EE-2025 Spring-2001

Lecture 8

D-to-A Conversion

9-February-01

Information

- Check the Bulletin Board for msgs
- Lab #4 is due next week
  - Notes file: marionnotes.mat (marionshort.mat)
  - Spectrogram image display info
    - New M-file: plotspec.m & spectgr.m
  - **FORMAL** Lab Report (150 points)
  - Auditions next week
- Problem Set #5 is also due next week

Quiz #1 comments

- Quiz Results
  - Average = 83
  - Median = 87 (163/221 above 80)
  - Below 60, watch out
- Solution will be posted for one version
  - Others are similar

LECTURE

READING ASSIGNMENTS

- This Lecture:
  - Chapter 4, pp. 100-111
- Other Reading:
  - Recitation: Chapter 4, pp. 90-100
    - Strobe Demo
  - Next Lecture: Chapter 5 (beginning)

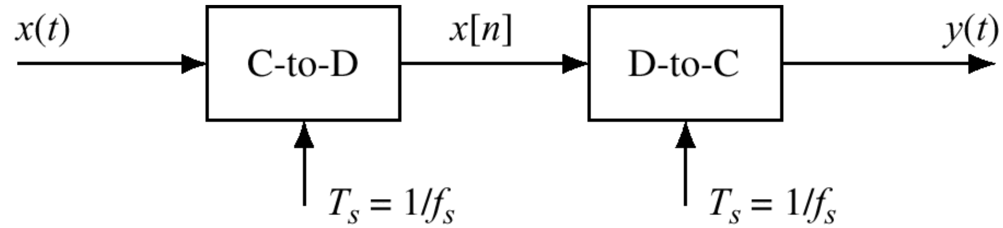
LECTURE OBJECTIVES

- FOLDING: a type of ALIASING
- DIGITAL-to-ANALOG CONVERSION is
  - Reconstruction from samples
    - SAMPLING THEOREM applies
  - Smooth Interpolation
- Mathematical Model of D-to-A
  - SUM of SHIFTED PULSES
    - Linear Interpolation example

SAMPLING THEOREM

Shannon Sampling Theorem

A continuous-time signal $x(t)$ with frequencies no higher than f_{max} can be reconstructed exactly from its samples $x[n] = x(nT_s)$, if the samples are taken at a rate $f_s = 1/T_s$ that is greater than $2f_{max}$.



$$y(t) = x(t) \quad \text{if } f_s > 2f_{max}$$

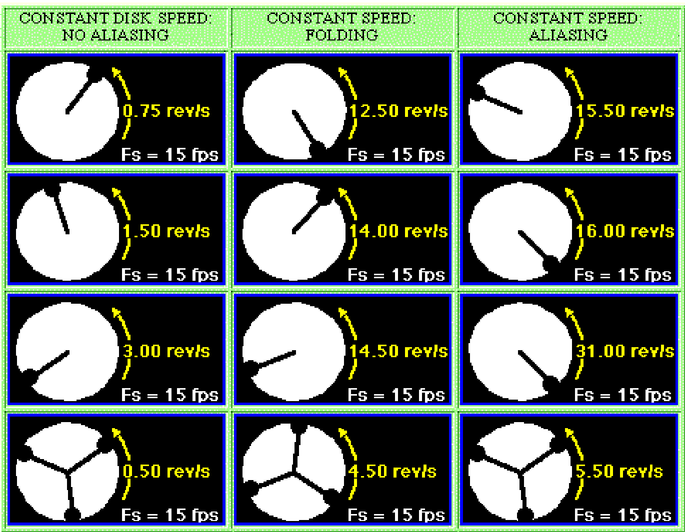
NYQUIST RATE

- "Nyquist Rate" Sampling
  - f_s = **TWICE** THE HIGHEST FREQUENCY in x(t)
  - "Sampling above the Nyquist rate"
- BANDLIMITED SIGNALS
  - DEF: x(t) has a HIGHEST FREQUENCY COMPONENT in its SPECTRUM
  - NON-BANDLIMITED EXAMPLE
    - TRIANGLE WAVE is NOT BANDLIMITED

