EE-2025 Spring-01

LECTURE #3

Phasor Addition Theorem

12-January-01

Web-CT Info

- Check the Bulletin Board for msgs
  - **MAKE YOUR OWN POSTINGS**
- Lectures are being posted
  - PDF format (4 per page)
- Get PDF file of Lab#1 from WebCT
  - Lab FAQs are also being posted
- On-Line HW #0 closes 15-Jan @ midnite
- HW #1 due next week (**in Recitation**)

Homework Formatting

- Include a Cover page with
  - Name
  - Lab section, i.e., L05, L21, etc.
  - Recitation Prof's name
  - **See example on Web-CT**
- Write on ONE side only
  - Use Engineer's paper or plain paper
- STAPLE

Lab Info

- NT passwd = SSN or old password
- Lab FAQs are also being posted
- Bring **Headphones** to Lab
- Lab #1 Verifications: Turn in at end of Lab
- MATLAB Help: Next week T & Wed 6PM VL-456

Lab Info: next week

- Lab #1 Report
  - Due week of 22-Jan
  - Turn in at **BEGINNING** of your lab time
  - Write-up sections 2 and 3
- Learn your Lab TA's format requirements
- No recitations on Monday 15-Jan. Attend recitation on 17-Jan in VL-361 during your lab time (No lab for these sections next week. Should have completed sign-offs this week. If not contact your Lab TA.).

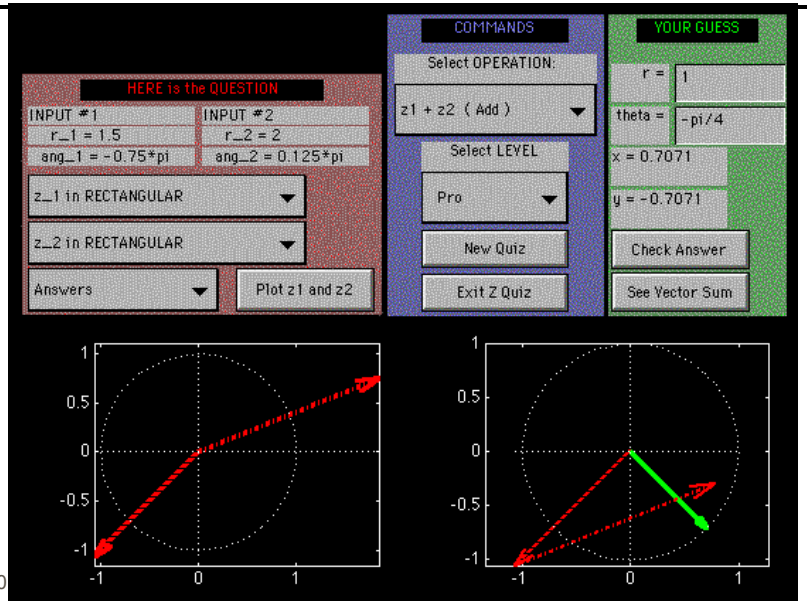
PRINTING QUOTA

- ECE Labs have printers, but...
- Limit your printing to essentials
  - Your account has a quota
- 10 pages/week
  - 2000 students
  - 3 courses/student
  - 15 weeks/semester
  - 900,000 pages !

READING ASSIGNMENTS

- This Lecture:
  - Chapter 2, pp. 31-43
- Other Reading:
  - Appendix A: Complex Numbers
  - Appendix B: MATLAB
  - Next Lecture: start Chapter 3

Z DRILL (Complex Arith)



