ECE 2025
FALL '00

HOMEWORK #3
SOLUTIONS

3.1) a) Freq. of DC component = 0 (by definition)

$$T \cong 0.033\,s \Rightarrow f = \frac{1}{T} \cong 30\,Hz$$

b) $$x(t) = 2 - 3\sin(2\pi \cdot 30t) =$$

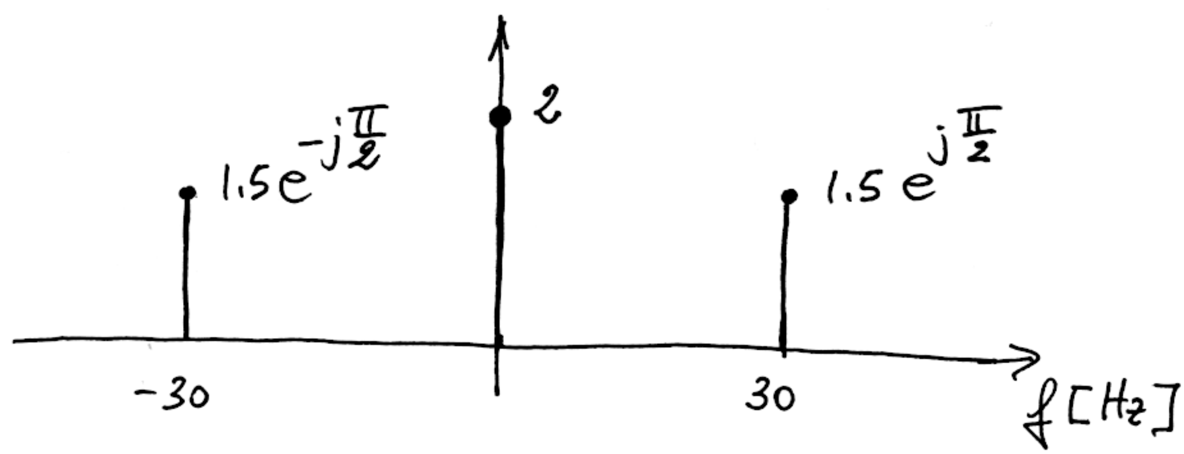
$$= 2 + 3\cos\left(2\pi \cdot 30t + \frac{\pi}{2}\right)$$

c) $$x(t) = 2 - 3\,\frac{e^{j2\pi \cdot 30t} - e^{-j2\pi \cdot 30t}}{2j} =$$

$$= 2 + \frac{3}{2}j\,e^{j2\pi \cdot 30t} - \frac{3}{2}j\,e^{-j2\pi \cdot 30t} =$$

$$= \frac{3}{2}e^{-j\frac{\pi}{2}}e^{-j2\pi \cdot 30t} + 2 + \frac{3}{2}e^{j\frac{\pi}{2}}e^{j2\pi \cdot 30t}$$

d)



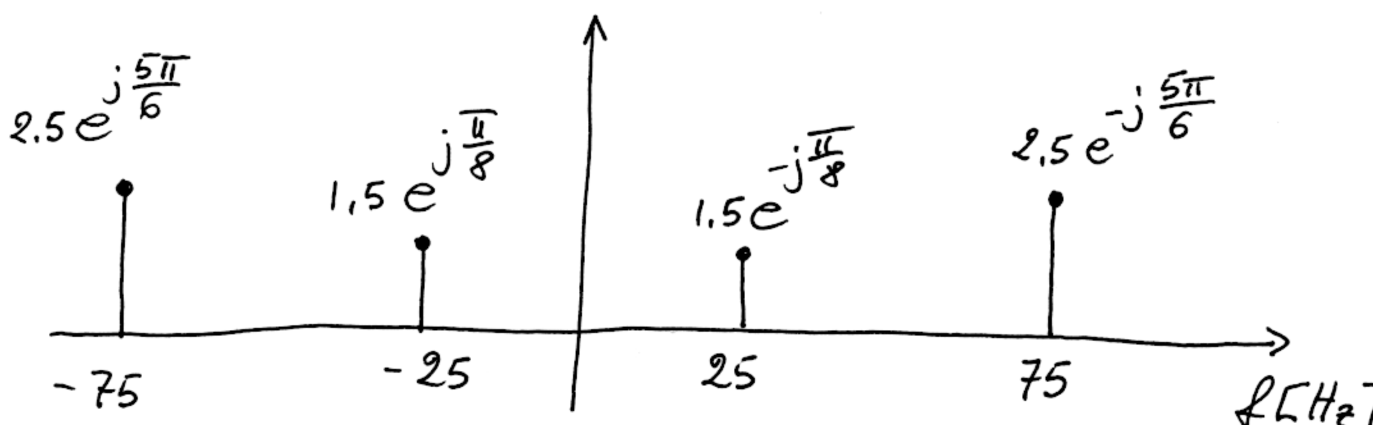
3.2) a)

