SOLUTIONS P.S. # 9

9.1 (a) "Star-ing" with $\delta(t-1)$ means delaying by one unit

So: $\delta(t+1) + 2e^{-(t-1)+1}\sin(\pi(t-1))\,u((t-1)+1)$

$= \delta(t+1) - 2e^{-t+2}\sin(\pi t)\,u(t)$

(b) $[u(4)-u(-1)]\delta(t+1)+[u(7)-u(-4)]\,\delta(t+4) = \delta(t+1)+\delta(t+4)$

(c) Use product rule

$$\frac{d}{dt}\sin(5\pi t)\,u(t-\tfrac{1}{2}) = 5\pi\cos\pi t . u(t-\tfrac{1}{2}) + \sin(5\pi t)\,\delta(t-\tfrac{1}{2})$$

$$= 5\pi\cos\pi t .\, u(t-\tfrac{1}{2}) + \sin\tfrac{5\pi}{2}\,\delta(t-\tfrac{1}{2})$$

(d) $$\int_{-\infty}^{t} e^{-2\tau-1}\underbrace{\delta(\tau-3)}_{\uparrow\ \text{peaks at } \tau=3}d\tau = \begin{cases} 0 & \text{if } t<3 \\ e^{-2.3-1} = e^{-7} & \text{if } t>3 \end{cases}$$

9.2 (a) $$h(t) = \int_{-\infty}^{t-2}\underbrace{\delta(\tau)}_{\uparrow\ \text{peaks at } \tau=0}d\tau = \begin{cases} 0 & \text{if } t-2<0, \text{ i.e. } t<2 \\ 1 & \text{if } t-2>0, \text{ or } t>2 \end{cases}$$

$\Rightarrow h(t) = u(t-2)$

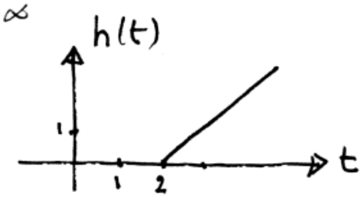
(b) $$\int_{-\infty}^{\infty}|h(t)|\,dt = \int_{-\infty}^{\infty}u(t-2)\,dt = \int_{2}^{\infty}dt \text{ diverges!}$$

UNSTABLE

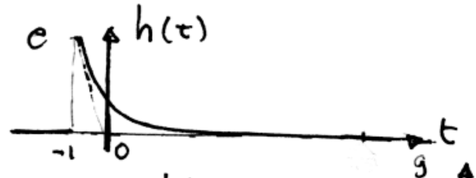
(c) CAUSAL since $h(t)=0$ for $t<0$

(d) $$y(t) = \int_{-\infty}^{\infty}h(\tau)\,u(t-\tau)\,d\tau = \int_{-\infty}^{\infty}u(t-2)\,u(t-\tau)\,d\tau$$

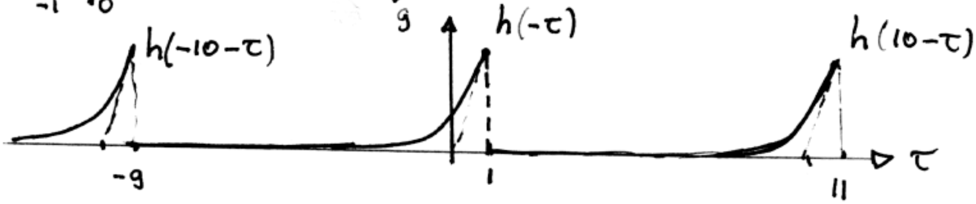
$$= \int_{2}^{\min(2,t)}d\tau = \begin{cases} 0, & t<2 \\ t-2, & t>0 \end{cases}$$



9.3



(a)



(b) $$y(t) = \int_{-\infty}^{\infty} h(\tau)\, \delta(t-\tau+10)\, d\tau = \begin{cases} 0 & \text{if } t+10 \notin [-1,9] \\ \exp[-(t+10)] & \text{if } t+10 \in [-1,9] \end{cases}$$

