EE-2025 Spring 2001

Lecture 5

Periodic Signals, Harmonics & Time-Varying Sinusoids

22-January-01

Web-CT Info

- Check the Bulletin Board for msgs
  - OFFICIAL
- Old Quizzes & Problems are linked
  - Quiz #1 on 2-Feb (Friday) Section B go to Room VL-456
- Prob Set #3 due NEXT WEEK

Lab Info

- Lab #1 Report due this week at Lab time.
- Lab #2
  - Write-up lab report on multipath due next week
  - Discuss lab report standards with your TA
- Miscellaneous
  - ERRORS ? ALWAYS Check Bulletin Board
  - Complete INSTRUCTOR VERIFICATION in Lab

Solving Mathematical Problems - G. Polya

1. Understand the problem
2. Devise a plan
3. Carry out the plan
4. Look back

How To Solve It, by G. Ploya, Princeton University Press

1. Understand the problem

- What is the unknown? What are the data? What is the condition?
  - Is the data sufficient? Or is it redundant?
  - Draw a figure.
  - Introduce suitable notation.
  - Express the condition in mathematical form

2. Devise a plan

- You must find a connection between what is given and what is to be found.
- Have you seen the same problem in a slightly different form?
- Can you use the solution to a related problem?
- Can you solve a special case first?

3. Carry out your plan

- Check each step.
  - Can you see clearly that each step is correct?
  - Can you PROVE that each step is correct?

4. Look back

- Can you check the result?
- Can you check your argument?
- Can you now see it at a glance?
- Can you now see another way to do the problem (probably more simply)?
- Can you use the method or the result in another problem?

The Rules

- Quizzes
  - NO make-ups given
  - Next Quiz would count for the one missed, IF excused
- Excused Absence
  - Must be written (by an "official")
  - Notify ahead of time via e-mail
- Consult Web-CT for more details

LECTURE

EE-2025 Georgia Tech

Lecture 5

Periodic Signals, Harmonics & Time-Varying Sinusoids

READING ASSIGNMENTS

- This Lecture:
  - Chapter 3, pp. 57-61
  - Chapter 3, pp. 66-77
- Next Lecture: **Notes**
  - **Fourier Series ANALYSIS**
  - Replaces pp.62-65 in DSP First

LECTURE OBJECTIVES

- Signals with HARMONIC Frequencies
  - Add Sinusoids with $f_k = kf_0$

$$x(t) = A_0 + \sum_{k=1}^{N} A_k \cos(2\pi k f_0 t + \varphi_k)$$

- FREQUENCY can change vs. TIME
  - Chirps: $x(t) = \cos(\alpha t^2)$
  - Introduce Spectrogram Visualization (**specgram.m**) (**plotspec.m**)

