EE-2025 Spring 2001

Lecture 9

FIR Filtering Intro

12-February-01

Information

- Music Listening this week
  - Have your song ready BEFORE lab
- HELP Sessions:
  - Mon & Tues @ 6pm, VL-361
  - Everything is fair game: labs, homework, MATLAB, …
- Problem Set #5 due THIS WEEK

LECTURE

Review: Aliasing and Folding in Sampling

- Consider a 100 Hz cosine signal

$$x(t) = \cos(200\pi t + \phi)$$

- Sample it with f_s = 125 Hz

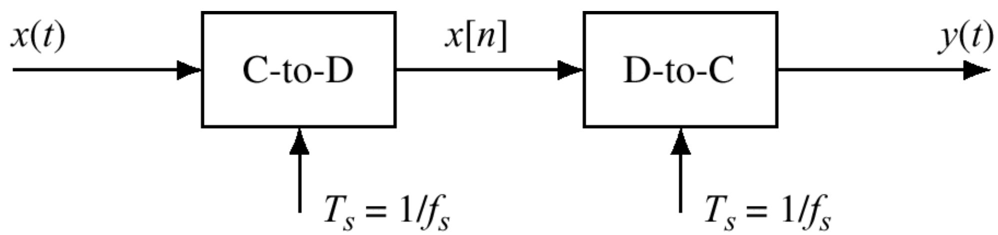
$$x[n] = x(n/125) = \cos(200\pi n/125 + \phi)$$

$$x[n] = \cos(1.6\pi n + \phi) = \cos(2\pi n - .4\pi n + \phi)$$

$$x[n] = \cos(-0.4\pi n + \phi) = \cos(0.4\pi n - \phi)$$

Reconstruction from Samples

$f_s = 125$ Hz



$$x(t) = \cos(200\pi t + \phi)$$

$$x[n] = \cos(0.4\pi n - \phi)$$

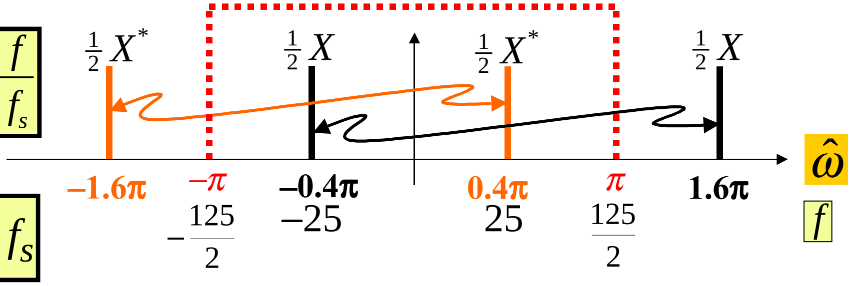
$$y(t) = \cos(0.4\pi f_s t + \phi) = \cos(0.4\pi(125)t + \phi)$$

$$y(t) = \cos(50\pi t - \phi)$$

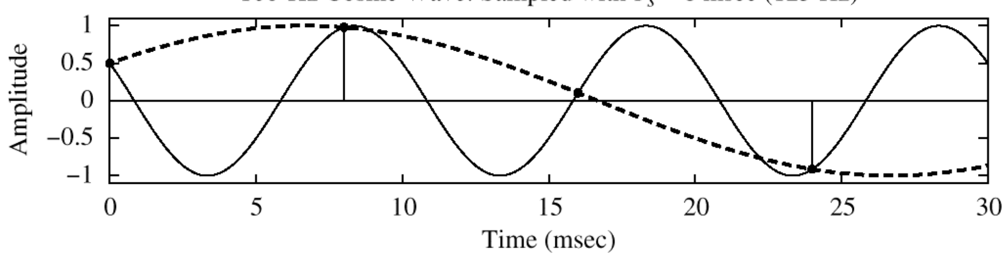
SPECTRUM $f_s = 125\text{ Hz}$

$$\hat{\omega} = 2\pi \frac{f}{f_s}$$

$$f = \frac{\hat{\omega}}{2\pi} f_s$$



100-Hz Cosine Wave: Sampled with $T_s = 8$ msec (125 Hz)



