EE-2025 Spring-2001

Lecture 10

Linearity & Time-Invariance

16-February-2001

Info: Web-CT, Lab, HW

- **Honor Code and Lab Reports !!!**
- Lab Quiz next week during Lab #6
  - MATLAB questions about the Labs
- Labs #5 and #6: Image Processing
  - Sampling, then Filtering: Blurring & Sharpening (De-Blur)
- Prob Set #7 posted this weekend
- Quiz #2 on 2-March
  - Prob Sets #3, #4, #5, #6 and #7

READING ASSIGNMENTS

- This Lecture:
  - Chapter 5, pp. 133-152
- Other Reading:
  - Recitation: Ch. 5, pp. 127-133, 142-146
    - CONVOLUTION
  - Next Lecture: Chapter 6, start

LECTURE OBJECTIVES

- BLOCK DIAGRAM REPRESENTATION
  - Components for Hardware
  - Connect Simple Filters Together to Build More Complicated Systems
- GENERAL PROPERTIES of FILTERS
  - LINEARITY
  - TIME-INVARIANCE
  - ==> CONVOLUTION

LTI SYSTEMS

OVERVIEW

- IMPULSE RESPONSE, h[n]
  - FIR case: same as $\{b_k\}$
- CONVOLUTION
  - GENERAL: y[n] = x[n]*h[n]
- GENERAL CLASS of SYSTEMS
  - LINEAR and TIME-INVARIANT
- ALL LTI have h[n] & use convolution !

GENERAL FIR FILTER

- FILTER COEFFICIENTS $\{b_k\}$
  - DEFINE THE FILTER

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

  - For example, $\{b_k\} = \{3, -1, 2, 1\}$

$$y[n] = \sum_{k=0}^{3} b_k x[n-k]$$

$$= 3x[n] - x[n-1] + 2x[n-2] + x[n-3]$$

MATLAB for FIR FILTER

- `yy = conv(bb,xx)`
  - VECTOR **bb** contains Filter Coefficients
    - For example bb = [3,-1,2,1];
  - DSP-First: `yy = firfilt(bb,xx)`
- FILTER COEFFICIENTS $\{b_k\}$

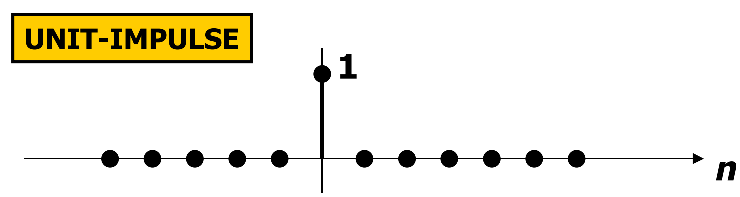
$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

conv2()
for images

The Unit Impulse Signal

- x[n] has only one NON-ZERO VALUE

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$



FIR IMPULSE RESPONSE

- Simply let x[n] = δ[n], then y[n] = h[n]
  - Filter Coeffs = Impulse Response

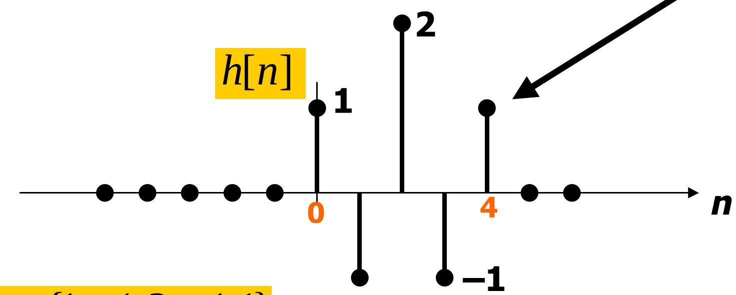
n	$n<0$	0	1	2	3	...	M	$M+1$	$n>M+1$
$x[n]=\delta[n]$	0	1	0	0	0	0	0	0	0
$y[n]=h[n]$	0	b_0	b_1	b_2	b_3	...	b_M	0	0

$$h[n]=\sum_{k=0}^{M} b_k \delta[n-k]$$

MATH FORMULA for h[n]