$$x(t) = \frac{3}{2} e^{-j\frac{\pi}{8}} e^{j50\pi t} + \frac{3}{2} e^{j\frac{\pi}{8}} e^{-j50\pi t} +$$

$$- \frac{5}{2} e^{j\frac{\pi}{6}} e^{j150\pi t} - \frac{5}{2} e^{-j\frac{\pi}{6}} e^{-j150\pi t} =$$

$$= \frac{5}{2} e^{j\frac{5\pi}{6}} e^{-j150\pi t} + \frac{3}{2} e^{j\frac{\pi}{8}} e^{-j50\pi t} +$$

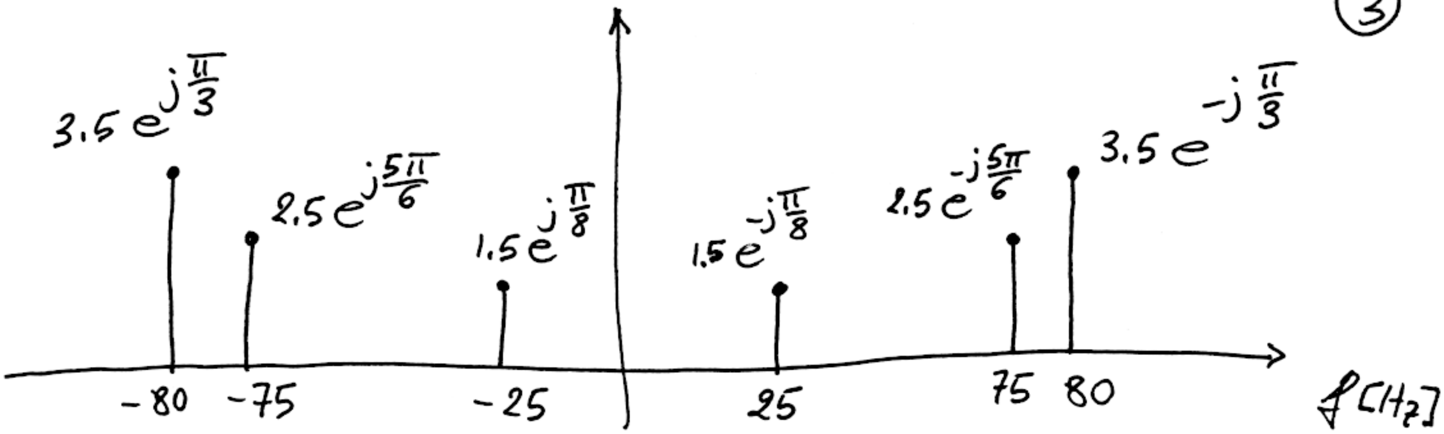
$$+ \frac{3}{2} e^{-j\frac{\pi}{8}} e^{j50\pi t} + \frac{5}{2} e^{-j\frac{5\pi}{6}} e^{j150\pi t}$$



b) Yes, $x(t)$ is periodic : $T = \frac{1}{25} = 40\,ms$

First and third harmonics are present.

c) The frequency of the new sinusoid is different from all the frequencies in the spectrum of $x(t)$.