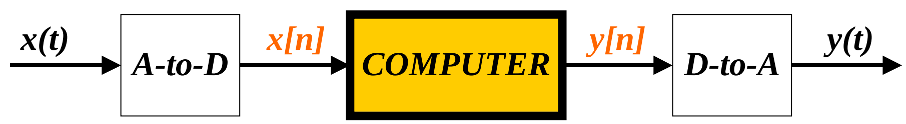
READING ASSIGNMENTS

- This Lecture:
  - Chapter 5, pp. 119-131
- Other Reading:
  - Recitation: Ch. 5, pp. 127-133, 142-146
    - CONVOLUTION
  - Next Lecture: Chapter 5, pp. 133-152

LECTURE OBJECTIVES

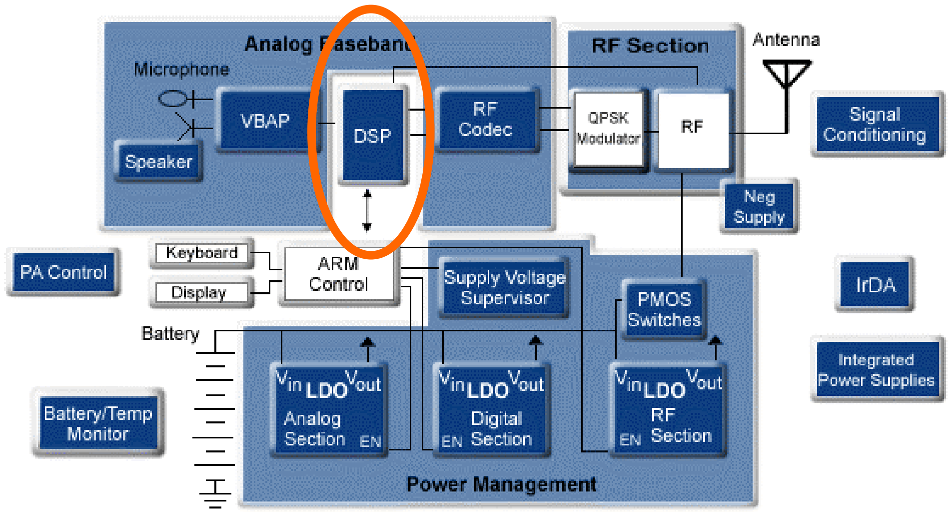
- INTRODUCE FILTERING IDEA
  - Weighted Average
  - Running Average
- FINITE IMPULSE RESPONSE FILTERS
  - **FIR** Filters
  - Show how to compute the output y[n] from the input signal, x[n]

DIGITAL FILTERING



- CONCENTRATE on the COMPUTER
  - PROCESSING ALGORITHMS
  - SOFTWARE (MATLAB)
  - HARDWARE: DSP chips, VLSI
- DSP: DIGITAL SIGNAL PROCESSING

Digital Cell Phone



Free (?) with 2 year contract

DISCRETE-TIME SYSTEM

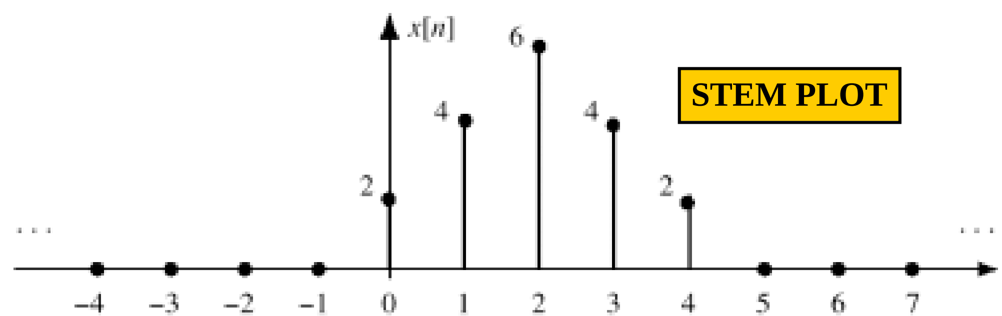
- OPERATE on x[n] to get y[n]
- WANT a GENERAL CLASS of SYSTEMS
  - ANALYZE the SYSTEM
    - TOOLS: TIME-DOMAIN & FREQUENCY-DOMAIN
  - SYNTHESIZE the SYSTEM