- Use SHIFTED IMPULSES to write h[n]

$$h[n]=\delta[n]-\delta[n-1]+2\delta[n-2]-\delta[n-3]+\delta[n-4]$$



$$\{b_k\}=\{1,-1,2,-1,1\}$$

LTI: Convolution Sum

- **Output = Convolution of x[n] & h[n]**
  - NOTATION: y[n] = x[n]*h[n]
  - Here is the FIR case:

$$y[n]=\sum_{k=0}^{M} h[k]x[n-k]$$

FINITE LIMITS

FINITE LIMITS

Same as b_k

CONVOLUTION Example

$$h[n]=\delta[n]-\delta[n-1]+2\delta[n-2]-\delta[n-3]+\delta[n-4]$$

$$x[n]=u[n]$$

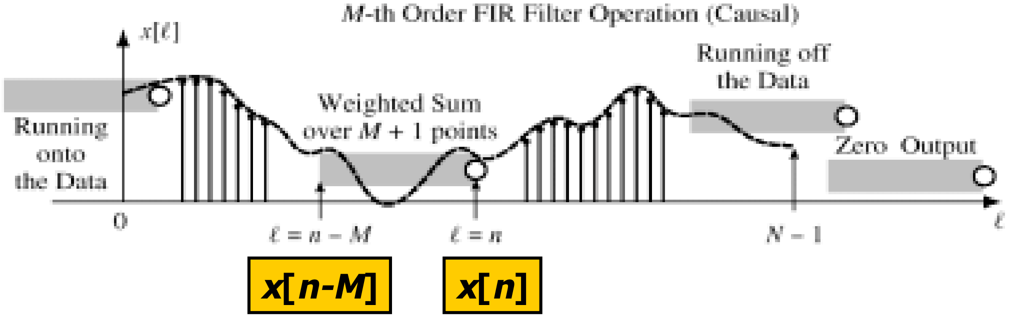
n	−1	0	1	2	3	4	5	6	7
$x[n]$	0	1	1	1	1	1	1	1	...
$h[n]$	0	1	−1	2	−1	1	0	0	0
$h[0]x[n]$	0	1	1	1	1	1	1	1	1
$h[1]x[n-1]$	0	0	−1	−1	−1	−1	−1	−1	−1
$h[2]x[n-2]$	0	0	0	2	2	2	2	2	2
$h[3]x[n-3]$	0	0	0	0	−1	−1	−1	−1	−1
$h[4]x[n-4]$	0	0	0	0	0	1	1	1	1
$y[n]$	0	1	0	2	1	2	2	2	...

GENERAL FIR FILTER

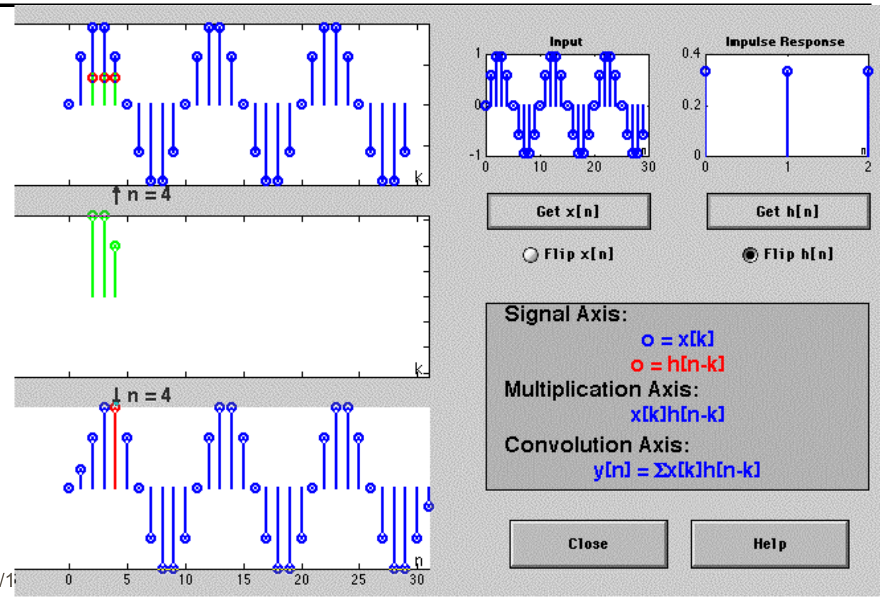
- SLIDE a Length-L WINDOW over x[n]

$$y[n] = \sum_{k=0}^{M} b_k\, x[n-k]$$

M-th Order FIR Filter Operation (Causal)