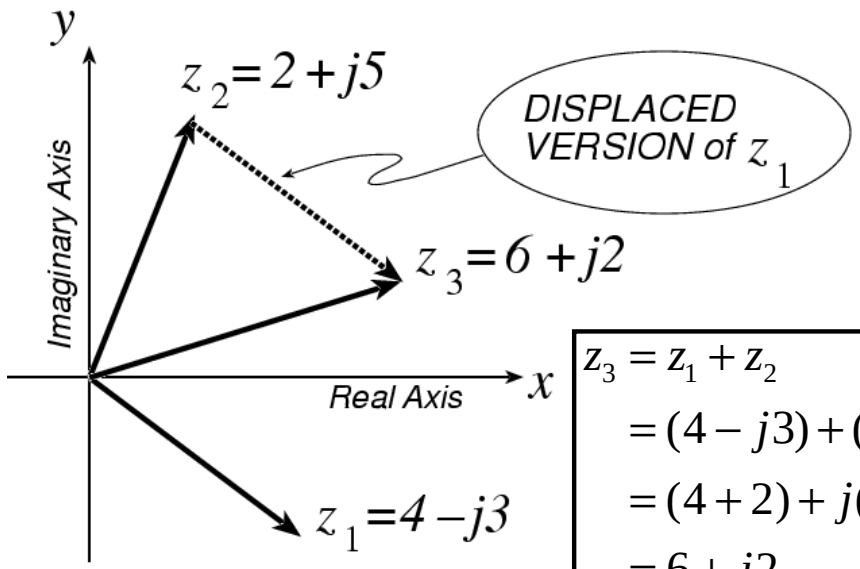
LECTURE OBJECTIVES

- Phasors = Complex Amplitude
  - Complex Numbers **represent** Sinusoids

$$z(t) = Xe^{j\omega t} = (Ae^{j\varphi})e^{j\omega t}$$

Develop the ABSTRACTION:
Adding Sinusoids = Complex Addition
PHASOR ADDITION THEOREM

COMPLEX ADDITION = VECTOR Addition

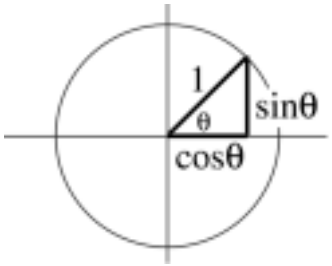


$$\begin{aligned} z_3 &= z_1 + z_2 \\ &= (4 - j3) + (2 + j5) \\ &= (4+2) + j(-3+5) \\ &= 6 + j2 \end{aligned}$$

Euler's Formula

$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$

- Can you recall $\cos(\theta_1+\theta_2)$?
- Use the real part of $e^{j(\theta_1+\theta_2)}$



$$e^{j(\theta_1+\theta_2)} = e^{j\theta_1}e^{j\theta_2}$$

$$= (\cos\theta_1 + j\sin\theta_1)(\cos\theta_2 + j\sin\theta_2)$$

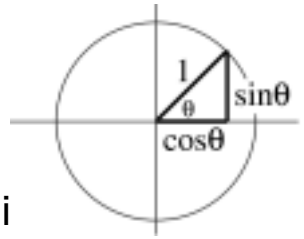
$$= \cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2 + j(\ldots)$$

$$\cos(\theta_1+\theta_2)$$

COMPLEX EXPONENTIAL SIGNAL

$$e^{j\omega t} = \cos(\omega t) + j\sin(\omega t)$$

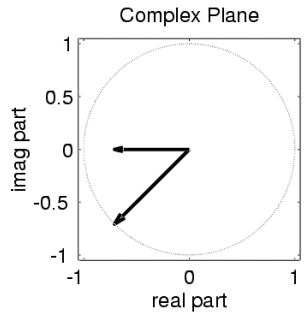
- **Rotating Vector**
  - Angle changes vs. time
  - **$\theta = \omega t$**
  - ex: $\omega = 10\pi$
  - Rotates one revolution (2π) i