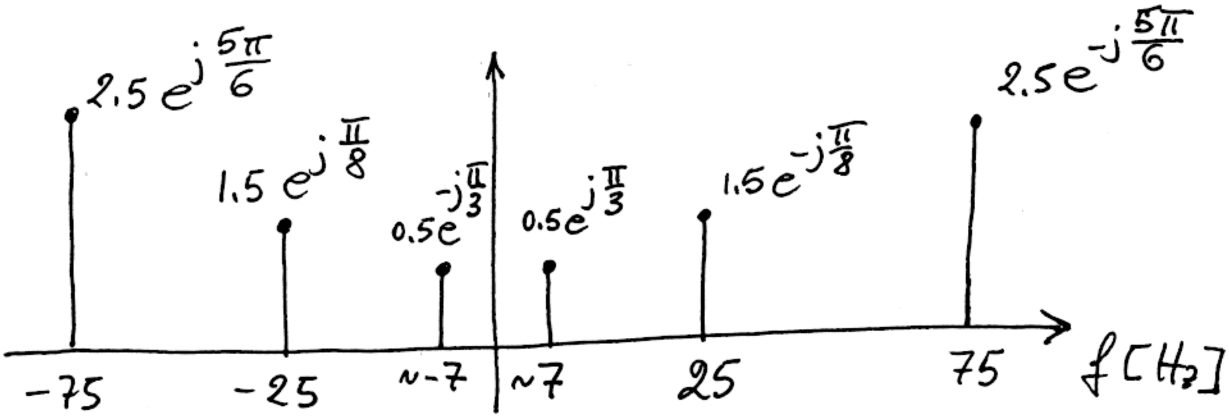


$y(t)$ is periodic : fundamental frequency =

$= \text{G.C.D.}(25, 75, 80) = 5\ Hz$

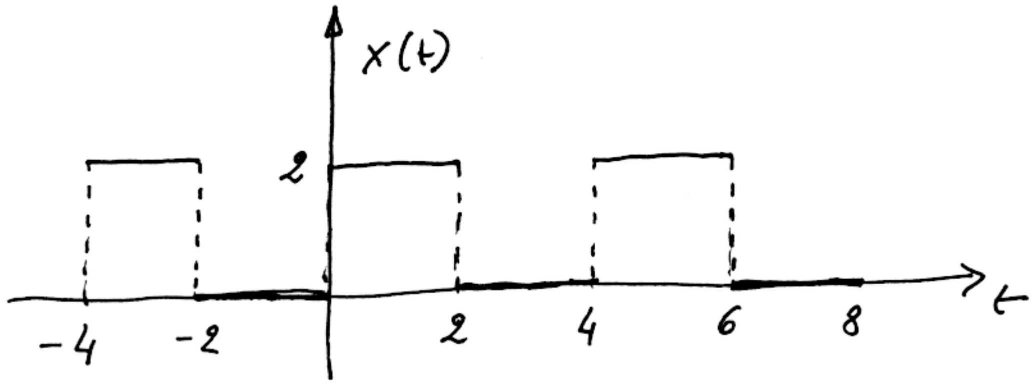
$T = \frac{1}{5} = 200\ ms$

d) Add a new frequency to the spectrum, as in (c):



$w(t)$ is not periodic because one of the three frequencies $\left(\frac{5\sqrt{2}}{2}\right)$ is not a rational multiple of the other two.