D-T SYSTEM EXAMPLES

- EXAMPLES:
  - POINTWISE OPERATORS
    - SQUARING: $y[n] = (x[n])^2$
  - RUNNING AVERAGE
    - RULE: "the output at time n is the average of three consecutive input values"

DISCRETE-TIME SIGNAL

- x[n] is a LIST of NUMBERS
  - INDEXED by "n"



3-PT AVERAGE SYSTEM

- ADD 3 CONSECUTIVE NUMBERS
  - Do this for each "n"

the following input–output equation

Make a TABLE

$$y[n] = \tfrac{1}{3}(x[n] + x[n+1] + x[n+2])$$

n	$n < -2$	-2	-1	0	1	2	3	4	5	$n > 5$
$x[n]$	0	0	0	2	4	**6**	**4**	**2**	0	0
$y[n]$	0	$\frac{2}{3}$	2	4	$\frac{14}{3}$	**4**	2	$\frac{2}{3}$	0	0

n=0 $y[0] = \tfrac{1}{3}(x[0] + x[1] + x[2])$

n=1 $y[1] = \tfrac{1}{3}(x[1] + x[2] + x[3])$

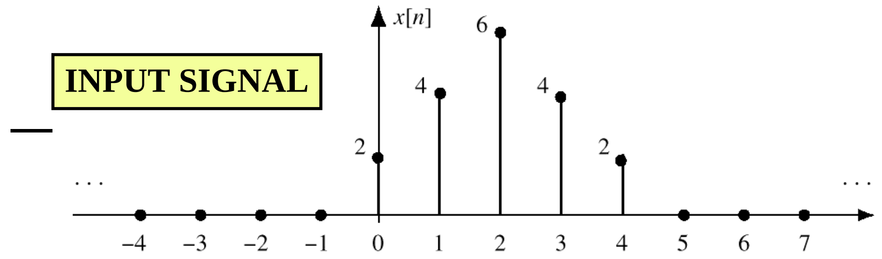


Figure 5.2 Finite-length input signal, $x[n]$.

$$y[n] = \frac{1}{3}(x[n] + x[n+1] + x[n+2])$$

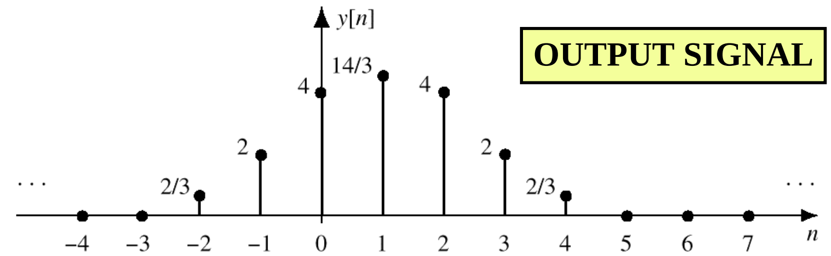


Figure 5.3 Output of running average, $y[n]$.

PAST, PRESENT, FUTURE

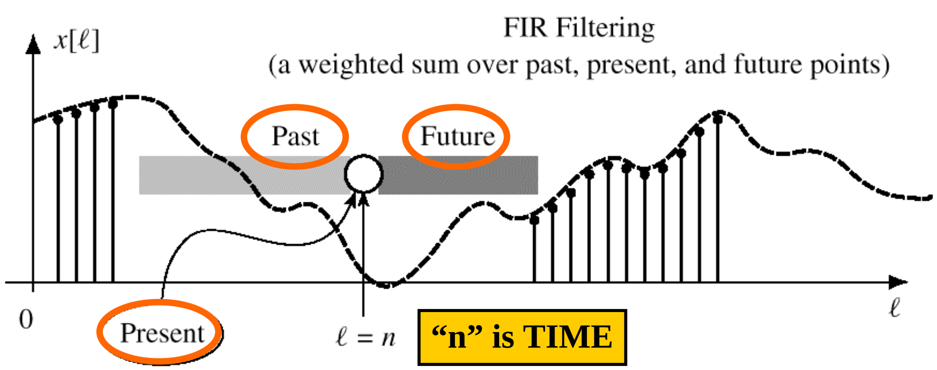


Figure 5.4 The running-average filter calculation at time index n uses values within a sliding window (shaded). Dark shading indicates the future ($\ell > n$); light shading, the past ($\ell < n$).