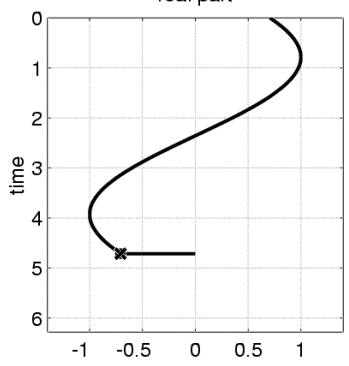


$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$

Rotating Phasor

See Demo on CD-ROM
Chapter 2





Cos = REAL PART

Real Part of Euler's

$$\cos(\omega t) = \Re e\{e^{j\omega t}\}$$

General Sinusoid

$$x(t) = A\cos(\omega t + \varphi)$$

So,

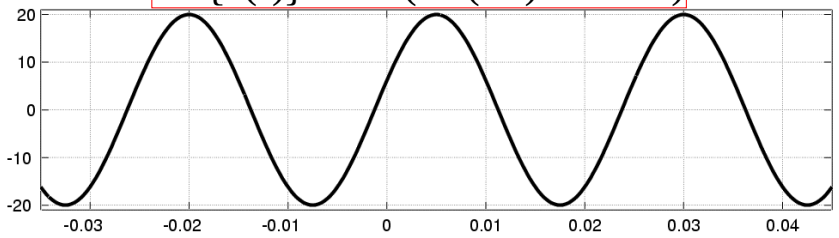
$$A\cos(\omega t + \varphi) = \Re e\{Ae^{j(\omega t+\varphi)}\}$$

$$= \Re e\{Ae^{j\varphi}e^{j\omega t}\}$$

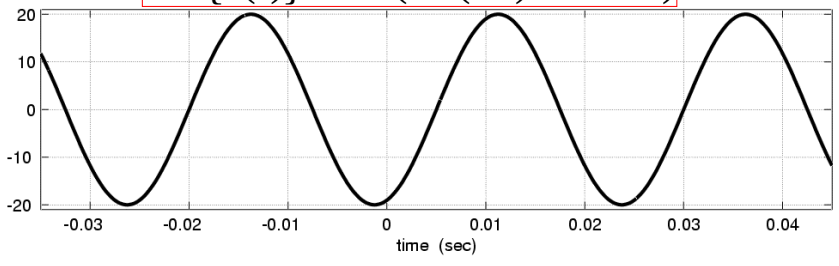
Real & Imaginary Part Plots

$$z(t) = 20e^{-j0.4\pi}e^{j2\pi(40)t}$$

$$\Re e\{z(t)\} = \cos(2\pi(40)t - 0.4\pi)$$



$$\Im m\{z(t)\} = \sin(2\pi(40)t - 0.4\pi)$$



COMPLEX AMPLITUDE

General Sinusoid

$$x(t) = A\cos(\omega t + \varphi) = \Re e\{Ae^{j\varphi}e^{j\omega t}\}$$

Sinusoid = REAL PART of $(Ae^{j\phi})e^{j\omega t}$

$$x(t) = \Re e\{Xe^{j\omega t}\} = \Re e\{z(t)\}$$

Complex AMPLITUDE = X

$$z(t) = Xe^{j\omega t} \qquad X = Ae^{j\varphi}$$

EXAMPLE: Find Complex Amp

- Find the COMPLEX AMPLITUDE for:

$$x(t) = \sqrt{3}\cos(77\pi t + 0.5\pi)$$

- Use EULER's FORMULA:

$$x(t) = \Re e\left\{\sqrt{3}e^{j(77\pi t + 0.5\pi)}\right\}$$

$$= \Re e\left\{\sqrt{3}e^{j0.5\pi}e^{j77\pi t}\right\}$$

$$X = \sqrt{3}e^{j0.5\pi}$$

WANT to ADD SINUSOIDS

- ALL SINUSOIDS HAVE SAME FREQUENCY
- HOW to GET {Amp,Phase} of RESULT ?