REVIEW: General Sum of Cosines

$$x(t) = A_0 + \sum_{k=1}^{N} A_k \cos(2\pi f_k t + \varphi_k)$$

$$X_0 = A_0 e^{j0}$$

$$x(t) = X_0 + \sum_{k=1}^{N} \Re e\{X_k e^{j2\pi f_k t}\}$$

$$X_k = A_k e^{j\varphi_k}$$

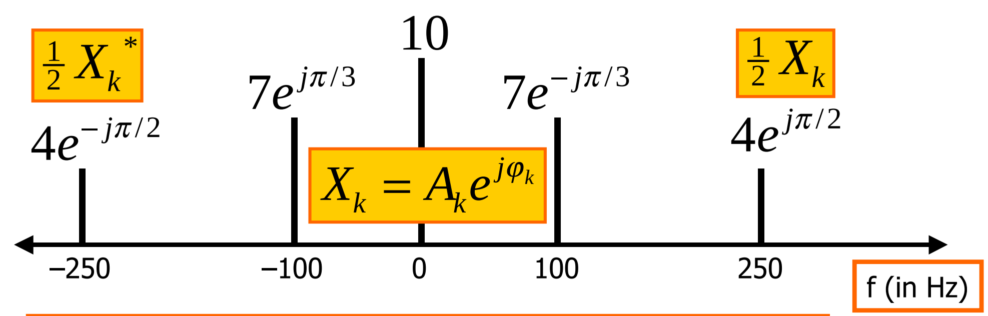
$$\text{Frequency} = f_k$$

$$\Re e\{z\} = \tfrac{1}{2} z + \tfrac{1}{2} z^*$$

$$x(t) = X_0 + \sum_{k=1}^{N} \left\{ \tfrac{1}{2} X_k e^{j2\pi f_k t} + \tfrac{1}{2} X_k^* e^{-j2\pi f_k t} \right\}$$

SPECTRUM DIAGRAM

- Recall Complex Amplitude vs. Freq



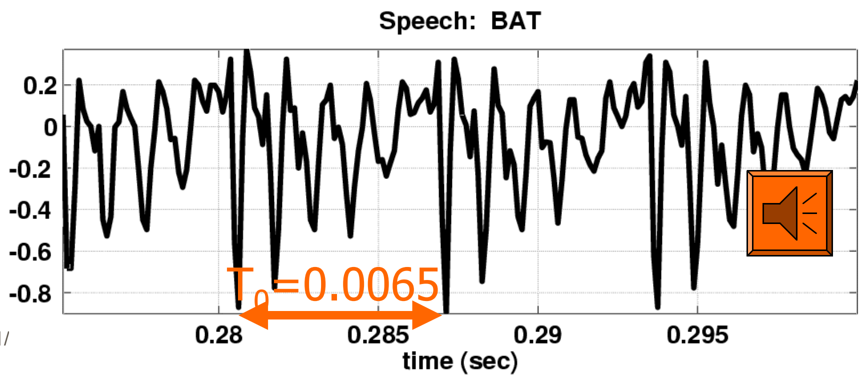
$$x(t) = 10 + 14\cos(2\pi(100)t - \pi/3) + 8\cos(2\pi(250)t + \pi/2)$$

Periodic Signals $x(t+T_0)=x(t)$

- Period signals repeat every T_0 sec

$$x(t + T_0) = x(t)$$

- Vowel sounds in speech are nearly Periodic



Period of Complex Exponential

$$x(t) = e^{j\omega t}$$

$$x(t + T_0) = x(t)\,?$$

Definition: Period is T_0

$$e^{j\omega(t+T_0)} = e^{j\omega t} e^{j\omega T_0} = e^{j\omega t}$$

$$\Rightarrow e^{j\omega T_0} = 1 = e^{j2\pi k} \quad \Rightarrow \omega T_0 = 2\pi k$$

$$\omega = \frac{2\pi k}{T_0} = \left(\frac{2\pi}{T_0}\right)k = \omega_0 k$$

k = integer

Harmonic Signal Spectrum

Therefore, $x(t+T_0) = x(t)$ requires: $f_k = k f_0$

$$x(t) = A_0 + \sum_{k=1}^{N} A_k \cos(2\pi k f_0 t + \varphi_k)$$

$$X_k = A_k e^{j\varphi_k}$$

$$f_0 = \frac{1}{T}$$

$$x(t) = X_0 + \sum_{k=1}^{N} \tfrac{1}{2} X_k e^{j2\pi k f_0 t} + \sum_{k=1}^{N} \tfrac{1}{2} X_k^* e^{-j2\pi k f_0 t}$$