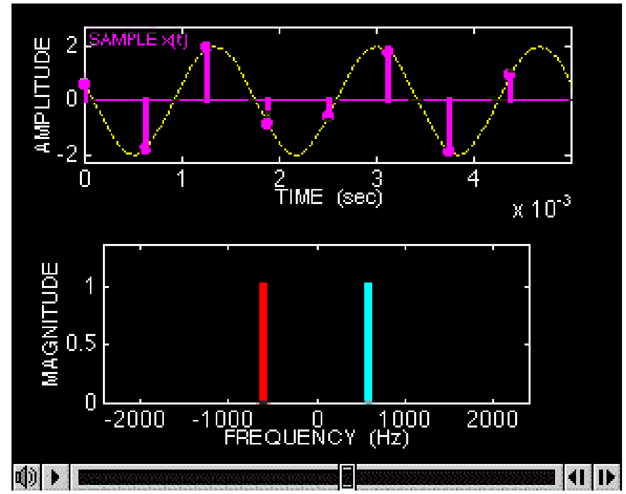
DEMOS from CHAPTER 4

- CD-ROM DEMOS
- SAMPLING DEMO
  - Different Sampling Rates
    - Aliasing of a Sinusoid
- STROBE DEMO
  - Synthetic vs. Real
  - Television SAMPLES at 30 fps
- Sampling & Reconstruction

STROBE DEMO (Synthetic)



SAMPLING DEMO (Ch. 4)



SPECTRUM for x[n]

- INCLUDE ALL SPECTRUM LINES
  - ALIASES
    - ADD INTEGER MULTIPLES of 2π and -2π
  - FOLDED ALIASES
    - ALIASES of NEGATIVE FREQS
- PLOT versus NORMALIZED FREQUENCY
  - i.e., DIVIDE f_o by f_s

$$\hat{\omega} = 2\pi \frac{f}{f_s} + 2\pi\ell$$

EXAMPLE: SPECTRUM

- $x[n] = A\cos(0.2\pi n+\phi)$
- FREQS @ 0.2π and -0.2π
- ALIASES:
  - $\{2.2\pi, 4.2\pi, 6.2\pi, \ldots\}$ & $\{-1.8\pi, -3.8\pi, \ldots\}$
  - EX: $x[n] = A\cos(4.2\pi n+\phi)$
- ALIASES of NEGATIVE FREQ:
  - $\{1.8\pi, 3.8\pi, 5.8\pi, \ldots\}$ & $\{-2.2\pi, -4.2\pi \ldots\}$

ALIASING DERIVATION

- Other Frequencies give the same $\hat{\omega}$

$$\text{If } \; x(t) = A\cos(2\pi(f + \ell f_s)t + \varphi)$$

$$\text{and we substitute: } \; t \leftarrow \tfrac{n}{f_s}$$

$$\text{then: } \; x[n] = A\cos(2\pi(f + \ell f_s)\tfrac{n}{f_s} + \varphi)$$

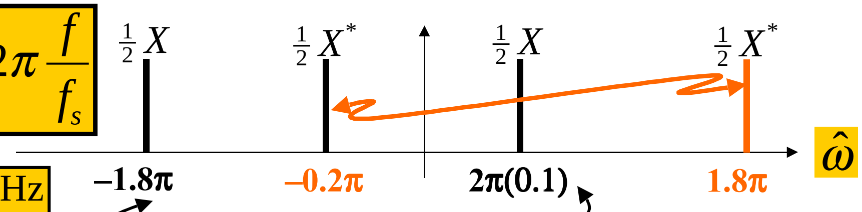
$$x[n] = A\cos(2\pi\tfrac{f}{f_s}n + 2\pi\ell n + \varphi) = A\cos(\hat{\omega}n + \varphi)$$

$$\hat{\omega} = \frac{2\pi(f + \ell f_s)}{f_s} = \frac{2\pi f}{f_s} + \frac{2\pi\ell f_s}{f_s} = \frac{2\pi f}{f_s} + 2\pi\ell$$

SPECTRUM (MORE LINES)