33) a)



b)

$$a_0 = \frac{1}{T_0}\int_0^{T_0} x(t)\,dt = \frac{1}{4}\int_0^{2} 2\,dt = 1$$

c)

$$a_1 = \frac{1}{T_0}\int_0^{T_0} x(t)e^{-j\omega_0 t}\,dt = \frac{1}{4}\int_0^{2} 2e^{-j\omega_0 t}\,dt =$$

$$= \frac{1}{2}\left[\frac{e^{-j\omega_0 t}}{-j\omega_0}\right]_0^2 = \frac{j}{2\omega_0}\left(e^{-j2\omega_0} - 1\right) =$$

$$= \frac{j}{\pi}\left(e^{-j\pi} - 1\right) = -\frac{2j}{\pi}$$

(d) If $x(t)$ has a Fourier Series

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} = a_0 + \sum_{k\neq 0} a_k e^{jk\omega_0 t}$$

then the new signal

$$\hat{x}(t) = x(t) + 1$$

can be written as $\hat{x}(t) = (1+a_0) + \sum_{k\neq 0} a_k e^{jk\omega_0 t}$

If we call the Fourier Series coefficients for $\hat{x}(t)$ $\hat{a}_k$, then

$$\hat{a}_0 = 1 + a_0$$
$$\hat{a}_k = a_k \quad \text{for } k \neq 0$$

d) Note that $\hat{x}(t) = x(t) + 1$. So:

$$\hat{a}_1 = \frac{1}{T_0} \int_0^{T_0} [x(t) + 1] e^{-j\omega_0 t}$$

$$= \frac{1}{T_0} \int_0^{T_0} x(t) e^{-j\omega_0 t} dt + \frac{1}{T_0} \int_0^{T_0} e^{-j\omega_0 t} dt$$

But: $$\int_0^{T_0} e^{-j\omega_0 t} dt = \left[\frac{e^{-j\omega_0 t}}{-j\omega_0} \right]_0^{T_0} =$$

$$= \frac{1}{-j\omega_0} \left(e^{-j\omega_0 T_0} - 1 \right) = \frac{1}{-j\omega_0} \left(e^{-j2\pi} - 1 \right) = 0$$

Therefore: $$\hat{a}_1 = \frac{1}{T_0} \int_0^{T_0} x(t) e^{-j\omega_0 t} dt = a_1 .$$

3.4) a) The ratio between the frequencies of two consecutive notes is constant. Let f_n be the frequency of the n-th note. Then:

$$f_{n+1} = k f_n$$

$$f_{n+2} = k f_{n+1} = k^2 f_n$$

$$\vdots$$

$$f_{n+12} = k f_{n+11} = k^{12} f_n$$

$$f_{n+12} = 2 f_n \Rightarrow k^{12} = 2 \Rightarrow k = 2^{1/12}$$

b)

C	52	524 Hz
$C^{\#}$	53	555.16
D	54	588.17
E^{b}	55	623.14
E	56	660.20
F	57	699.46
$F^{\#}$	58	741.05
G	59	785.11
$G^{\#}$	60	831.80
A	61	881.26
B^{b}	62	933.66
B	63	989.18
C	64	1048.00

If we use A-440 as the reference note, then $f_k = (440)\, 2^{(k-49)/12}$ because key #49 is A-440

The freqs are then

k	52	53	54	55	56	57	58
f_k	523.3 Hz	554.4	587.3	622.3	659.3	698.5	740 Hz

k	59	60	61	62	63	64
f_k	784 Hz	830.6	880	932.3	987.8	1046.5 Hz

c) From (a): $f_{52+m} = (2)^{m/12} f_{52}$

Let: $n = 52 + m$; then:

$$f_n = (2)^{\frac{n-52}{12}} \cdot 524 \quad [Hz]$$

3.5) a)

$$\omega_1 = \left.\frac{d\psi}{dt}\right|_{t=0} = \beta$$

$$\omega_2 = \left.\frac{d\psi}{dt}\right|_{t=T_2} = 2\alpha T_2 + \beta$$

b) $\psi(t) = 2\pi(30t^2 - 30t)$

$$\omega(t) = \frac{d\psi}{dt} = 2\pi(60t - 30) \quad [rad/s]$$

According to this definition, the angular frequency ω can be a negative number.