EE-2025 Spring-2001

Lecture 11

Frequency Response of FIR

19-February-01

Info: Web-CT, Lab, HW

- Lab #6: FIR Filtering of Images
  - **Lab Quiz during Lab #6**
  - **You can consult your lab reports and also use MATLAB during the Lab Quiz**
- Prob Set #6 due this week
- Prob Set #7 posted
- Quiz #2 on 2-March (Friday)

DEBUGGING

- "Any Fool" can write code
- Debugging is the interesting part
  - It takes talent !!!
- HOWEVER,
  - Assume the **stupid** mistake is the problem

LECTURE

READING ASSIGNMENTS

- This Lecture:
  - Chapter 6, pp. 157-165, 169-176
- Other Reading:
  - Recitation: Ch. 6, pp. 176-188
    - FREQUENCY RESPONSE EXAMPLES
  - Next Lecture: Chapter 6, pp. 188-194

LECTURE OBJECTIVES

- SINUSOIDAL INPUT SIGNAL
  - DETERMINE the FIR FILTER OUTPUT
- FREQUENCY RESPONSE of FIR
  - PLOTTING vs. Frequency
  - MAGNITUDE vs. Freq
  - PHASE vs. Freq

MAG

PHASE

$$H(\hat{\omega}) = |H(\hat{\omega})| e^{j\varphi(\hat{\omega})}$$

DOMAINS: Time & Frequency

- **Time-Domain: "n" = time**
  - x[n] discrete-time signal
  - x(t)
- **Frequency Domain (sum of sinusoids)**
  - Spectrum vs. f (Hz)
    - ANALOG vs. DIGITAL
  - Spectrum vs. omega-hat
- Move back and forth QUICKLY

LTI SYSTEMS

- LTI: Linear & Time-Invariant
- COMPLETELY CHARACTERIZED by:
  - **IMPULSE RESPONSE** h[n]
  - **CONVOLUTION**: y[n] = x[n]*h[n]
    - The "rule"defining the system can ALWAYS be re-written as convolution
- FIR Example: h[n] is same as b_k

Example

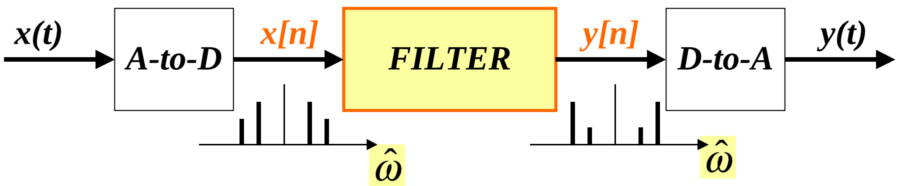
- FIR Filter is "FIRST DIFFERENCE"
  - y[n] = x[n] - x[n-1]
- Write output as a convolution
  - Need impulse response

$$h[n] = \delta[n] - \delta[n-1]$$

  - Then, another way to compute the output is :

$$\begin{aligned} y[n] &= (\delta[n] - \delta[n-1]) * x[n] \\ &= \delta[n] * x[n] - \delta[n-1] * x[n] = x[n] - x[n-1] \end{aligned}$$

DIGITAL "FILTERING"



- CONCENTRATE on the **SPECTRUM**
- SINUSOIDAL INPUT
  - INPUT x[n] = SUM of SINUSOIDS
  - Then, OUTPUT y[n] = SUM of SINUSOIDS

GENERAL FIR FILTER

- FILTER COEFFICIENTS $\{b_k\}$
  - DEFINE THE FILTER

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

  - For example, $\{b_k\} = \{3, -1, 2, 1\}$

$$y[n] = \sum_{k=0}^{3} b_k x[n-k]$$

$$= 3x[n] - x[n-1] + 2x[n-2] + x[n-3]$$

FILTERING EXAMPLE

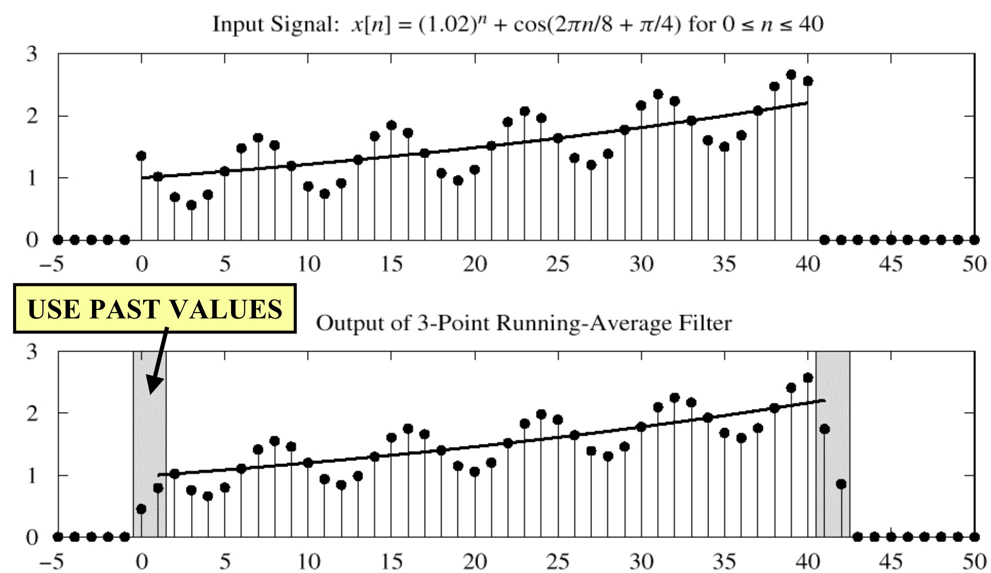
- 7-point AVERAGER
  - Removes cosine
    - By making its amplitude (A) smaller

