GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Fall 2000
Problem Set #8

Assigned: 15-Oct-00
Due Date: Week of 30-Oct-00

Quiz #2 will be on 20-October (Friday). Coverage is Homeworks #3 − #7 One page of notes will be allowed.

Review session: 7:30pm on Thursday in the ECE Auditorium.

Reading: In *DSP First*, Chapter 7 on *The z-Transform.*

$\Longrightarrow$ **Please check the "Bulletin Board" often. All official course announcements are posted there.**

ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.

PROBLEM 8.1:

A linear time-invariant filter is described by the difference equation

$$y[n] = \frac{1}{5}\{x[n] + x[n-1] + x[n-2] + x[n-3] + x[n-4]\} = \frac{1}{5}\sum_{k=0}^{4} x[n-k]$$

(a) Find the output $y_1[n]$ when the input is

$$x_1[n] = 10\delta[n-50].$$

(b) Find the output $y_2[n]$ when the input is

$$x_2[n] = \begin{cases} 1 & 0 \le n \le 10 \\ 0 & \text{otherwise.} \end{cases}$$

(c) Find the output $y_3[n]$ when the input is

$$x_3[n] = 4\cos(0.1\pi n + \pi/2) + 3\cos(0.4\pi n - \pi) \qquad \text{for } -\infty < n < \infty.$$

PROBLEM 8.2*:

Listed below are four linear time-invarinat systems that are described either in terms of its input/output (difference equation), or its impuse response, or its frequency response.

(a) $y[n] = 4(x[n] - x[n-4])$.

(b) $h[n] = -\delta[n] - \delta[n-1] - \delta[n-2] - \delta[n-3]$.

(c) $H(e^{j\hat{\omega}}) = [4j\sin(4\hat{\omega})]e^{-j5\hat{\omega}}$.

(d) $h[n] = \delta[n] + \delta[n-5]$.

For each of the four systems above, determine the poles and zeros of the system function, and make a plot of the pole-zero locations in the z-plane. Show the unit circle for reference.

PROBLEM 8.3*:

The diagram in Figure 1 depicts a *cascade connection* of two linear time-invariant systems; i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

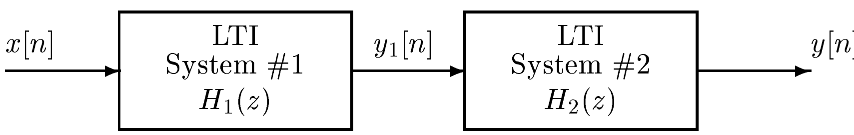


Figure 1: Cascade connection of two LTI systems.

Assume that the two systems are described by the difference equations

$$y_1[n] = x[n] + x[n-2] \quad \text{and} \quad y[n] = y_1[n] - y_1[n-1].$$

(a) Determine the system function $H(z) = H_1(z)H_2(z)$ for the overall system.

(b) Plot the poles and zeros of $H(z)$ in the z-plane.

(c) From $H(z)$, determine the impulse response $h[n]$ of the overall system.

(d) From $H(z)$, obtain an expression for the frequency response $H(e^{j\hat{\omega}})$ of the overall cascade system.

PROBLEM 8.4*:

The system function of a linear time-invariant filter is given by the formula

$$H(z) = (1 + z^{-1})(1 - e^{j2\pi/5}z^{-1})(1 - e^{-j2\pi/5}z^{-1})$$

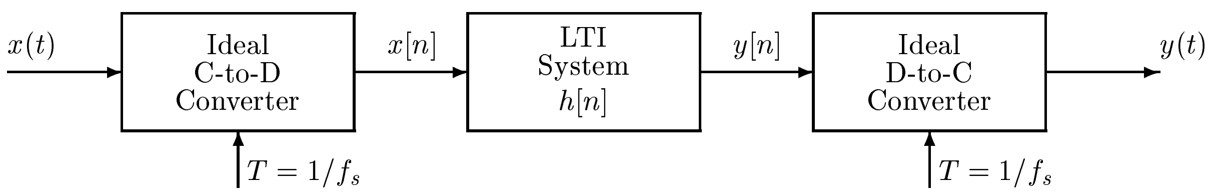
(a) Use multiplication of z-transform polynomials to find the output when the input is

$$x[n] = -\delta[n-2] - \delta[n-3] - \delta[n-4].$$

(b) If the input is of the form $x[n] = Ae^{j\phi}e^{j\hat{\omega}n}$, for what values of frequency $\hat{\omega}$ will the output signal be zero for all n (i.e., $y[n] = 0$) ? Find all possible frequencies in the range $-\pi \leq \hat{\omega} \leq \pi$. *Hint: Take a look at the locations of the zeros of $H(z)$.*

PROBLEM 8.5*:

We would like to build a system that will remove 60 Hz line noise using an A-D converter, a digital filter, and a D-A converter as illustrated in the following diagram.



The input $x[n]$ and output $y[n]$ of the digital filter are related by the difference equation

$$y[n] = x[n] + ax[n-1] + bx[n-2]$$

If the sampling frequency of the A-D and D-A converters is $f_s = 8$ kHz, find the values for a and b so that if $x(t) = \cos(2\pi(60)t)$ then $y(t) = 0$.

PROBLEM 8.6*:

Evaluate the following integrals.

(a) $\displaystyle\int_{-\infty}^{t} e^{-(\tau-1)}d\tau =$

(b) $\displaystyle\int_{0}^{t} e^{-2(t-\tau)}e^{\tau}d\tau =$

(c) $\displaystyle\int_{0}^{t} \tau e^{(t-\tau)}d\tau =$