$$x_1(t) = 1.7\cos(2\pi(10)t + 70\pi/180)$$

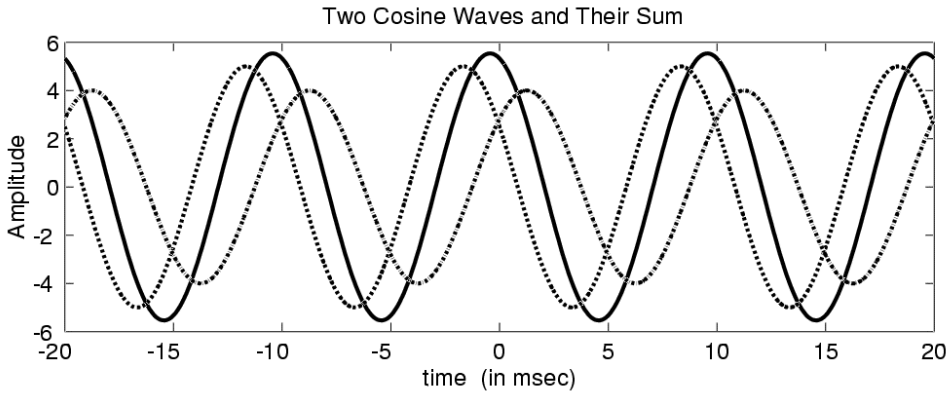
$$x_2(t) = 1.9\cos(2\pi(10)t + 200\pi/180)$$

$$x_3(t) = x_1(t) + x_2(t)$$

$$= 1.532\cos(2\pi(10)t + 141.79\pi/180)$$

ADD SINUSOIDS

- Sum Sinusoid has **SAME** Frequency



PHASOR ADDITION RULE

$$x(t) = \sum_{k=1}^{N} A_k\cos(\omega_0 t + \phi_k)$$

$$= A\cos(\omega_0 t + \phi)$$

Get the new complex amplitude by complex addition

$$\sum_{k=1}^{N} A_k e^{j\phi_k} = Ae^{j\phi}$$

Phasor Addition Proof

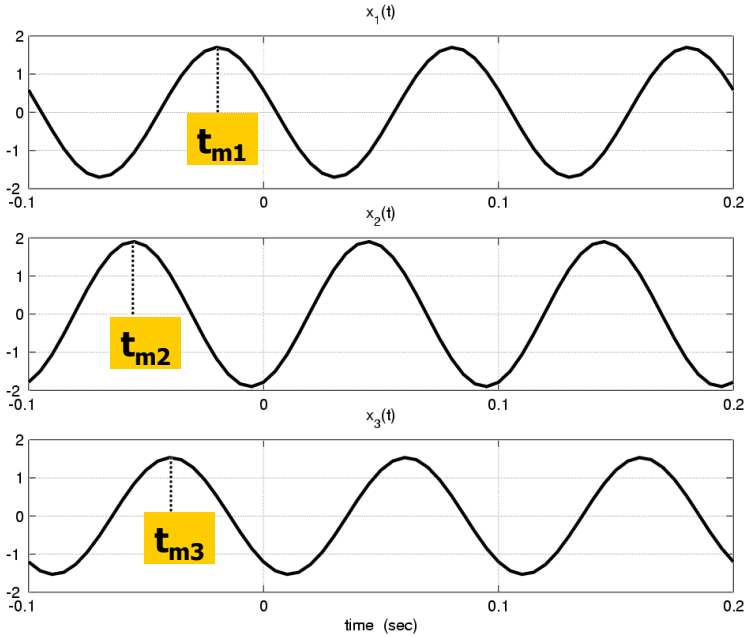
$$\sum_{k=1}^{N} A_k \cos(\omega_0 t + \phi_k) = \sum_{k=1}^{N} \Re e\left\{A_k e^{j(\omega_0 t+\phi_k)}\right\}$$

$$= \Re e\left\{\sum_{k=1}^{N} A_k e^{j\phi_k} e^{j\omega_0 t}\right\}$$

$$= \Re e\left\{\left(\sum_{k=1}^{N} A_k e^{j\phi_k}\right) e^{j\omega_0 t}\right\}$$

$$= \Re e\left\{\left(Ae^{j\phi}\right) e^{j\omega_0 t}\right\} = A\cos(\omega_0 t + \phi)$$

