EE-2025 Spring-01

LECTURE #2

Phase & Time-Shift

Complex Exponentials

8-January-01

INFORMATION

- MATLAB Sessions: Tues and Weds in VL-456 (6 - 7:30 pm)
- LABS start this week **(TUESDAY)**
  - Attend correct section (in VanLeer-252)
  - Computer acct: gtxxxx, password: SSN
  - Verification must be signed during Lab
- RECITATIONS
  - Attend your assigned time

Homework Info

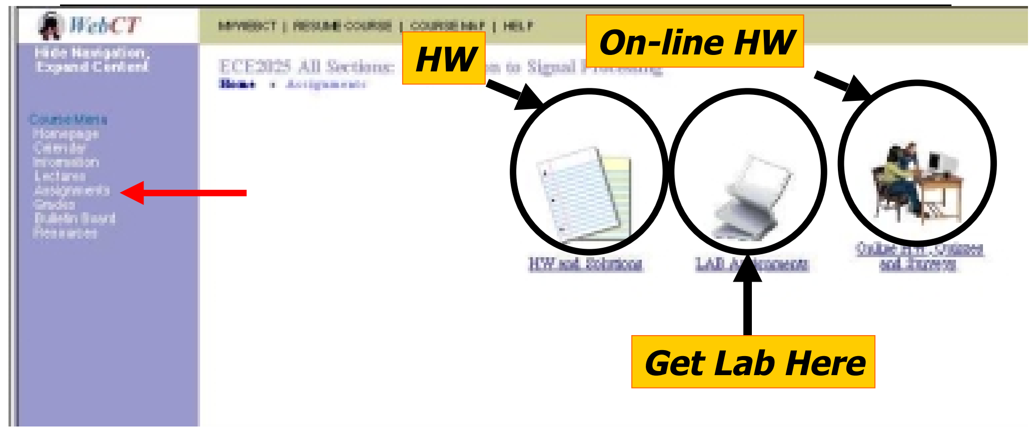
- On-Line HW #0 ends Weds. nite
  - Last attempt is scored
- HWs will be posted on Friday/Sat
  - Covered in Rec during the following Week
  - Due the week after that (9+ days later)
- Prob Set #1 due in RECITATION (rm 361)
  - At the beginning of class next week
  - Solutions will be posted after last Recitation
  - Monday Recitations turn in at Weds. Lab (next week only)

Lab Info

- NT passwd = SSN
- Lab #1 has been posted
- Lab FAQs are being posted
- Lab #1 Report
  - Due week of 22-Jan.
  - Turn in during your lab time
  - Write-up sections 2 and 3
  - Include INSTRUCTOR VERIFICATION

Web-CT Info

- Check the Bulletin Board for msgs
  - **MAKE YOUR OWN POSTINGS**
- Web-CT Password:
  - Last 4 digits of student number; change it soon
- PDF Files on WebCT
  - Lectures are being posted (4 slides per page)
  - Get PDF file of Lab#1 from WebCT
    - Hard copy of Instructor Verification Sheet
  - HW #1



READING ASSIGNMENTS

- This Lecture:
  - Chapter 2, pp. 17-32
- Appendix A: Complex Numbers
- Appendix B: MATLAB
- Next Lecture: finish Chap. 2, pp. 31-43

LECTURE OBJECTIVES

- Define Sinusoid from a plot
- Relate TIME-SHIFT to PHASE
- Introduce an **ABSTRACTION**:
  - Complex Numbers **represent** Sinusoids
  - Complex Exponential Signal

$$z(t) = Xe^{j\omega t}$$

SINUSOIDAL SIGNAL

$$A\cos(\omega t+\varphi)$$

- **FREQUENCY** ω
  - Radians/sec
  - Hertz (cycles/sec)

$$\omega=(2\pi)f$$

- **PERIOD** (in sec)