↑ peaks for $\tau = t+10$

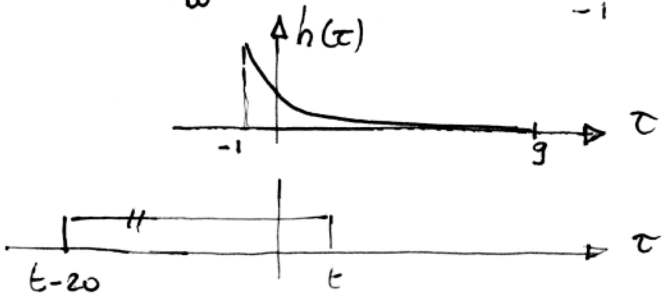
$$\therefore\ y(t) = \begin{cases} \exp[-(t+10)] & \text{if } t \in (-11,-1) \\ 0 & \text{else} \end{cases}$$

Also note that $\delta(t+10)$ is an impulse at $t=-10$, hence $y(t)$ is the corresponding shift of the impulse response.

$$y(t) = h(t+10)$$

(c) For $x(t) = \begin{cases} 1 & 0 \le t < 20 \\ 0 & \text{else} \end{cases}$

$$y(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau = \int_{-1}^{9} e^{-\tau}\, x(t-\tau)\, d\tau$$



Prob 9.3(c)

5 Regions

Use:
$$\int_0^x e^{-t}\,dt = 1 - e^{-x}$$

1. for $t < -1$, $\quad y(t) = 0$

2. for $-1 \le t < 9$ $\quad y(t) = \int_{-1}^{t} e^{-\tau}\,d\tau = \left.\frac{e^{-\tau}}{-1}\right|_{-1}^{t} = e - e^{-t}$

3. for $t \ge 9$ and $t - 20 < -1$

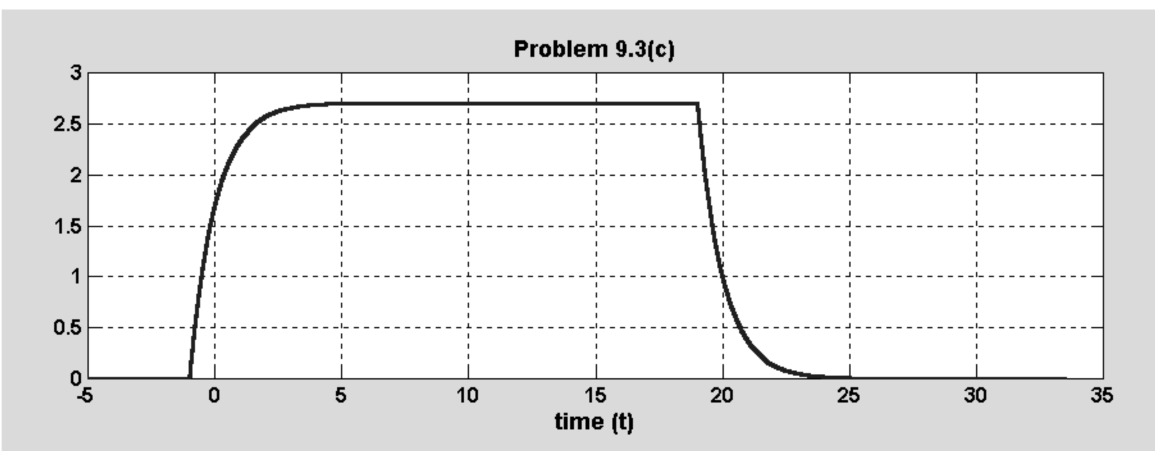
$9 \le t < 19$ $\quad y(t) = \int_{-1}^{9} e^{-\tau}\,d\tau = e - e^{-9}$