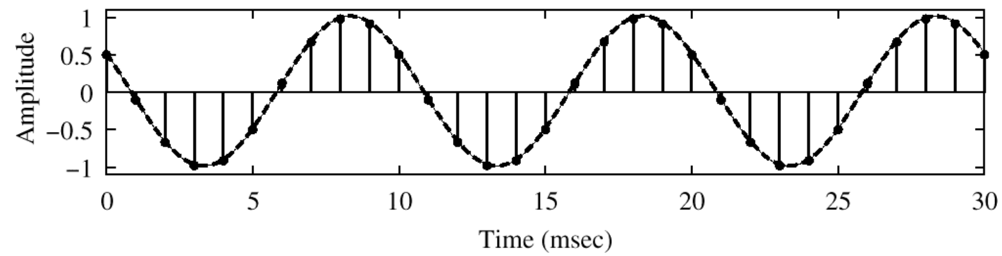
$$\hat{\omega} = 2\pi\frac{f}{f_s}$$

$f_s = 1\text{kHz}$



$$\omega = \hat{\omega}/T_s = \hat{\omega} f_s = 0.2\pi(1000) = 2\pi(100)$$

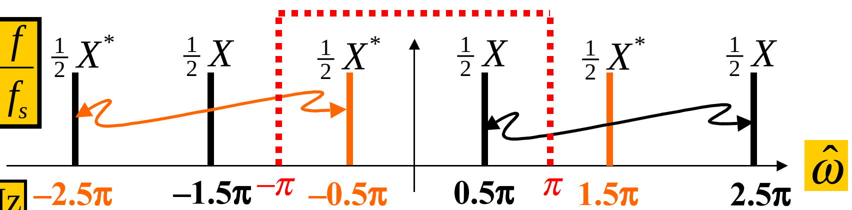
100-Hz Cosine Wave: Sampled with T_s = 1 msec (1000 Hz)



SPECTRUM (ALIASING CASE)

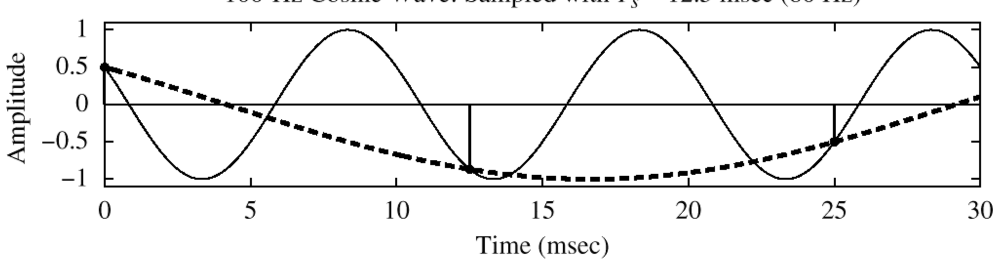
$$\hat{\omega} = 2\pi\frac{f}{f_s}$$

$f_s = 80\text{Hz}$



$$\omega = \hat{\omega}/T_s = \hat{\omega} f_s = 0.5\pi(80) = 2\pi(20)$$

100-Hz Cosine Wave: Sampled with T_s = 12.5 msec (80 Hz)



FOLDING DERIVATION

- Negative Freqs can give the same $\hat{\omega}$

$$x(t) = A\cos(2\pi(-f + \ell f_s)t - \varphi)$$

$$x[n] = x(nT_s) = A\cos(2\pi(-f + \ell f_s)nT_s - \varphi)$$

$$x[n] = A\cos((-2\pi f T_s)n + (2\pi\ell f_s T_s)n - \varphi)$$

$$x[n] = A\cos((2\pi f T_s)n - 2\pi\ell n + \varphi)$$

$\cos(-\theta) = \cos\theta$

$$x[n] = A\cos(\hat{\omega}n + \varphi)$$

SAME DIGITAL SIGNAL

FOLDING (a type of ALIASING)

- EXAMPLE: 3 different x(t); same x[n]
- 900 Hz "folds" to 100 Hz when f_s=1kHz

$$f_s = 1000 \text{ Hz}$$

$$\hat{\omega} = 2\pi \frac{100}{1000} = 2(0.1)$$

$$\cos(2\pi(100)t) \rightarrow \cos[2\pi(0.1)n]$$

$$\cos(2\pi(1100)t) \rightarrow \cos[2\pi(1.1)n] = \cos[2\pi(0.1)n]$$

$$\cos(2\pi(900)t) \rightarrow \cos[2\pi(0.9)n]$$

$$= \cos[2\pi(0.9)n - 2\pi n] = \cos[2\pi(-0.1)n] = \cos[2\pi(0.1)n]$$

DIGITAL FREQ $\hat{\omega}$ AGAIN

Normalized Radian Frequency

$$\hat{\omega} = \omega T_s = \frac{2\pi f}{f_s} + 2\pi\ell$$

ALIASING

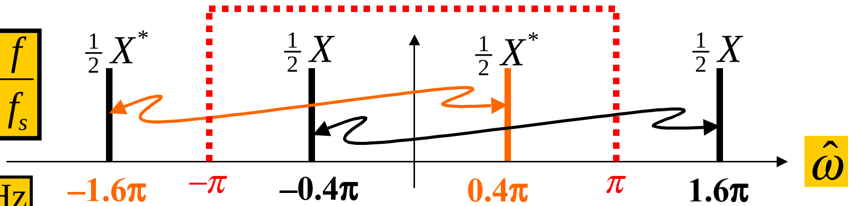
$$\hat{\omega} = \omega T_s = -\frac{2\pi f}{f_s} + 2\pi\ell$$