$$T=\frac{1}{f}=\frac{2\pi}{\omega}$$

- **AMPLITUDE** A
  - Magnitude
- **PHASE** φ

PLOTTING COSINE SIGNAL from the FORMULA

$$5\cos(0.3\pi t+1.2\pi)$$

- Determine **period**:

$$T=2\pi/\omega=2\pi/0.3\pi=20/3$$

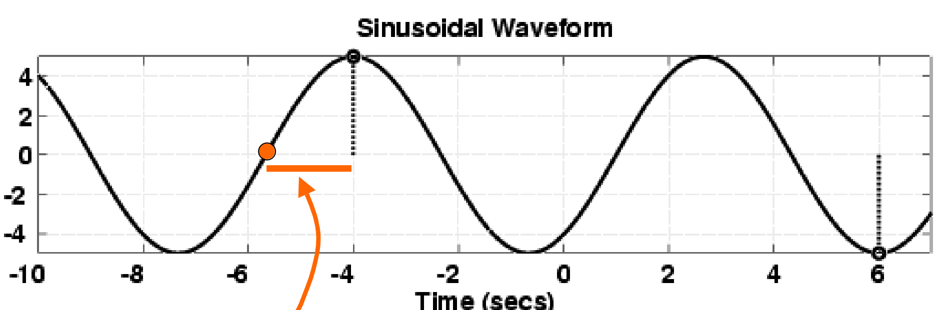
- Determine a **peak** location by solving

$$(\omega t+\varphi)=0$$

- **Peak at t = -4**
- **Zero** crossing is T/4 before or after

ANSWER for the PLOT

$$5\cos(0.3\pi t+1.2\pi)$$

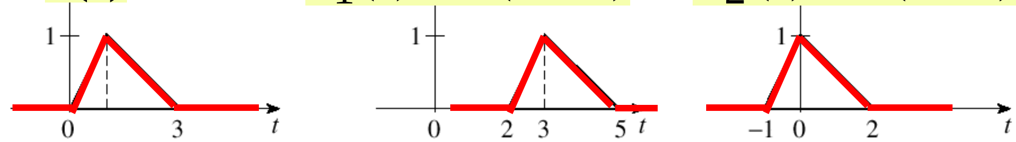


$$T/4=(20/3)/4=5/3$$

TIME-SHIFT

- In a mathematical formula replace t with $t\text{-}t_m$ $\quad x(t)=s(t-t_m)$

$s(t)$ $\quad x_1(t)=s(t-2)$ $\quad x_2(t)=s(t+1)$



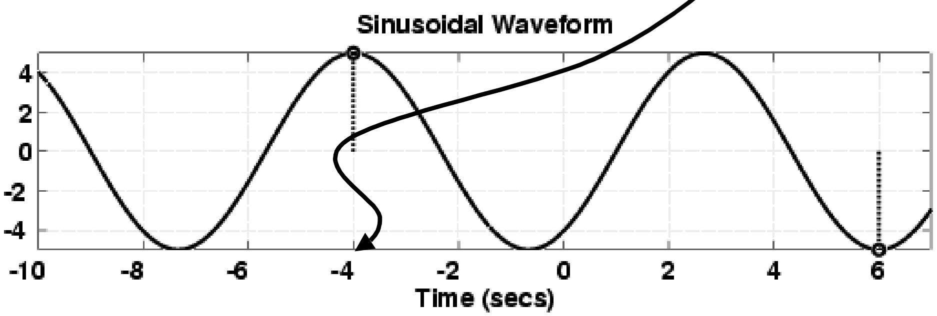
- Now try it on a cosine signal

$$x(t-t_m)=A\cos(\omega(t-t_m))$$

TIME-SHIFTED SINUSOID

- Then the t=0 point moves to $t=t_m$

$$x(t+4)=5\cos(0.3\pi(t+4))=5\cos(0.3\pi(t-(-4)))$$



PHASE <--> TIME-SHIFT

- Equate the formulas:

$$A\cos(\omega(t-t_m)) = A\cos(\omega t+\varphi)$$

- and we obtain: $-\omega t_m = \varphi$
- or, $t_m = \dfrac{-\varphi}{\omega}$

EX: Time-Shift from Phase

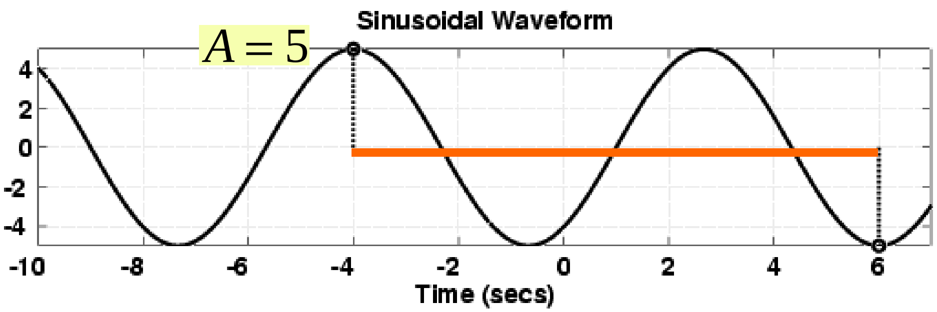
- Frequency : $\omega = .3\pi$ rad/s
- Phase: $\phi = 1.2\pi$ radians
- What is the time shift?
  - Also called the "time delay"
  - $t_m = -\phi/\omega = -(1.2\pi)/.3\pi$
  - **t_m = -4 sec.**
  - Note: $T = 2\pi/\omega$ =20/3 sec. (period)

SINUSOID from a PLOT

- Measure the period, T
  - Between peaks or zero crossings
  - Compute frequency: $\omega = 2\pi/T$
- Measure time of peak: t_m
  - Compute phase: $\phi = -\omega t_m$
- Measure height of positive peak: A

3 steps

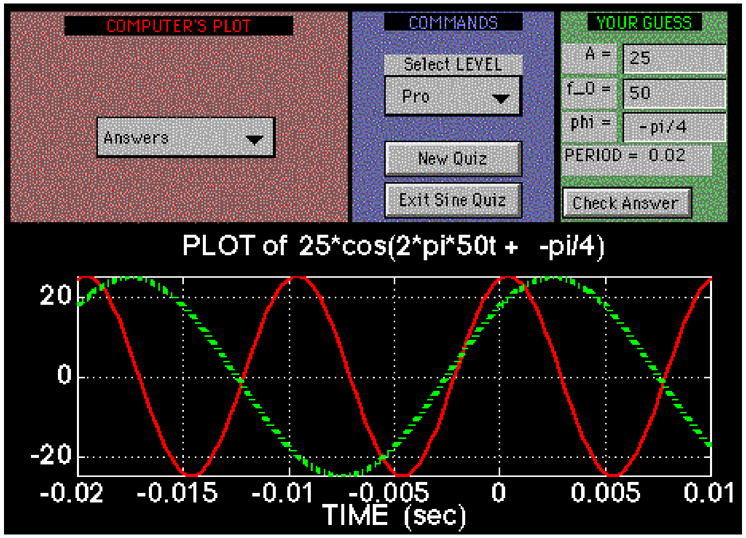
(A, ω, ϕ) from a PLOT



$T = 10/(1.5) = 20/3 \rightarrow \omega = 2\pi/T = 0.3\pi$

$t_m = -4 \rightarrow \phi = -(-4)(0.3\pi) = 1.2\pi$

SINE DRILL (MATLAB GUI)