CONVDEMO: MATLAB GUI



HARDWARE STRUCTURES

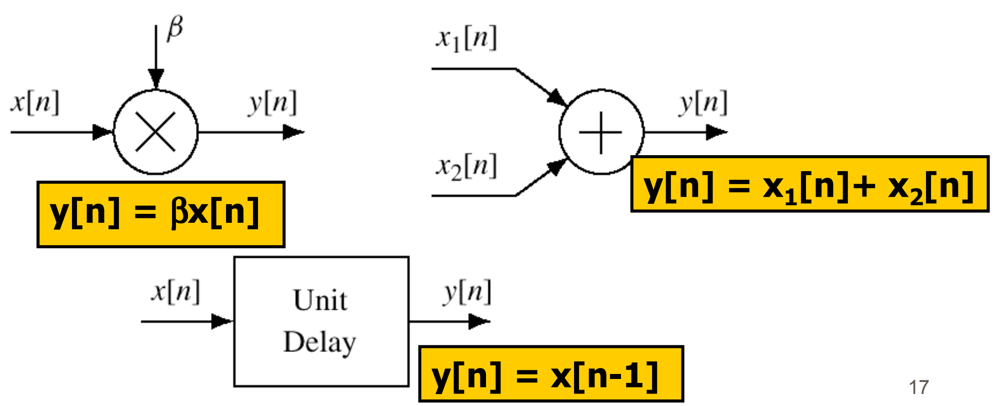
$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

- INTERNAL STRUCTURE of “FILTER”
  - WHAT COMPONENTS ARE NEEDED?
  - HOW DO WE “HOOK” THEM TOGETHER?
- SIGNAL FLOW GRAPH NOTATION

HARDWARE ATOMS

- Add, Multiply & Store

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$



FIR STRUCTURE

- Direct Form

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

SIGNAL FLOW GRAPH

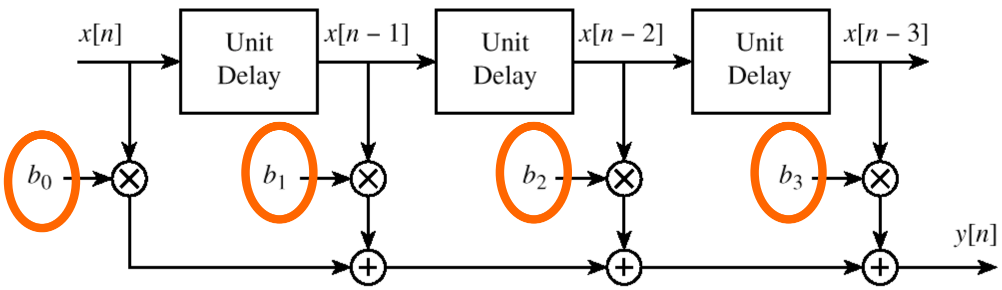


Figure 5.13 Block-diagram structure for the Mth order FIR filter.

SYSTEM PROPERTIES

- MATHEMATICAL DESCRIPTION
- TIME-INVARIANCE
- LINEARITY
- CAUSALITY
  - "No output prior to input"

TIME-INVARIANCE

- IDEA:
  - "Time-Shifting the input will cause the same time-shift in the output"
- EQUIVALENTLY,
  - We can prove that
    - The time origin (n=0) is picked arbitrary