FOLDED ALIAS

SPECTRUM (FOLDING CASE)

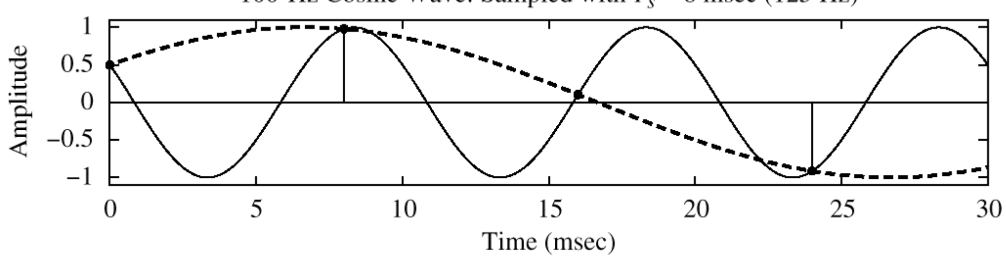
$$\hat{\omega} = 2\pi \frac{f}{f_s}$$

$$f_s = 125\text{Hz}$$

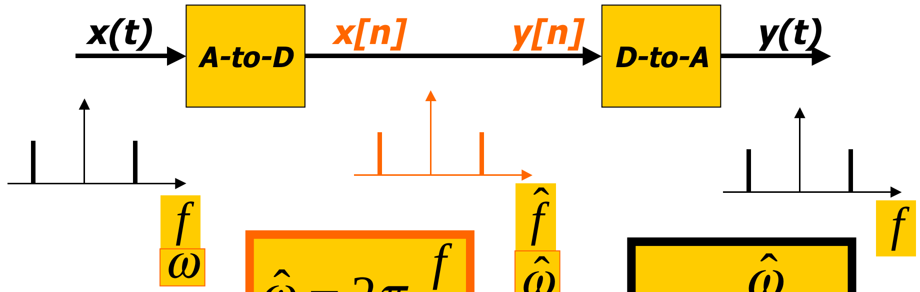


$$\omega = \hat{\omega}/T_s = \hat{\omega} f_s = 0.4\pi(125) = 2\pi(25)$$

100-Hz Cosine Wave: Sampled with T_s = 8 msec (125 Hz)



