GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Fall 2000
Problem Set #6

Assigned: 30-Sept-00
Due Date: Week of 9-Oct-00

There will be a lab quiz at the beginning of Lab #6 (3–9 Oct).

Quiz #2 on 20-October (Friday).

Reading: In *DSP First*, Chapter 5 on *FIR Filters* and Chapter 6 on *Frequency Response.*

$\Longrightarrow$ **Please check the "Bulletin Board" often. All official course announcements are posted there.**

ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web. Some problems have solutions similar to those found on the CD-ROM.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.

PROBLEM 6.1:

For each of the following systems, determine if they are (1) linear; (2) time-invariant; (3) causal.

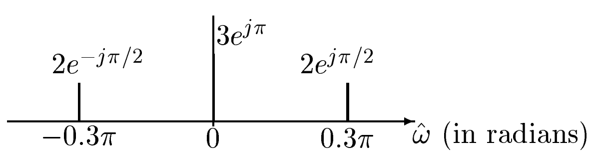
(a) $y[n] = 3x[n-1] + x[n] + 3x[n+1]$

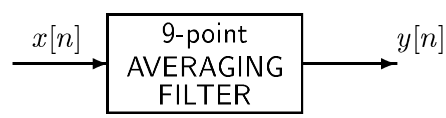
(b) $y[n] = x[n]\cos(.3\pi n)$

(c) $y[n] = |x[-n]|$

PROBLEM 6.2:

A discrete-time signal $x[n]$ has the two-sided spectrum representation shown below.





(a) Write an equation for $x[n]$. Make sure to express $x[n]$ as a real-valued signal.

(b) Determine the formula for the output signal $y[n]$.

See Problem 6.1 of Spring 1999 for solution to this problem.

PROBLEM 6.3*:

A discrete-time system is defined by the input/output relation

$$y[n] = 2x[n] - 5x[n-1] + 2x[n-2]. \tag{1}$$

(a) Determine whether or not the system defined by Equation (1) is (i) linear; (ii) time-invariant; (iii) causal. Explain your answers.

(b) Obtain an expression for the frequency response of this system.

(c) Make a sketch of the frequency response (magnitude and phase) as a function of frequency. *Hint: Use symmetry to simplify your expression before determining the magnitude and phase.*

(d) For the system of Equation (1), determine the output $y_1[n]$ when the input is

$$x_2[n] = 4 + 4\cos(0.5\pi(n-1))$$

Hint: use the linearity and time-invariance properties.

PROBLEM 6.4*:

A second discrete-time system is defined by the input/output relation

$$y[n] = (x[n-1])^2. \tag{2}$$

(a) Determine whether or not the system defined by (2) is (i) linear; (ii) time-invariant; (iii) causal.

(b) For the system of Equation (2), determine the output $y_1[n]$ when the input is

$$x_1[n] = 2\cos(0.75\pi n) = e^{j0.75\pi n} + e^{-j0.75\pi n}.$$

Express your answer in terms of cosine functions. Do not leave any squared powers of cosine functions in your answers. Note that this system produces output contain frequencies that are not present in the input signal. Explain how this system might cause “aliasing” of sinusoidal components of the input.

PROBLEM 6.5*:

The diagram in Fig. 1 depicts a *cascade connection* of two linear time-invariant systems; i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

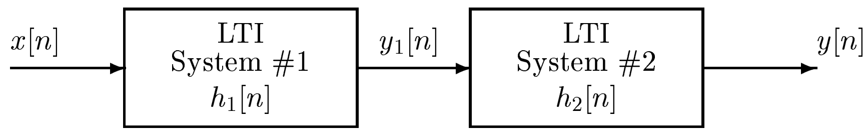


Figure 1: Cascade connection of two LTI systems.

(a) Suppose that System #1 is a "first-difference" filter described by the difference equation

$$y_1[n] = x[n] - x[n-1],$$

and System #2 is described by the impulse response

$$h_2[n] = u[n] - u[n-10]$$

Determine the impulse response sequence, $h[n] = h_1[n] * h_2[n]$, of the overall cascade system.

(b) Obtain a single difference equation that relates $y[n]$ to $x[n]$ in Fig. 1.

PROBLEM 6.6*:

A linear time-invariant discrete-time system is described by the difference equation

$$y[n] = x[n] - 2x[n-1] + 3x[n-2] - 4x[n-3] + 2x[n-4].$$

(a) Draw a block diagram that represents this system in terms of unit-delay elements, coefficient multipliers, and adders as in Figure 5.13 in the text.

(b) Determine the impulse response $h[n]$ for this system.

(c) Use convolution to determine the output due to the input

$$x[n] = \delta[n] - \delta[n-1] + \delta[n-2] = \begin{cases} 1 & n = 0, 1, 2 \\ 0 & \text{otherwise} \end{cases}$$

Plot the output sequence $y[n]$ for $-3 \le n \le 10$.

PROBLEM 6.7*:

The frequency response of a linear time-invariant filter is given by the formula

$$\mathcal{H}(\hat{\omega}) = (1 + e^{-j\hat{\omega}})(1 - e^{-j\pi/4}e^{-j\hat{\omega}})(1 - e^{j\pi/4}e^{-j\hat{\omega}}). \qquad (3)$$

(a) Write the difference equation that gives the relation between the input $x[n]$ and the output $y[n]$.

(b) What is the output if the input is $x[n] = \delta[n]$?

(c) If the input is of the form $x[n] = Ae^{j\phi}e^{j\hat{\omega}n}$, for what values of $-\pi \le \hat{\omega} \le \pi$ will $y[n] = 0$ for all n?