TESTING Time-Invariance

$$w[n] = y[n - n_0]$$

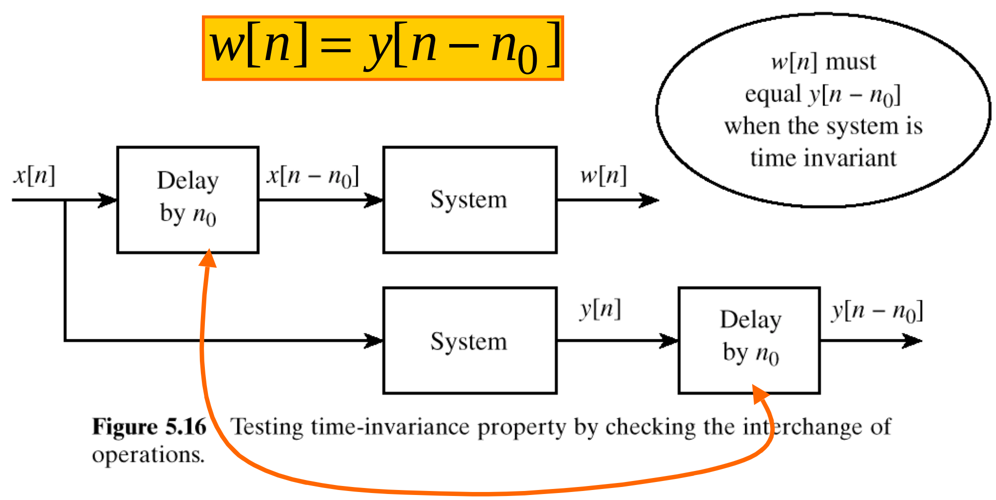


Figure 5.16 Testing time-invariance property by checking the interchange of operations.

LINEAR SYSTEM

- LINEARITY = Two Properties
- SCALING
  - "Doubling x[n] will double y[n]"
- SUPERPOSITION:
  - "Adding two inputs gives an output that is the sum of the individual outputs"

TESTING LINEARITY

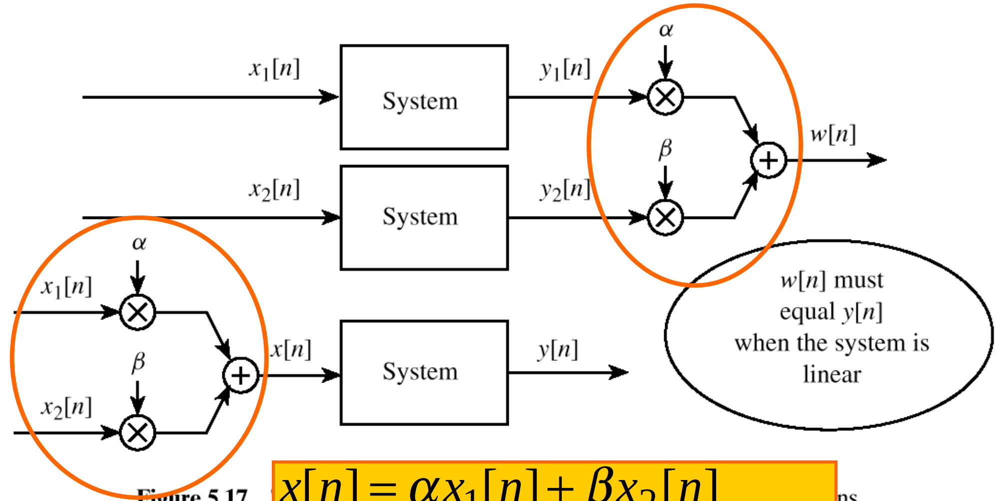


Figure 5.17 ... ns.

$$x[n] = \alpha x_1[n] + \beta x_2[n]$$

$$\rightarrow y[n] = \alpha y_1[n] + \beta y_2[n]$$

FIR Filters are Linear

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

$$y[n] = \sum_{k=0}^{M} b_k \left(\alpha x_1[n-k] + \beta x_2[n-k]\right)$$

$$y[n] = \alpha\left(\sum_{k=0}^{M} b_k x_1[n-k]\right) + \beta\left(\sum_{k=0}^{M} b_k x_2[n-k]\right)$$