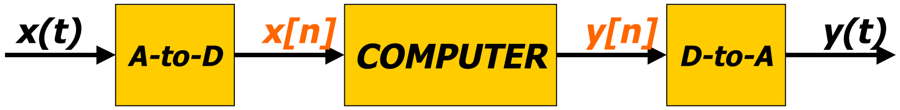
FREQUENCY DOMAINS



$$\hat{\omega} = 2\pi \frac{f}{f_s}$$

$$f = \frac{\hat{\omega}}{2\pi} f_s$$

D-to-A Reconstruction

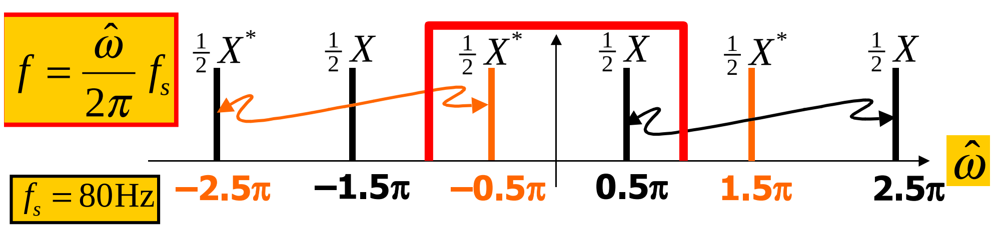


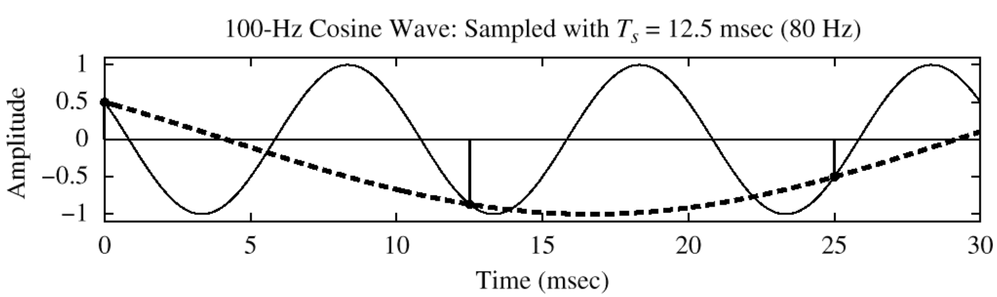
- Create continuous y(t) from y[n]
  - **IDEAL**
    - If you have formula for y[n]
  - Replace n in y[n] with $f_s t$
  - $y[n] = A\cos(0.2\pi n + \phi)$ with $f_s = 8000$ Hz
  - $y(t) = A\cos(2\pi(800)t + \phi)$

D-to-A is AMBIGUOUS !

- ALIASING
  - Given y[n], which y(t) do we pick ? ? ?
  - INFINITE NUMBER of y(t)
    - PASSING THRU THE SAMPLES, y[n]
  - D-to-A RECONSTRUCTION MUST CHOOSE ONE OUTPUT
- RECONSTRUCT THE SMOOTHEST ONE
  - THE LOWEST FREQ, if y[n] = sinusoid

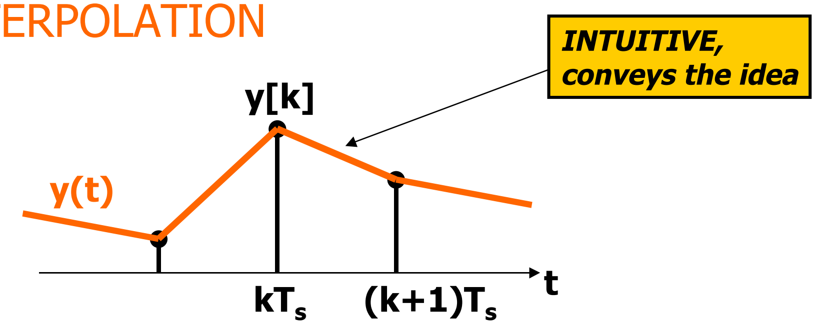
SPECTRUM (ALIASING CASE)





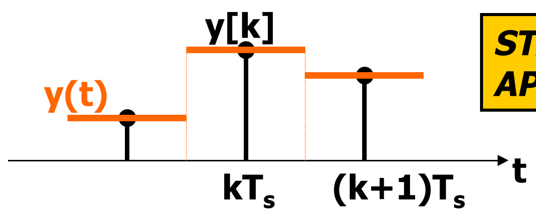
Reconstruction (D-to-A)

