ECE 2025 FALL 2000

PROBLEM SET #7 SOLUTIONS

**7.1** (a) Best approach: #1

STEP 1: Find the difference equation for the filter.

STEP 2: Compute $y[n]$ by iterating this difference equation using software (e.g., MATLAB) on the input data from $n=0$ to $n=9999$.

(b) Best approach using current knowledge: #1

STEP 1: Find the difference equation for the filter.

STEP 2: Compute $y[n]$ for $n \geq 0$.

Better approach: #2

STEP 1: Find $X(z)$ (need to know how to find the z-transform of an infinite length sequence.)

STEP 2: Find $H(z)$

STEP 3: $Y(z) = H(z)X(z)$

STEP 4: Inverse transform $Y(z)$ to get $y[n]$.

(c) Best approach: #3

STEP 1: Find $H(e^{j\hat{\omega}})$

STEP 2: Compute $H(e^{j0.1\pi})$ and $H(e^{j0.4\pi})$.

STEP 3: Find $y[n]$ by multiplying the amplitudes and adding the phases for each pair of cosine/frequency-response terms.

(d) Best approach: #2

STEP 1: Find $X(z)$

STEP 2: Find $H(z)$

STEP 3: $Y(z) = H(z)X(z)$

STEP 4: Inverse transform $Y(z)$ to get $y[n]$.

(e) Best approach: Convolution

STEP 1: Find $h[n]$

STEP 2: Compute $y[n] = h[n] * 10\,\delta[n-50]$

$$= 10\,h[n-50]$$

7.2

(a) $\{b_k\} = \{1, 0, 0, 0, -1\}$

$$H(z) = \sum_{k=0}^{4} b_k z^{-k} = 1 - z^{-4}$$

(b) $$H(e^{j\hat{\omega}}) = H(z)\Big|_{z=e^{j\hat{\omega}}} = 1 - e^{-j4\hat{\omega}}$$

$$= e^{-j2\hat{\omega}}\left(e^{+j2\hat{\omega}} - e^{-j2\hat{\omega}}\right)$$

$$= 2je^{-j2\hat{\omega}} \sin(2\hat{\omega})$$

(c) Using superposition:

$$3 \longmapsto 3H(e^{j0}) = 0$$

$$2\cos\left(\tfrac{3\pi}{4}n - \pi/4\right) \longmapsto H(e^{j\frac{3\pi}{4}}) \cdot 2\cos\left(\tfrac{3\pi}{4}n - \pi/4\right)$$

$$= 2j\underbrace{e^{-j\frac{3\pi}{2}}}_{j}\underbrace{\sin\left(\tfrac{3\pi}{2}\right)}_{-1} 2\cos\left(\tfrac{3\pi}{4}n - \pi/4\right)$$

$$= 4\cos\left(\tfrac{3\pi n}{4} - \pi/4\right)$$

$$11\cos\left(\tfrac{3\pi}{2}n - \pi/3\right) \longmapsto H(e^{j\frac{3\pi}{2}}) \cdot 11\cos\left(\tfrac{3\pi}{2}n - \pi/3\right)$$

$$= 0$$

$$\Rightarrow y[n] = 4\cos\left(\tfrac{3\pi n}{4} - \pi/4\right)$$

With $f_s = 8000$ Hz, $y(t) = 4\cos(6000\pi t - \pi/4)$

7.3 (a) Difference Eqn: $y[n] = \frac{1}{3}x[n] + \frac{1}{3}x[n-6]$

Impulse Response: $h[n] = \frac{1}{3}\delta[n] + \frac{1}{3}\delta[n-6]$

Freq. Response: $H(e^{j\hat{\omega}}) = \frac{1}{3} + \frac{1}{3}e^{-j6\hat{\omega}}$

$$= \frac{1}{3}e^{-j3\hat{\omega}}\left(e^{+j3\hat{\omega}} + e^{-j3\hat{\omega}}\right)$$

$$= \frac{2}{3}e^{-j3\hat{\omega}}\cos(3\hat{\omega})$$

System Function: $H(z) = \frac{1}{3} + \frac{1}{3}z^{-6}$

(b) Diff. Eqn: $y[n] = x[n] - 2x[n-1] + x[n-2] - 2x[n-3] + x[n-4]$

Impulse Response: $h[n] = \delta[n] - 2\delta[n-1] + \delta[n-2]$
$- 2\delta[n-3] + \delta[n-4]$