DEFINE FUNDAMENTAL

$$x(t) = A_0 + \sum_{k=1}^{N} A_k \cos(2\pi k f_0 t + \varphi_k)$$

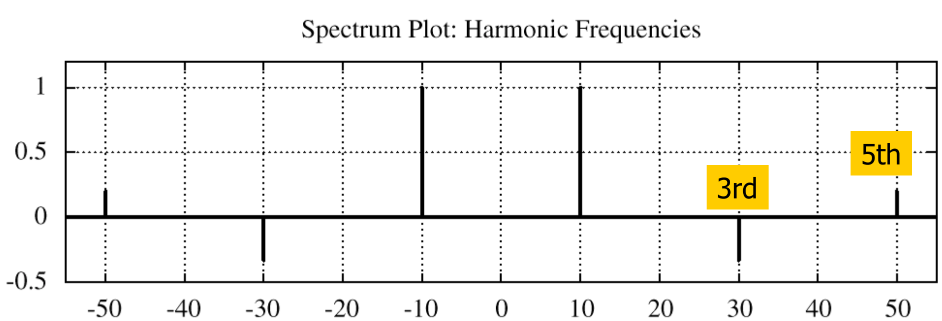
$$f_k = k f_0 \qquad (\omega_0 = 2\pi f_0)$$

$$f_0 = \text{fundamental frequency}$$

$$T_0 = \text{fundamental Period}$$

$$f_0 = \frac{1}{T_0}$$

Harmonic Signal (3 Freqs)

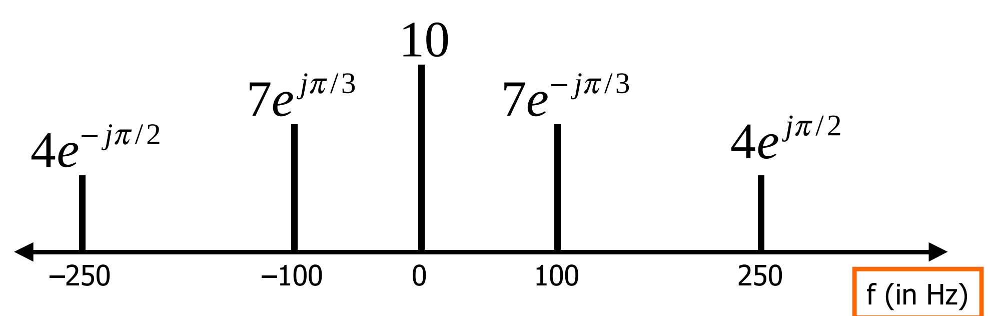


What is the fundamental frequency?

10 Hz

POP QUIZ: FUNDAMENTAL

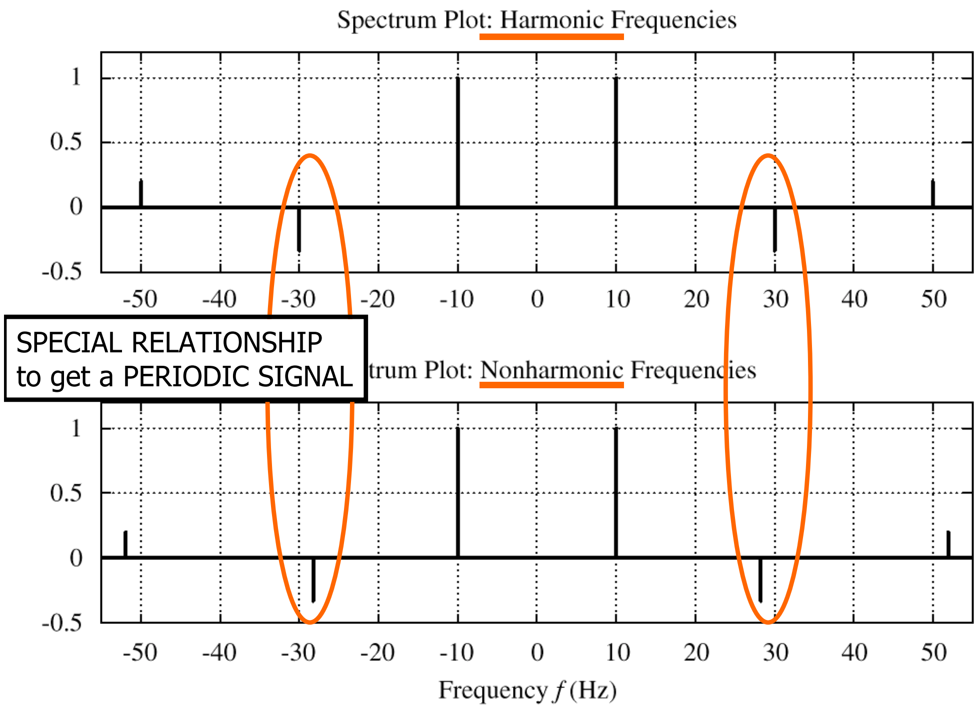
- Here's another spectrum:



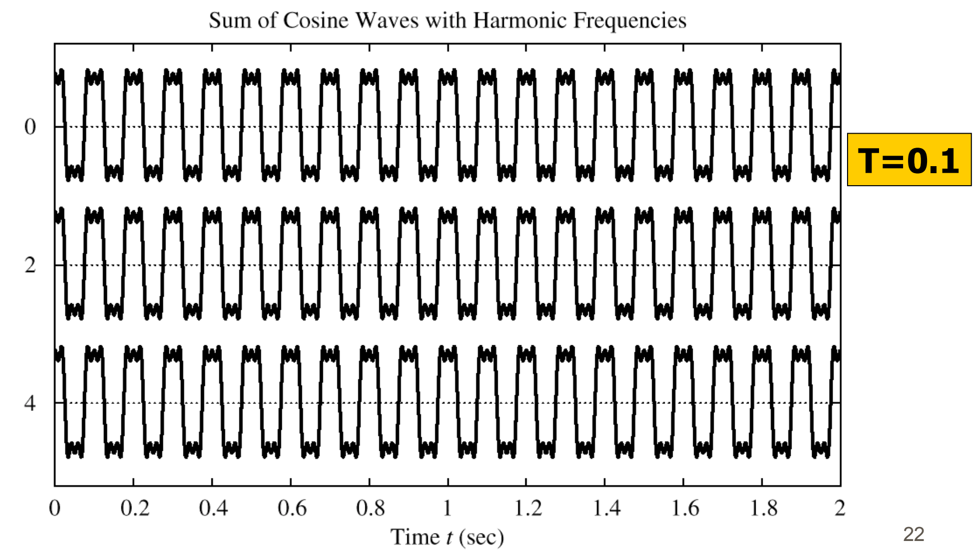
What is the fundamental frequency?

100 Hz ?

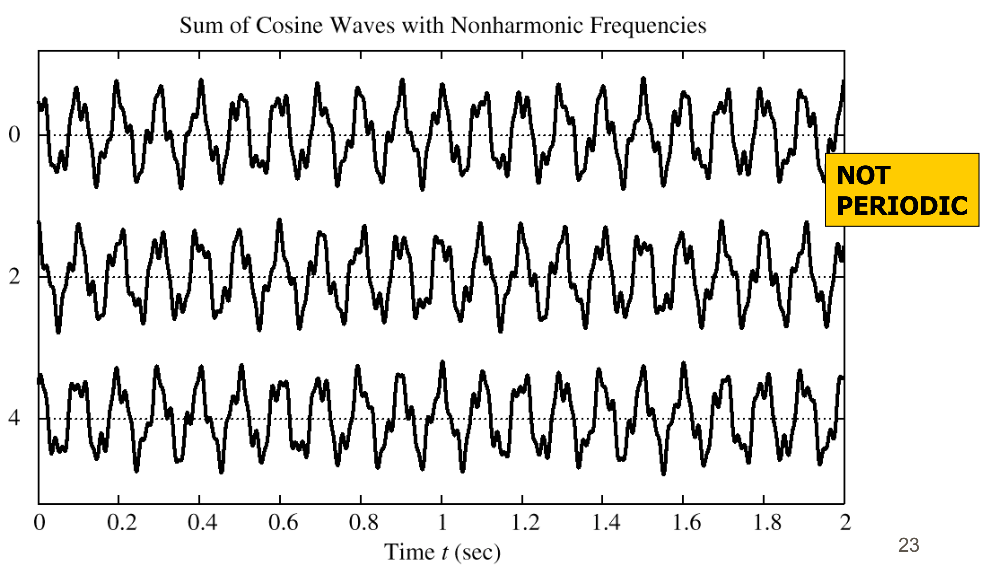
50 Hz ?