- CONVERT STREAM of NUMBERS to x(t)
- "CONNECT THE DOTS"
- INTERPOLATION



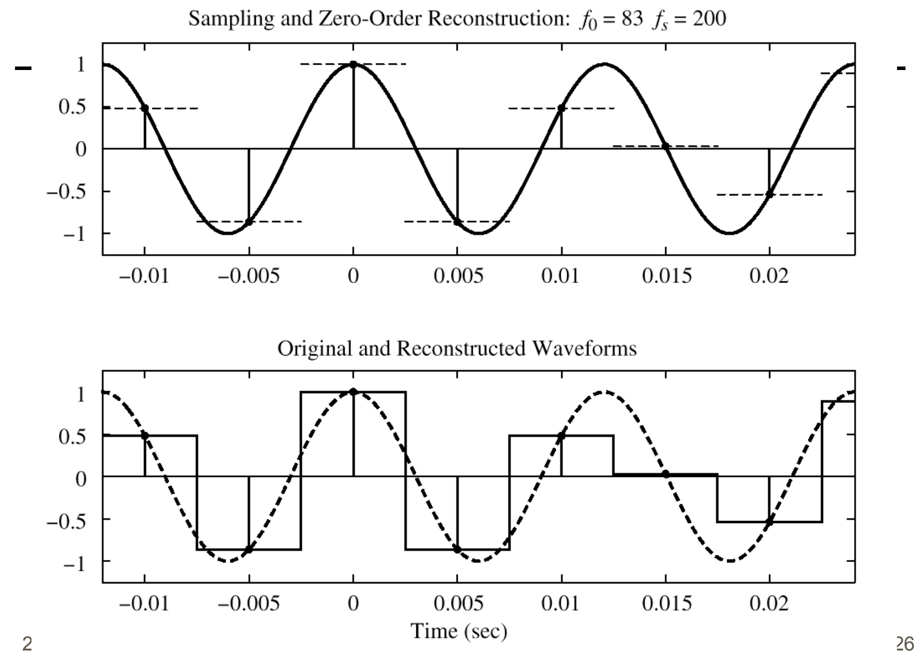
SAMPLE & HOLD DEVICE

- CONVERT y[n] to y(t)
  - y[k] should be the value of y(t) at $t = kT_s$
  - Make y(t) equal to y[k] for
    - $kT_s - 0.5T_s < t < kT_s + 0.5T_s$



STAIR-STEP APPROXIMATION

SQUARE PULSE CASE



MATH MODEL for D-to-A

$$y(t) = \sum_{n=-\infty}^{\infty} y[n]p(t - nT_s)$$

SQUARE PULSE:

$$p(t) = \begin{cases} 1 & -\frac{1}{2}T_s < t \leq \frac{1}{2}T_s \\ 0 & \text{otherwise} \end{cases}$$

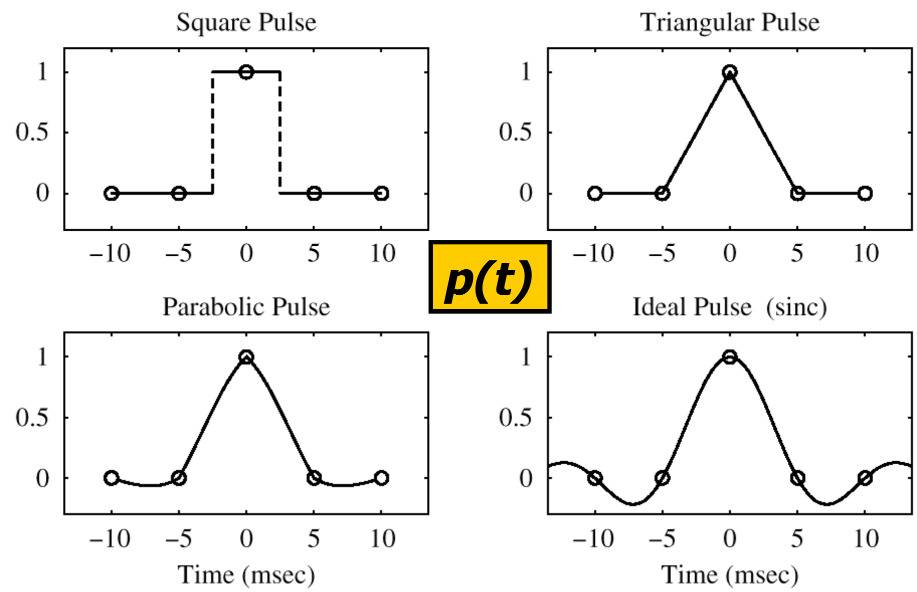


Figure 4.17 Four different pulses for D-to-C conversion. The sampling period is $T_s = 0.005$, i.e., $f_s = 200$ Hz. Note that the duration of each pulse is approximately one or two times T_s.