Freq. Response: $H(e^{j\hat{\omega}}) = 1 - 2e^{-j\hat{\omega}} + e^{-j2\hat{\omega}}$
$- 2e^{-j3\hat{\omega}} + e^{-j4\hat{\omega}}$

$$= e^{-j2\hat{\omega}}\left(1 - 4\cos(\hat{\omega}) + 2\cos(2\hat{\omega})\right)$$

System Function: $H(z) = 1 - 2z^{-1} + z^{-2} - 2z^{-3} + z^{-4}$

(c) Diff. Eqn.: $y[n] = \frac{3}{2}x[n] + x[n-2] + \frac{3}{2}x[n-4]$

Impulse Response: $h[n] = \frac{3}{2}\delta[n] + \delta[n-2] + \frac{3}{2}\delta[n-4]$

Freq. Response: $H(e^{j\hat{\omega}}) = e^{-j2\hat{\omega}}[1 + 3\cos(2\hat{\omega})]$

$$= \frac{3}{2} + e^{-j2\hat{\omega}} + \frac{3}{2}e^{-j4\hat{\omega}}$$

System Function: $H(z) = \frac{3}{2} + z^{-2} + \frac{3}{2}z^{-4}$

(d) Diff. Eqn: $y[n] = 2x[n] + x[n-2] + x[n-4] + 3x[n-7]$

Impulse Response: $h[n] = 2\delta[n] + \delta[n-2] + \delta[n-4] + 3\delta[n-7]$

Freq. Response: $H(e^{j\hat{\omega}}) = 2 + e^{-j2\hat{\omega}} + e^{-j4\hat{\omega}} + 3e^{-j7\hat{\omega}}$

System Function: $H(z) = 2 + z^{-2} + z^{-4} + 3z^{-7}$

7.4 $H(e^{j\hat{\omega}}) = \frac{1}{3}\left(1 + e^{-j3\hat{\omega}}\right) = \frac{1}{3} e^{-j\frac{3\hat{\omega}}{2}} \left(e^{j\frac{3\hat{\omega}}{2}} + e^{-j\frac{3\hat{\omega}}{2}}\right)$

$= \frac{2}{3} e^{-j\frac{3\hat{\omega}}{2}} \cos\left(3\hat{\omega}/2\right)$

For $f_s = 8000$ samples/sec,

$x[n] = 1 + \cos\left(\frac{5000\pi n}{8000} - \pi/8\right) + 6\cos\left(\frac{11{,}000\pi n}{8000} - \pi/3\right)$

$= 1 + \cos\left(\frac{5}{8}\pi n - \pi/8\right) + \underbrace{6\cos\left(\frac{11}{8}\pi n - \pi/3\right)}_{6\cos\left(-\frac{5}{8}\pi n - \pi/3\right)}$

$\Rightarrow x[n] = 1 + \cos\left(\frac{5}{8}\pi n - \pi/8\right) + 6\cos\left(\frac{5}{8}\pi n + \pi/3\right)$

$= 1 + 6.2106\cos\left(\frac{5}{8}\pi n + 0.8869\right)$

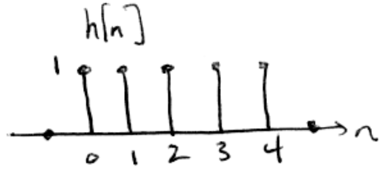
$y[n] = H(e^{j0}) \cdot 1 + \left|H(e^{j\frac{5}{8}\pi})\right| 6.2106\cos\left(\frac{5}{8}\pi n + 0.8869 + \angle H(e^{j\frac{5}{8}\pi})\right)$

$H(e^{j0}) = \frac{2}{3}, \quad H(e^{j\frac{5}{8}\pi}) = \frac{2}{3} e^{-j\frac{15}{16}\pi} \cos\left(\frac{15}{16}\pi\right)$