PHASE is AMBIGUOUS

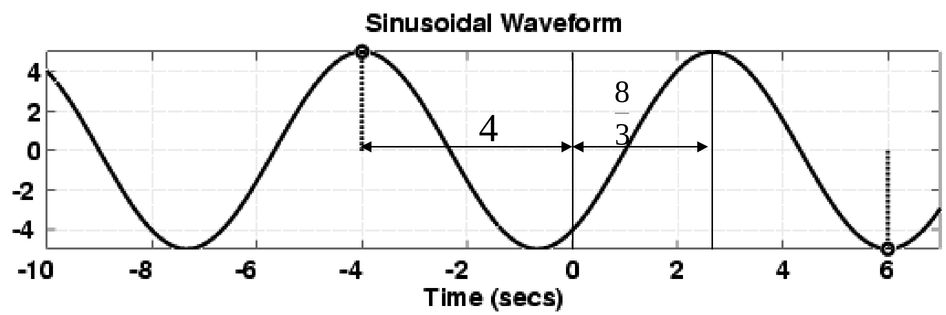
- The cosine signal is periodic
  - Period is 2π

    $$A\cos(\omega t+\varphi+2\pi)=A\cos(\omega t+\varphi)$$

  - Thus adding any multiple of 2π leaves x(t) unchanged
  - $$\text{if } t_m=\frac{-\varphi}{\omega}, \text{ then}$$

    $$t_{m_2}=\frac{-(\varphi+2\pi)}{\omega}=\frac{-\varphi}{\omega}-\frac{2\pi}{\omega}=t_m-T$$

Illustration of Phase Ambiguity



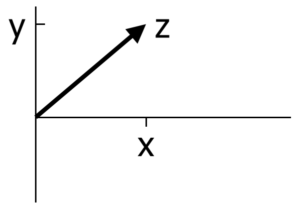
$$x(t)=5\cos[.3\pi(t+4)]=5\cos(.3\pi t+1.2\pi)$$

$$x(t)=5\cos(.3\pi t+1.2\pi-2\pi)=5\cos(.3\pi t-.8\pi)$$

$$x(t)=5\cos[.3\pi(t-\frac{.8\pi}{.3\pi})]=5\cos[.3\pi(t-\frac{8}{3})]$$

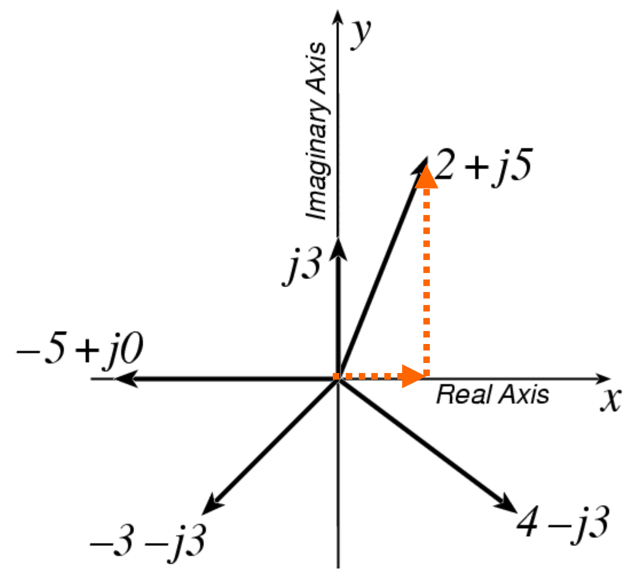
COMPLEX NUMBERS

- To solve: $z^2 = -1$
  - $z = \boldsymbol{j}$
  - Math and Physics use $z = \boldsymbol{i}$
- Complex number: $z = x + \boldsymbol{j}\,y$