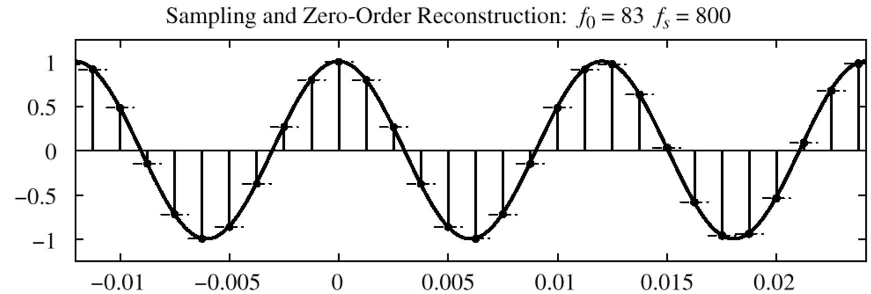
EXPAND the SUMMATION

$$\sum_{n=-\infty}^{\infty} y[n]p(t-nT_s) =$$

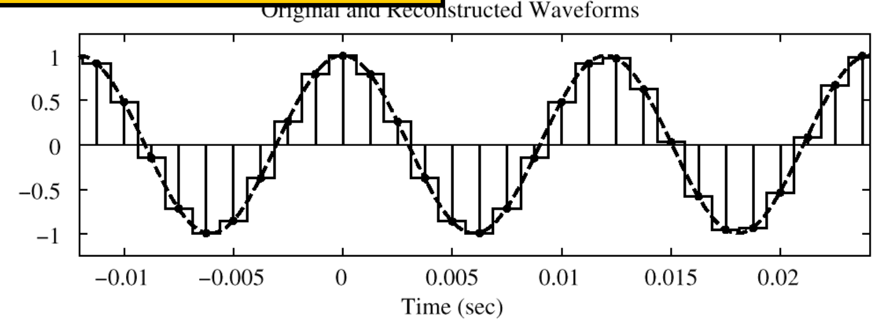
$$\ldots + y[0]p(t) + y[1]p(t-T_s) + y[2]p(t-2T_s) + \ldots$$

- SUM of SHIFTED PULSES $p(t-nT_s)$
  - "WEIGHTED" by y[n]
  - CENTERED at $t=nT_s$
  - SPACED by T_s
    - RESTORES "REAL TIME"

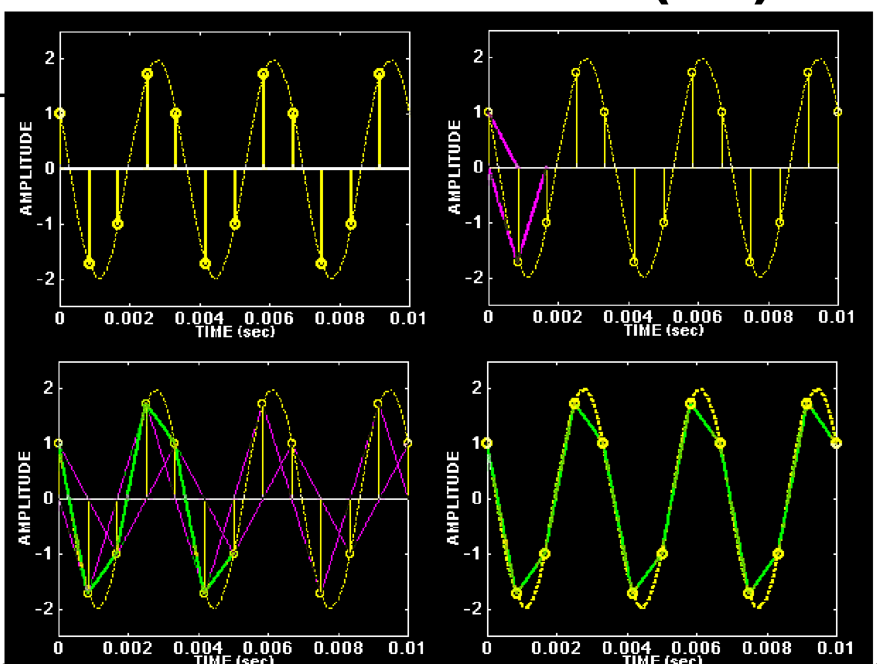
OVER-SAMPLING CASE



EASIER TO RECONSTRUCT

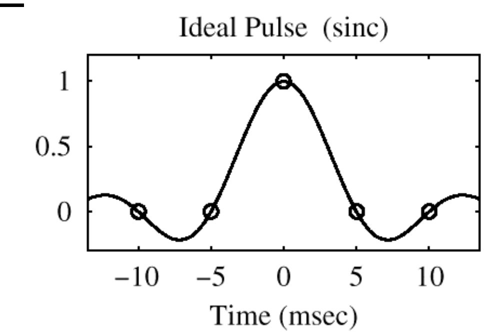


TRIANGULAR PULSE (2X)



OPTIMAL PULSE ?

CALLED "BANDLIMITED INTERPOLATION"



$$p(t) = \frac{\sin\frac{\pi}{T_s}t}{\frac{\pi}{T_s}t} \qquad \text{for } -\infty < t < \infty$$

$$p(t) = 0 \qquad \text{for } t = 0, \pm T_s, \pm 2T_s$$