$\Rightarrow y[n] = \frac{2}{3} + 4.0606\cos\left(\frac{5}{8}\pi n + 1.0832\right)$

$y(t) = \frac{2}{3} + 4.0606\cos\left(5000\pi t + 1.0832\right)$

7.5 (a) $h[n] = \sum_{k=0}^{4} \delta[n-k]$




(b) $H(z) = \sum_{k=0}^{4} z^{-k} = \frac{1-z^{-5}}{1-z^{-1}}$

(c) Solve $1-z^{-5}=0 \Rightarrow z^{-5}=1$

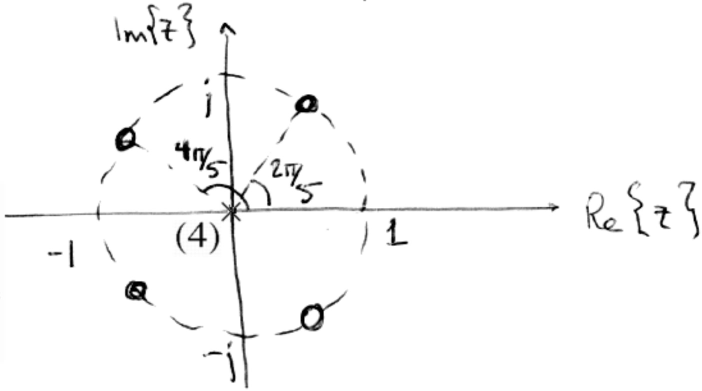
$\Rightarrow$ roots are $e^{-j\frac{2\pi k}{5}}$, $k=0,1,2,3,4$

Note that $k=0$ root cancels with $(1-z^{-1})$ in denominator of $H(z)$.

$\Rightarrow$ Zeros at $e^{-j\frac{2\pi k}{5}}$, $k=1,2,3,4$

$$\frac{1-z^{-5}}{1-z^{-1}} = \frac{z^5-1}{z^4(z-1)}$$

implies that there are also 4 poles at $z=0$



7.6

(1) $H(e^{j\hat{\omega}}) = \sum_{k=0}^{4} e^{-jk\hat{\omega}} = \dfrac{1-e^{-j5\hat{\omega}}}{1-e^{-j\hat{\omega}}}$

(2) $H(e^{j\hat{\omega}}) = \dfrac{e^{-j\frac{5}{2}\hat{\omega}}\left(e^{+j\frac{5}{2}\hat{\omega}} - e^{-j\frac{5}{2}\hat{\omega}}\right)}{e^{-j\hat{\omega}/2}\left(e^{+j\hat{\omega}/2} - e^{-j\hat{\omega}/2}\right)} \cdot \dfrac{2j}{2j}$