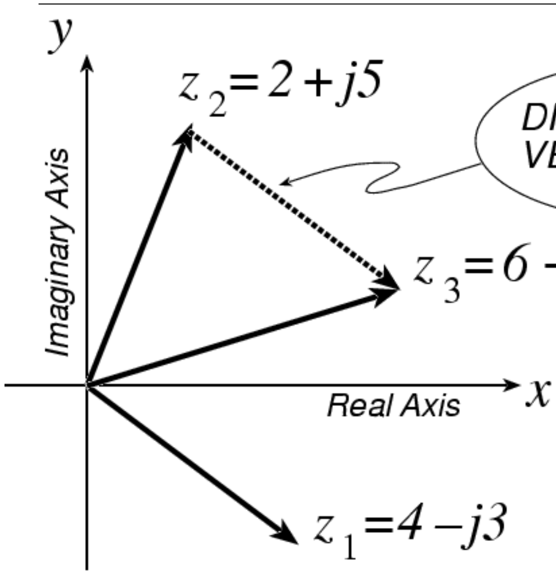


Cartesian coordinate system

PLOT COMPLEX NUMBERS



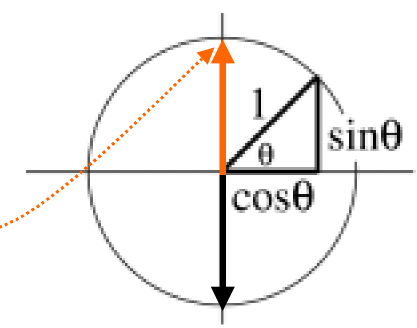
COMPLEX ADDITION = VECTOR Addition



$$
\begin{aligned}
z_3 &= z_1 + z_2 \\
&= (4 - j3) + (2 + j5) \\
&= (4+2) + j(-3+5) \\
&= 6 + j2
\end{aligned}
$$

*** POLAR FORM ***

- Vector Form
  - Length =1
  - Angle = θ
- Common Values
  - **j** has angle of 0.5π
  - -1 has angle of π
  - **– j** has angle of 1.5π
  - also, its angle is $-0.5\pi = 1.5\pi - 2\pi$
  - AMBIGUOUS PHASE



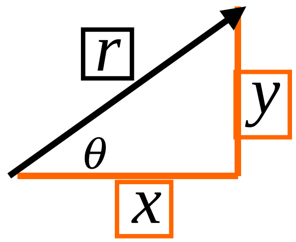
POLAR <--> RECTANGULAR

- Relate (x,y) to (r,θ)

$$z = x + jy = re^{j\theta}$$

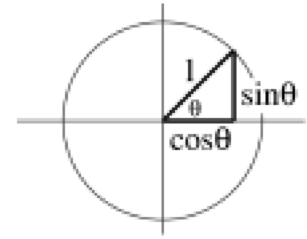
$$r^2 = x^2 + y^2$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$



Euler's FORMULA

- Complex Exponential
  - Real part is cosine
  - Imaginary part is sine
  - Magnitude is one



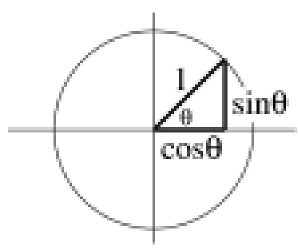
$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$

$$re^{j\theta} = r\cos(\theta) + jr\sin(\theta)$$

COMPLEX EXPONENTIAL

$$e^{j\omega t} = \cos(\omega t) + j\sin(\omega t)$$

- **Rotating Vector**
  - Angle changes vs. time
  - **θ=ω*t***
  - ex: $\omega = 10\pi$
  - Rotates 0.1π in 0.01 secs



$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$

Cos = REAL PART

Real Part of Euler's

$$\cos(\omega t) = \Re e\left\{e^{j\omega t}\right\}$$

General Sinusoid

$$x(t) = A\cos(\omega t + \varphi)$$

So,

$$A\cos(\omega t + \varphi) = \Re e\left\{Ae^{j(\omega t + \varphi)}\right\}$$

$$= \Re e\left\{Ae^{j\varphi}e^{j\omega t}\right\}$$

COMPLEX AMPLITUDE

General Sinusoid

$$x(t) = A\cos(\omega t + \varphi) = \Re e\left\{Ae^{j\varphi}e^{j\omega t}\right\}$$

Sinusoid = REAL PART of $(Ae^{j\phi})e^{j\omega t}$

$$x(t) = \Re e\left\{Xe^{j\omega t}\right\} = \Re e\{z(t)\}$$

Complex AMPLITUDE = X

$$z(t) = Xe^{j\omega t} \qquad X = Ae^{j\varphi}$$