ANOTHER 3-pt AVERAGER

- Uses "PAST" VALUES of x[n]
  - IMPORTANT IF "n" represents REAL TIME
    - WHEN x[n] & y[n] ARE STREAMS

$$y[n] = \frac{1}{3}(x[n] + x[n-1] + x[n-2])$$

n	$n < -2$	-2	-1	0	1	2	3	4	5	6	7	$n > 7$
$x[n]$	0	0	0	2	4	**6**	**4**	**2**	0	0	0	0
$y[n]$	0	0	0	$\frac{2}{3}$	2	4	$\frac{14}{3}$	**4**	2	$\frac{2}{3}$	0	0

GENERAL FIR FILTER

- FILTER COEFFICIENTS $\{b_k\}$
  - DEFINE THE FILTER

$$y[n] = \sum_{k=0}^{M} b_k \, x[n-k]$$

  - For example, $\{b_k\} = \{3, -1, 2, 1\}$

$$y[n] = \sum_{k=0}^{3} b_k x[n-k]$$

$$= 3x[n] - x[n-1] + 2x[n-2] + x[n-3]$$

GENERAL FIR FILTER

- FILTER COEFFICIENTS $\{b_k\}$

$$y[n] = \sum_{k=0}^{M} b_k \, x[n-k]$$

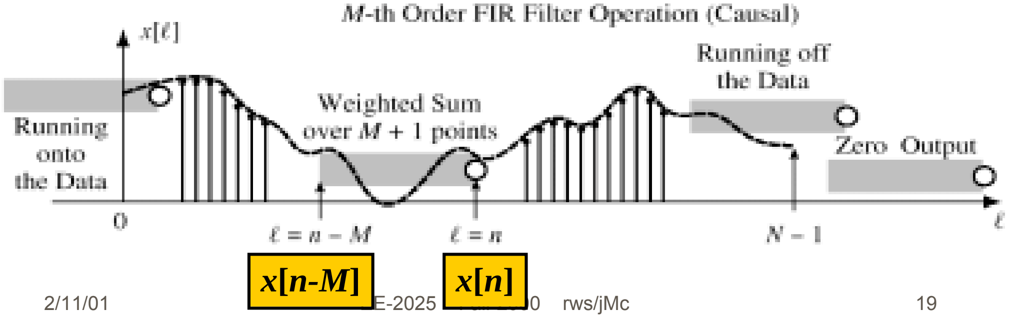
- FILTER ORDER is M
- FILTER LENGTH is L = M+1
  - NUMBER of FILTER COEFFS is L

GENERAL FIR FILTER

- SLIDE a WINDOW across x[n]

$$y[n] = \sum_{k=0}^{M} b_k \, x[n-k]$$

M-th Order FIR Filter Operation (Causal)



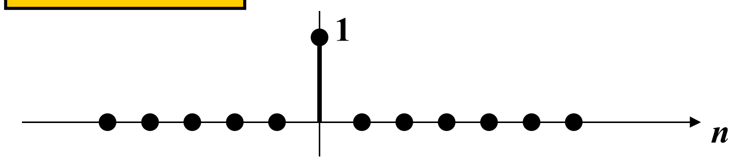
FILTERED STOCK SIGNAL

SPECIAL INPUT SIGNALS

- x[n] = SINUSOID **FREQUENCY RESPONSE**
- x[n] has only one NON-ZERO VALUE

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

UNIT-IMPULSE



UNIT IMPULSE SIGNAL δ[n]

n	...	−2	−1	0	1	2	3	4	5	6	...
$\delta[n]$	0	0	0	1	0	0	0	0	0	0	0
$\delta[n-3]$	0	0	0	0	0	0	1	0	0	0	0