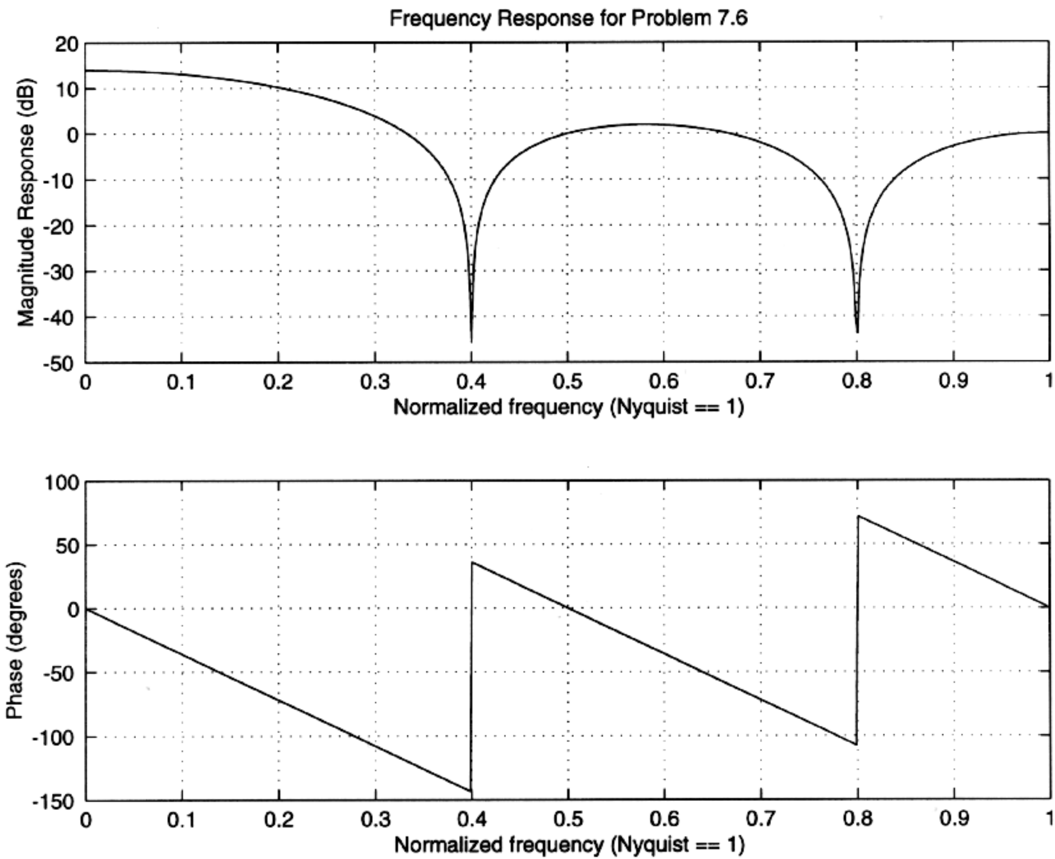
$$= e^{-j2\hat{\omega}} \frac{\sin(5\hat{\omega}/2)}{\sin(\hat{\omega}/2)}$$

(3) See freqz() plot on next page.

(4) Choose $\hat{\omega}$ to correspond to a zero of $H(e^{j\hat{\omega}})$.

$\Rightarrow$ Either $\hat{\omega} = \frac{2\pi}{5}$ or $\hat{\omega} = 4\pi/5$ will work.

$\Rightarrow y[n] = H(e^{j0}) \cdot 1 = 5.$



Frequency Response for Problem 7.6
Magnitude Response (dB)
20
10
0
-10
-20
-30
-40
-50
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Normalized frequency (Nyquist == 1)
Phase (degrees)
100
50
0
-50
-100
-150
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Normalized frequency (Nyquist == 1)

7.7

(a) $H(z) = (1-z^{-1})(1+2z^{-1}-z^{-2})(1+z^{-1})$

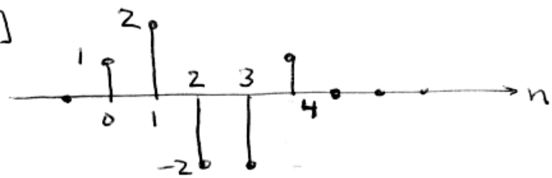
$= (1+z^{-1}-3z^{-2}+z^{-3})(1+z^{-1})$

$H(z) = 1+2z^{-1}-2z^{-2}-2z^{-3}+z^{-4}$

$\Rightarrow y[n] = x[n]+2x[n-1]-2x[n-2]-2x[n-3]+x[n-4]$

(b) $y[n] = h[n] = \delta[n]+2\delta[n-1]-2\delta[n-2]$
$-2\delta[n-3]+\delta[n-4]$

h[n]




(c) $X(z) = z^{-1}+3z^{-3}-z^{-6}$

$Y(z) = H(z)X(z) = (1+2z^{-1}-2z^{-2}-2z^{-3}+z^{-4})(z^{-1}+3z^{-3}-z^{-6})$

$= z^{-1}+2z^{-2}-2z^{-3}-2z^{-4}+z^{-5}+3z^{-3}+6z^{-4}-6z^{-5}-6z^{-6}$
$+3z^{-7}-z^{-6}-2z^{-7}+2z^{-8}+2z^{-9}-z^{-10}$

$Y(z) = z^{-1}+2z^{-2}+z^{-3}+4z^{-4}-5z^{-5}-7z^{-6}+z^{-7}$
$+2z^{-8}+2z^{-9}-z^{-10}$

y[n]