$$y_7[n] = \sum_{k=0}^{6} (\tfrac{1}{7}) x[n-k]$$

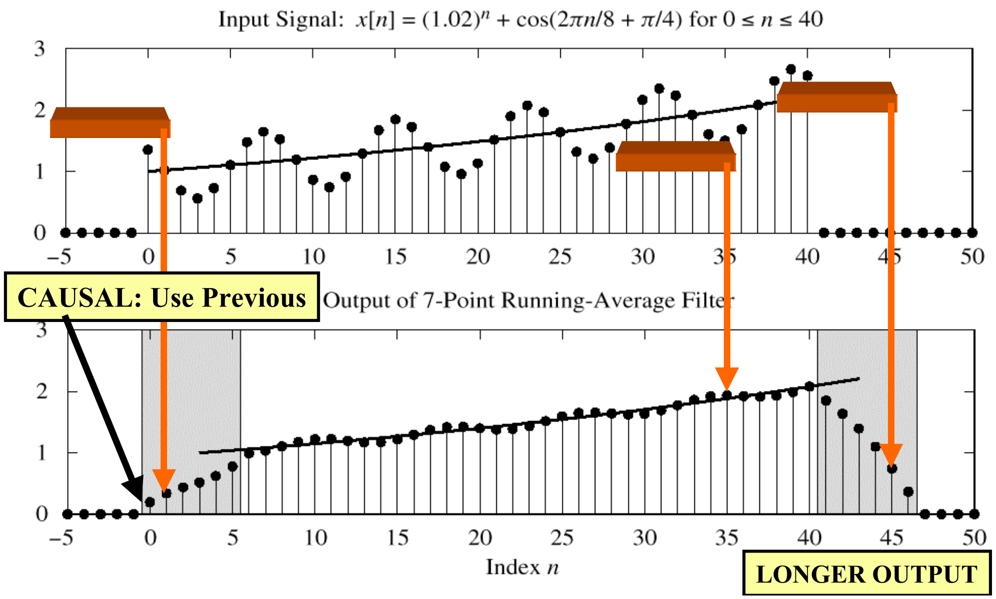
- 3-point AVERAGER
  - Changes A slightly

$$y_3[n] = \sum_{k=0}^{2} (\tfrac{1}{3}) x[n-k]$$

3-pt AVG EXAMPLE



7-pt FIR EXAMPLE (AVG)



SINUSOIDAL RESPONSE

- INPUT: x[n] = SINUSOID
- OUTPUT: y[n] will also be a SINUSOID
  - Different Amplitude and Phase
  - **SAME** Frequency
- AMPLITUDE & PHASE CHANGE
  - Called the **FREQUENCY RESPONSE**

COMPLEX EXPONENTIAL

$$x[n] = Ae^{j\phi}e^{j\hat{\omega}n} \qquad -\infty < n < \infty$$

x[n] is the input signal—a complex exponential

$$y[n] = \sum_{k=0}^{M} b_k x[n-k] = \sum_{k=0}^{M} h[k]x[n-k]$$

FIR DIFFERENCE EQUATION

COMPLEX EXP OUTPUT

- Use the FIR "Difference Equation"

$$y[n] = \sum_{k=0}^{M} b_k x[n-k] = \sum_{k=0}^{M} b_k Ae^{j\phi}e^{j\hat{\omega}(n-k)}$$

$$= \left(\sum_{k=0}^{M} b_k e^{j\hat{\omega}(-k)} \right) Ae^{j\phi}e^{j\hat{\omega}n}$$

$$= H(\hat{\omega})Ae^{j\phi}e^{j\hat{\omega}n}$$

FREQUENCY REPONSE

- At each frequency, we can DEFINE

$$H(\hat{\omega}) = \sum_{k=0}^{M} b_k e^{-j\hat{\omega}k}$$

← **FREQUENCY RESPONSE**

- Complex-valued formula
  - Has MAGNITUDE vs. frequency
  - And PHASE vs. frequency