Harmonic Signal (3 Freqs)



NON-Harmonic Signal



FREQUENCY ANALYSIS

- **Now, a much HARDER problem**
- Given a recording of a song, have the computer write the music
- Can a machine extract frequencies?
  - Yes, if we COMPUTE the spectrum for x(t)
    - During short intervals

Time-Varying FREQUENCIES Diagram

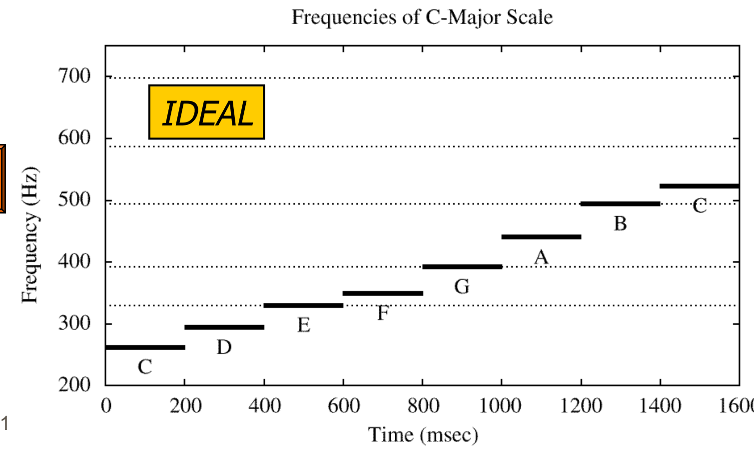
Frequency is the vertical axis

Figure 3.18 Sheet-music notation is a time–frequency diagram.

Time is the horizontal axis

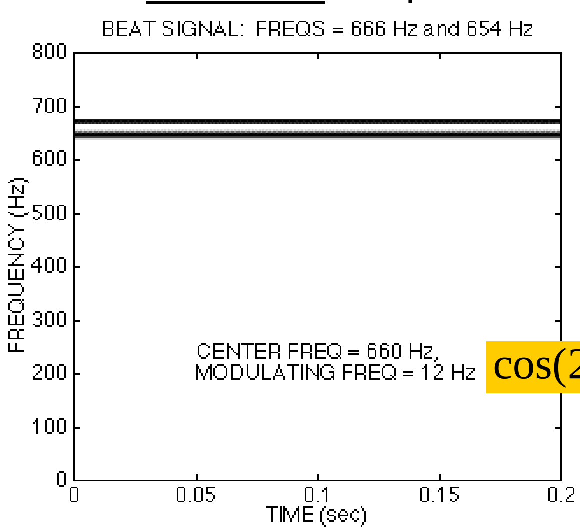
SIMPLE TEST SIGNAL

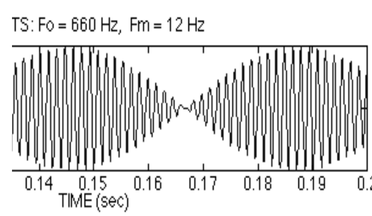
- C-major SCALE: stepped frequencies
  - Frequency is constant for each note