$$y[n] = \alpha y_1[n] + \beta y_2[n]$$

$$\Rightarrow \quad \text{Linear}$$

FIR Filters are Time-Invariant

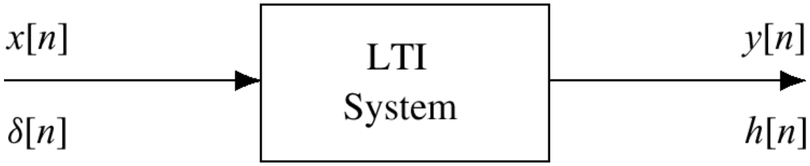
$$x_1[n] = x[n-n_0]$$

$$w[n] = \sum_{k=0}^{M} b_k x_1[n-k] = \sum_{k=0}^{M} b_k x[(n-k)-n_0]$$

$$w[n] = \sum_{k=0}^{M} b_k x[(n-n_0)-k] = y[n-n_0]$$

$$\Rightarrow \quad \text{Time - invariant}$$

LTI (Linear & Time Invariant) SYSTEMS



- COMPLETELY CHARACTERIZED by h[n]

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$$

- FIR Example: h[n] is same as b_k

$$y[n] = x[n] * h[n] = \sum_{k=0}^{M} b_k x[n-k] = \sum_{k=0}^{M} h[k]x[n-k]$$

POP QUIZ

- FIR Filter is "FIRST DIFFERENCE"
  - y[n] = x[n] - x[n-1]
- Write output as a convolution
  - Need impulse response
  - Then, anoth $h[n] = \delta[n] - \delta[n-1]$ utput:

$$y[n] = (\delta[n] - \delta[n-1]) * x[n]$$

CASCADE SYSTEMS

- Does the order of S_1 & S_2 matter?
  - NO, LTI SYSTEMS can be rearranged !!!
  - WHAT ARE THE FILTER COEFFS? $\{b_k\}$

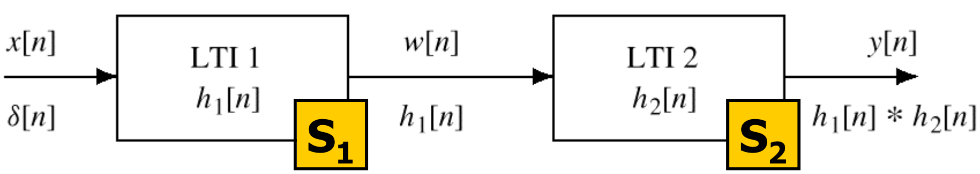


Figure 5.19 A Cascade of Two LTI Systems.

CASCADE EQUIVALENT

- Find "overall" h[n] for a cascade ?

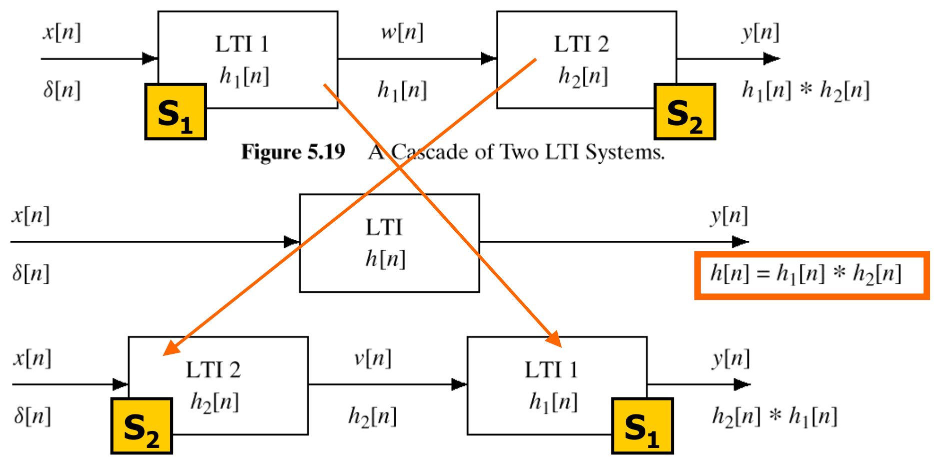


Figure 5.19 A Cascade of Two LTI Systems.

Figure 5.20 Switching the order of cascaded LTI systems.