that the piano keyboard is divided into octaves, with the tones in each octave being twice the frequency of the corresponding tones in the next lower octave. To calibrate the frequency scale, the reference tone is the A above middle-C, which is usually called A440 since its frequency is 440 Hz. Each octave contains 12 tones, and the ratio between the frequencies of successive tones is constant. Since middle C is 9 tones below A440, its frequency is approximately $(440)2^{-9/12} \approx 262$ Hz. High C is one octace above this frequency, at 524 Hz. The names of the tones (notes) of the octave starting with high C (not middle C) are:

| note name | C | $C\#$ | $D$ | $E^\flat$ | $E$ | $F$ | $F\#$ | $G$ | $G\#$ | $A$ | $B^\flat$ | $B$ | $C$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| note number | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 | 61 | 62 | 63 | 64 |
| frequency | | | | | | | | | | | | | |

(a) Explain why the ratio of the frequencies of successive notes must be $2^{1/12}$.

(b) Make a table of the frequencies of the tones of the octave beginning with high C, and ending at C-1048.

(c) The notes (from part (b)) on a piano ketboard are numbered 52 through 64. If $n$ denotes the note number, and $f$ denotes the frequency of the corresponding tone in hertz, give a formula for the frequency of the tone as a function of the note number.

## PROBLEM 3.5*:

A linear-FM "chirp" signal is one that sweeps in frequency from $\omega_1 = 2\pi f_1$ to $\omega_2 = 2\pi f_2$ as time goes from $t = 0$ to $t = T_2$. We can define the *instantaneous frequency* of the chirp as the derivative of the phase of the sinusoid:

$$x(t) = A\cos(\alpha t^2 + \beta t + \phi) \tag{1}$$

where the cosine function operates on a time-varying argument

$$\psi(t) = \alpha t^2 + \beta t + \phi$$

The derivative of the argument $\psi(t)$ is the *instantaneous frequency* which is also the audible frequency heard from the chirp *if the chirping frequency does not change too rapidly.*

$$\omega_i(t) = \frac{d}{dt}\psi(t) \qquad \text{radians/sec} \tag{2}$$

There are examples on the CD-ROM in the Chapter 3 demos.

(a) For the linear-FM "chirp" in (1), determine formulas for the beginning instantaneous frequency ($\omega_1$) and the ending instantaneous frequency ($\omega_2$) in terms of $\alpha$, $\beta$ and $T_2$. For this problem, assume that the starting time of the "chirp" is $t = 0$.

(b) For the "chirp" signal

$$x(t) = \Re e\left\{ e^{j2\pi(30t^2 - 30t)} \right\}$$

derive a formula for the *instantaneous* frequency versus time. Should your answer for the frequency be a positive number?