SPECTROGRAM EXAMPLE

- Two **Constant** Frequencies: Beats





$$\cos(2\pi(660)t)\cos(2\pi(12)t)$$

AM Radio Signal

- Same as BEAT Notes

$$\cos(2\pi(660)t)\cos(2\pi(12)t)$$

BEATS: Fo = 660 Hz, Fm = 12 Hz

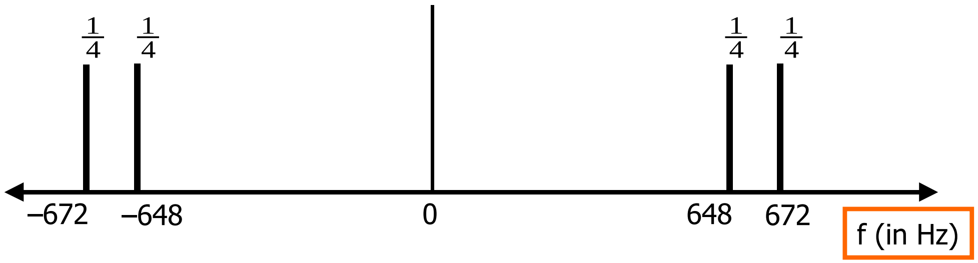
$$\tfrac{1}{2}\left(e^{j2\pi(660)t}+e^{-j2\pi(660)t}\right)\tfrac{1}{2}\left(e^{j2\pi(12)t}+e^{-j2\pi(12)t}\right)$$

$$\tfrac{1}{4}\left(e^{j2\pi(672)t}+e^{-j2\pi(672)t}+e^{j2\pi(648)t}+e^{-j2\pi(648)t}\right)$$

$$\tfrac{1}{2}\cos(2\pi(672)t)+\tfrac{1}{2}\cos(2\pi(648)t)$$

SPECTRUM of AM (Beat)

- 4 complex exponentials in AM:



What is the fundamental frequency?

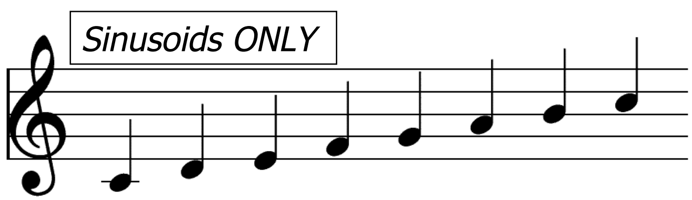
648 Hz ?

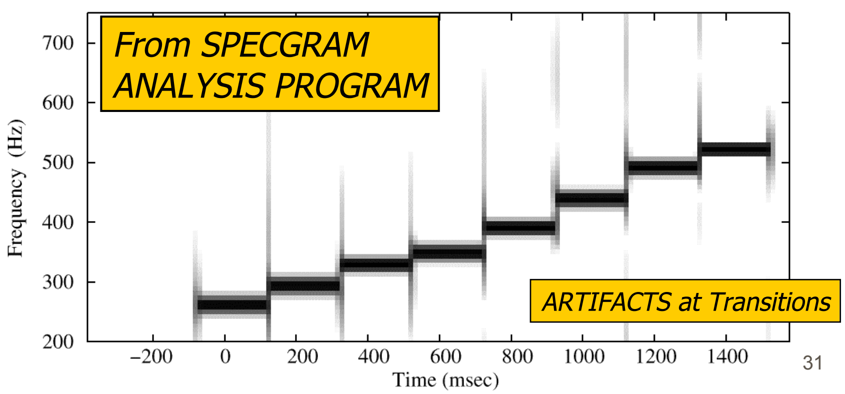
24 Hz ?

SPECTROGRAMS - visualizing frequency variation

- SPECTROGRAM Tool
  - MATLAB function is specgram.m
  - DSP First has spectgr.m (no plotting)
- ANALYSIS program
  - Takes x(t) as input
  - Produces spectrum values X_k
  - Breaks x(t) into SHORT TIME SEGMENTS
    - Then uses the FFT (Fast Fourier Transform)

SPECTROGRAM of C-Scale





Spectrogram of LAB SONG