GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Fall 2000
Problem Set #9

Assigned: 27-Oct-00
Due Date: Week of 6-Nov-00

Any grading questions on Quiz #2 must be resolved no later than 7-Nov; after that date the scores will not be changed.

Quiz #3 will be on 20-Nov (Monday). Coverage will be Homeworks #8, #9 and #10, which correspond to the Continuous-Time Signals & Systems notes.

Reading: Chapter 10 in Notes.

$\Longrightarrow$ **Please check the "Bulletin Board" often. All official course announcements are posted there.**

ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.

PROBLEM 9.1*:

Try your hand at expressing each of the following in a simpler form:

(a) Convolution: $\delta(t-1) * [\delta(t+2) + 2e^{-t+1}\sin(\pi t)u(t+1)] =$

(b) Multiplication: $[u(-t+3) - u(t)][\delta(t+1) + \delta(t+4)] =$

(c) $\dfrac{d}{dt}\left[\sin(5\pi t)u(t-\tfrac{1}{2})\right] =$

(d) $\displaystyle\int_{-\infty}^{t} e^{-2\tau-1}\delta(\tau-3)d\tau =$

Note: use properties of the impulse signal $\delta(t)$ and the unit-step signal $u(t)$ to perform the simplifications. For example, recall

$$\delta(t) = \frac{d}{dt}u(t) \qquad \text{where} \qquad u(t) = \begin{cases} 1 & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$$

Be careful to distinguish between multiplication and convolution; convolution is denoted by a "star", as in $u(t) * \delta(t-2)$.

PROBLEM 9.2*:

A continuous-time system is defined by the input/output relation

$$y(t) = \int_{-\infty}^{t-2} x(\tau)d\tau$$

(a) Determine the impulse response, $h(t)$, of this system.

(b) Is this a stable system? Explain with a proof or counter-example.

(c) Is it a causal system? Explain with a proof or counter-example.

(d) Use the convolution integral to determine the output of the system when the input is $u(t)$, the unit step signal:

$$u(t) = \begin{cases} 1 & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$$

Plot the output signal $y(t)$ versus t.

PROBLEM 9.3*:

A linear time-invariant system has impulse response:

$$h(t) = \begin{cases} e^{-t} & -1 \leq t < 9 \\ 0 & \text{otherwise} \end{cases}$$

(a) Plot $h(t - \tau)$ as a functions of τ for $t = -10, 0,$ and 10.

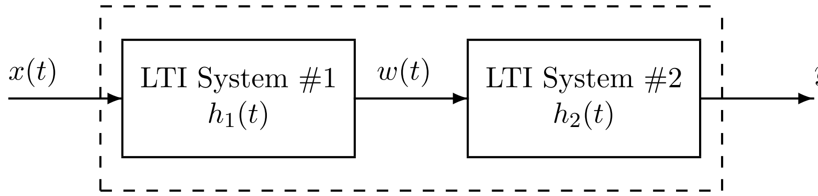
(b) Find the output $y(t)$ when the input is $x(t) = \delta(t + 10)$.

(c) Use the convolution integral to determine the output $y(t)$ when the input is

$$x(t) = \begin{cases} 1 & 0 \leq t < 20 \\ 0 & \text{otherwise} \end{cases}$$

Hint: You may want to check your answer with the results derived in Problem 6 (below).

PROBLEM 9.4*:



In the cascade of two LTI systems shown in the figure above, the first system has an impulse response

$$h_1(t) = u(t+3)$$

and the second system is described by the input/output relation

$$y(t) = \frac{dw(t)}{dt} - \pi w(t)$$

(a) Find the impulse response of the overall system; i.e., find the output $y(t) = h(t)$ when the input is $x(t) = \delta(t)$.

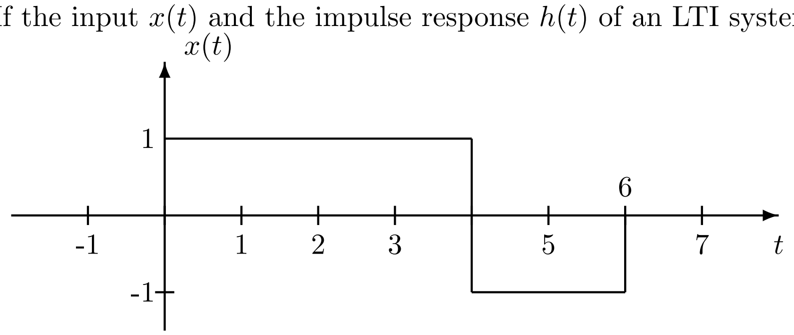
(b) Give a general expression for $y(t)$ in terms of $x(t)$ that holds for any input signal. Express your answer in terms of integrals, derivatives and delays. For example, you should try to get a form like:

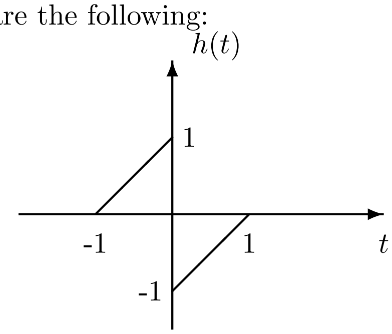
$$y(t) = Ax(t - t_1) + B\frac{d}{dt}x(t - t_2) + C\int_{-\infty}^{t-t_3} x(\tau)d\tau$$

where the parameters A, B, C, t_1, t_2 and t_3 have specific numeric values.

PROBLEM 9.5*:

If the input $x(t)$ and the impulse response $h(t)$ of an LTI system are the following:





(a) Determine $y(0)$, the value of the output at $t = 0$.

(b) Find all the values of t for which the output $y(t) = 0$. There will be regions where $y(t) = 0$, but there might also be isolated points where $y(t) = 0$. *Note: You do not need to find $y(t)$ at any other values of t.*

PROBLEM 9.6:

This is Problem 1.2 of Problem Set #1 of EE2201 from the Spring of 1999. The impulse response of an LTI continuous-time system is such that $h(t) = 0$ for $t \le T_1$ and for $t \ge T_2$. By drawing appropriate figures as recommended for evaluating convolution integrals, show that if $x(t) = 0$ for $t \le T_3$ and for $t \ge T_4$ then $y(t) = x(t) * h(t) = 0$ for $t \le T_5$ and for $t \ge T_6$. In the process of proving this result you should obtain expressions for T_5 and T_6 in terms of T_1, T_2, T_3, and T_4.