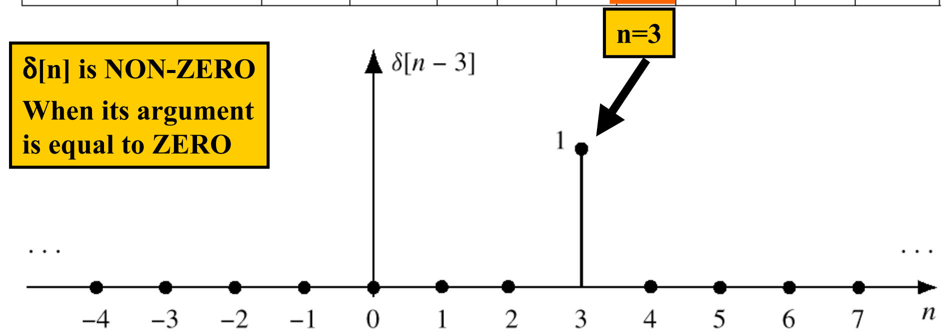
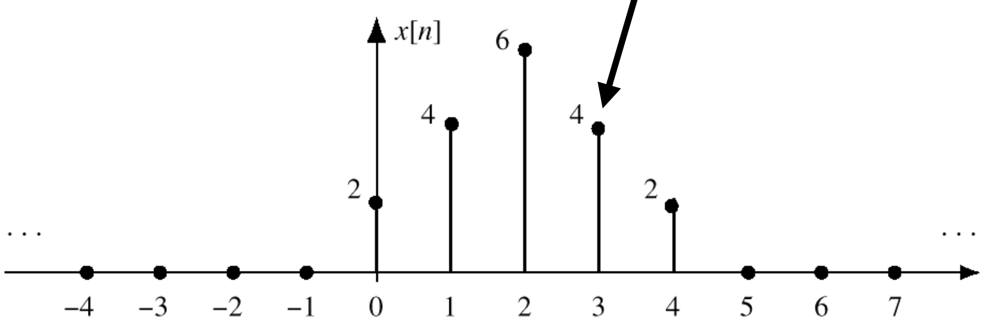


Figure 5.7 Shifted impulse sequence, $\delta[n-3]$.

MATH FORMULA for x[n]

- Use SHIFTED IMPULSES to write x[n]

$$x[n] = 2\delta[n] + 4\delta[n-1] + 6\delta[n-2] + 4\delta[n-3] + 2\delta[n-4]$$



SUM of SHIFTED IMPULSES

n	...	−2	−1	0	1	2	3	4	5	6	...
$2\delta[n]$	0	0	0	2	0	0	0	0	0	0	0
$4\delta[n-1]$	0	0	0	0	4	0	0	0	0	0	0
$6\delta[n-2]$	0	0	0	0	0	6	0	0	0	0	0
$4\delta[n-3]$	0	0	0	0	0	0	4	0	0	0	0
$2\delta[n-4]$	0	0	0	0	0	0	0	2	0	0	0
$x[n]$	0	0	0	2	4	6	4	2	0	0	0

$$x[n] = \sum_k x[k]\delta[n-k]$$ ← **This formula ALWAYS works**

$$= \ldots + x[-1]\delta[n+1] + x[0]\delta[n] + x[1]\delta[n-1] + \ldots \quad (5.3.6)$$

4-pt AVERAGER

- CAUSAL SYSTEM: USE PAST VALUES
  - y[n] = (x[n]+x[n-1]+x[n-2]+x[n-3])/4
- INPUT = UNIT IMPULSE SIGNAL = **δ[n]**
  - x[n] = **δ[n]**
  - **y[n] = 0.25 δ[n] + 0.25δ[n-1] + 0.25δ[n-2] + 0.25δ[n-3]**
- OUTPUT is called "IMPULSE RESPONSE"
  - h[n] = {…,0,0,0.25,0.25,0.25,0.25,0,0,…}

FIR IMPULSE RESPONSE

- Convolution = Filter Definition
  - Filter Coeffs = Impulse Response

n	$n<0$	0	1	2	3	$\ldots$	M	$M+1$	$n>M+1$
$x[n]=\delta[n]$	0	1	0	0	0	0	0	0	0
$y[n]=h[n]$	0	b_0	b_1	b_2	b_3	$\ldots$	b_M	0	0

$$y[n]=\sum_{k=0}^{M} b_k x[n-k]$$

$$y[n]=\sum_{k=0}^{M} h[k]x[n-k]$$

CONVOLUTION