4. for $t - 20 \ge -1$ and $t - 20 < 9$

$19 \le t < 29$ $\quad y(t) = \int_{t-20}^{9} e^{-\tau}\,d\tau = e^{-(t-20)} - e^{-9}$

5. for $t - 20 \ge 9 \Rightarrow t \ge 29$ $\quad y(t) = 0$

The signal $y(t)$ is given in the plot below:



9.4 (a) If $x(t) = \delta(t) \Rightarrow w(t) = u(t+3)$

$$\Rightarrow y(t) = \frac{d}{dt} u(t+3) - \pi u(t+3)$$

$$= \delta(t+3) - \pi u(t+3)$$

$\therefore$ Impulse response of overall system $= \delta(t+3) - \pi u(t+3)$

(b) From (a): $y(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$

$$= \int_{-\infty}^{\infty} \delta(\tau+3)\, x(t-\tau)\, d\tau - \pi \int_{-\infty}^{\infty} u(\tau+3)\, x(t-\tau)\, d\tau$$

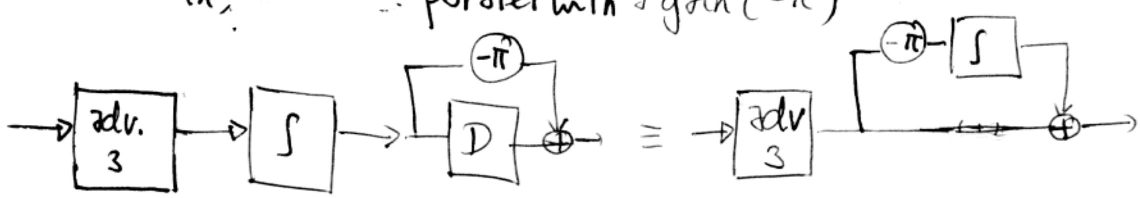
$$= x(t+3) - \pi \int_{-3}^{\infty} x(t-\tau)\, d\tau$$

letting $t-\tau = \tau'$ gives for the integral part:

$$\int_{\tau' = t+3}^{-\infty} x(\tau')\, d(-\tau') = \int_{-\infty}^{t+3} x(\tau')\, d\tau'$$

So: $y(t) = x(t+3) - \pi \int_{-\infty}^{t+3} x(\tau)\, d\tau$ (*)

Remark: System 1 is an advance of 3 units, concatenated with an integrator. System 2 is a differentiator in paralel with a gain $(-\pi)$



from which (*) can easily be derived.

9.5 Note that $x(t) = u(t) - 2u(t-4) + u(t-6)$

So, let's solve first $(h * u)(t) = g(t)$

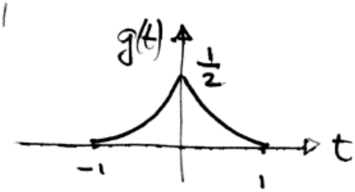
$$\int_{-\infty}^{\infty} h(\tau)\, u(t-\tau)\, d\tau = \int_{-\infty}^{t} h(\tau)\, d\tau$$

the integral of $h(\cdot)$!

$$\Rightarrow \quad g(t) = \begin{cases} 0 , & t < -1 \\ \frac{(t+1)^2}{2} , & -1 < t < 0 \\ \frac{1}{2} - \left[\frac{1}{2} - \frac{(1-t)^2}{2}\right] , & 0 < t < 1 \\ 0 & t > 1 \end{cases}$$

$\rightarrow$ area of little triangle

$\frac{(t-1)^2}{2} =$



Thus: from time-invariance and linearity,

$$y(t) = g(t) - 2g(t-4) + g(t-6)$$

(a) In particular: for $t=0$

$$y(0) = g(0) - 2g(-4) + g(-6) = \frac{1}{2}$$

(b) $y(t) = 0$ for $\begin{cases} t < -1 \\ 1 < t < 3 \\ t = 5 \\ t > 7 \end{cases}$