ADD SINUSOIDS EXAMPLE



Adding two sinusoids using phasors

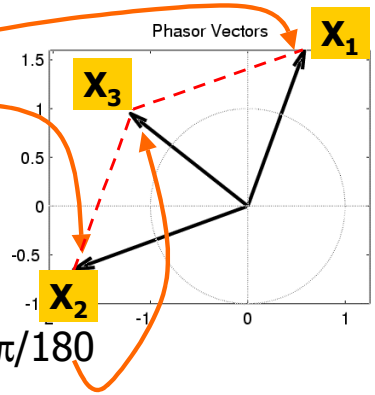
- Measure peak times:
  - t_{m1}=-0.0194, t_{m2}=-0.0556, t_{m3}=-0.0394
- Convert to phase (T=0.1)
  - $\phi_1 = -\omega t_{m1} = -2\pi(t_{m1}/T) = 70\pi/180$,
  - $\phi_2 = 200\pi/180$
- Pick off amplitudes
  - A_1=1.7, A_2=1.9

$$X_1 = 1.7e^{j70\pi/180}$$

$$X_2 = 1.9e^{j200\pi/180}$$

Phasor Add: Numerical

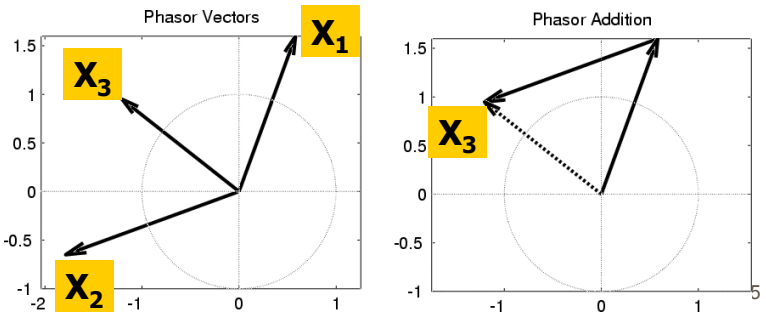
- Convert Polar to Cartesian
  - X_1 = 0.5814 + j1.597
  - X_2 = -1.785 - j0.6498
  - X_3 = -1.204 + j0.9476
- Convert back to Polar
  - X_3 = 1.532 at angle 141.79π/180
  - This is the sum



ADD SINUSOIDS

$$x_1(t) = 1.7\cos(2\pi(10)t + 70\pi/180)$$
$$x_2(t) = 1.9\cos(2\pi(10)t + 200\pi/180)$$
$$x_3(t) = x_1(t) + x_2(t)$$
$$= 1.532\cos(2\pi(10)t + 141.79\pi/180)$$

VECTOR (PHASOR) ADD



POP QUIZ: Add Sinusoids

- ADD THESE 2 SINUSOIDS:

$$x_1(t) = \cos(77\pi t)$$
$$x_2(t) = \sqrt{3}\cos(77\pi t + 0.5\pi)$$

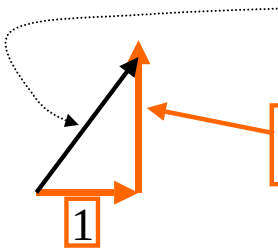
- COMPLEX ADDITION:

$$1e^{j0} + \sqrt{3}e^{j0.5\pi}$$

POP QUIZ (answer)

- COMPLEX ADDITION:

$$1 + j\sqrt{3} = 2e^{j\pi/3}$$



$$j\sqrt{3} = \sqrt{3}e^{j0.5\pi}$$

- CONVERT back to cosine form:

$$x_3(t) = 2\cos(77\pi t + \frac{\pi}{3})$$