$$H(\hat{\omega}) = |H(\hat{\omega})| e^{j\angle H(\hat{\omega})}$$

EXAMPLE 6.1

Example 6.1

$$\{b_k\} = \{1,\ 2,\ 1\}$$

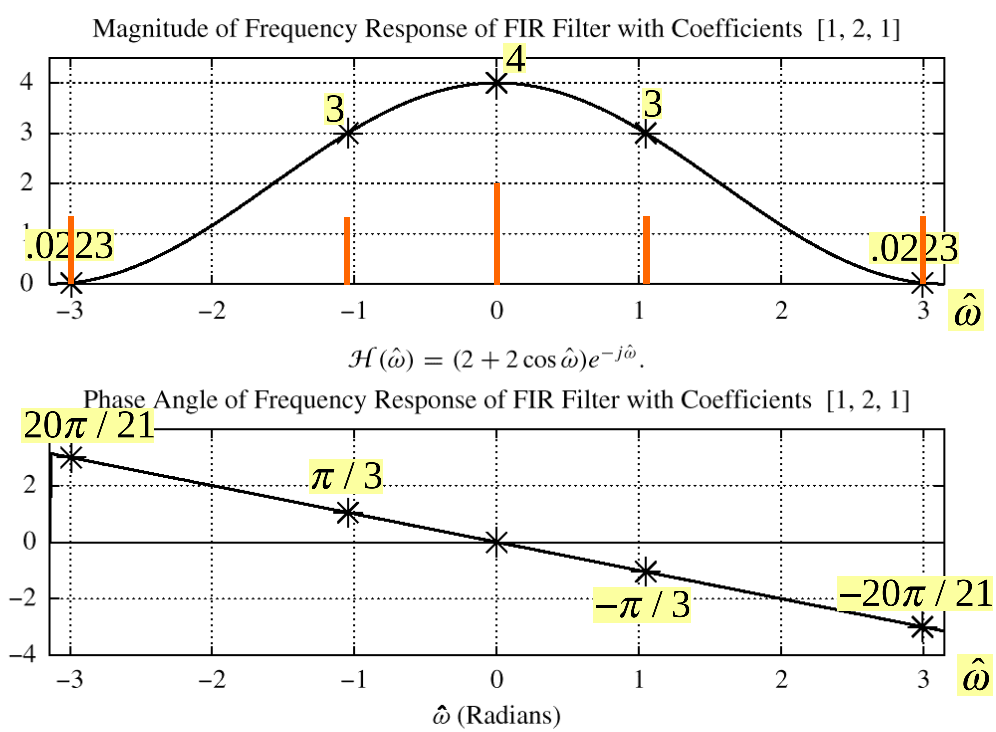
$$\mathcal{H}(\hat{\omega}) = 1 + 2e^{-j\hat{\omega}} + e^{-j\hat{\omega}2}$$

To obtain formulas for the magnitude and phase of the frequency response

$$\begin{aligned}\mathcal{H}(\hat{\omega}) &= 1 + 2e^{-j\hat{\omega}} + e^{-j\hat{\omega}2} \\ &= e^{-j\hat{\omega}}\left(e^{j\hat{\omega}} + 2 + e^{-j\hat{\omega}}\right) \\ &= e^{-j\hat{\omega}}\left(2 + 2\cos\hat{\omega}\right)\end{aligned}$$

EXPLOIT SYMMETRY

Since $(2 + 2\cos\hat{\omega}) \geq 0$ for frequencies $-\pi < \hat{\omega} \leq \pi$, the magnitude is $|\mathcal{H}(\hat{\omega})| = (2 + 2\cos\hat{\omega})$ and the phase is $\angle\mathcal{H}(\hat{\omega}) = -\hat{\omega}$.



Using the Frequency Response to Find the Ouput

- Consider the input signal

$$x[n] = 4 + 3\cos\left(\frac{\pi}{3}n - \frac{\pi}{2}\right) + 3\cos\left(\frac{20\pi}{21}n\right)$$

- The corresponding output is

$$y[n] = 4\cdot 4 + 3\cdot 3\cos\left(\frac{\pi}{3}n - \frac{\pi}{3} - \frac{\pi}{2}\right)$$

$$+ 3\cdot 0.0223\cos\left(\frac{20\pi}{21}n - \frac{20\pi}{21}\right)$$

- Thus the third term is "removed" by the filter.

MATLAB: FREQUENCY RESPONSE

- HH = freqz(bb,1,ww)
  - VECTOR bb contains Filter Coefficients
  - DSP-First: HH = freekz(bb,1,ww)
- FILTER COEFFICIENTS $\{b_k\}$

$$H(\hat{\omega}) = \sum_{k=0}^{M} b_k e^{-j\hat{\omega}k}$$

LTI SYSTEMS

- LTI: Linear & Time-Invariant
- COMPLETELY CHARACTERIZED by:
  - FREQUENCY RESPONSE, or
  - IMPULSE RESPONSE h[n]
- Sinusoid IN -----> Sinusoid OUT
  - At the SAME Frequency

Time & Frequency Relation

- Get Frequency Response from Diff. Eq.

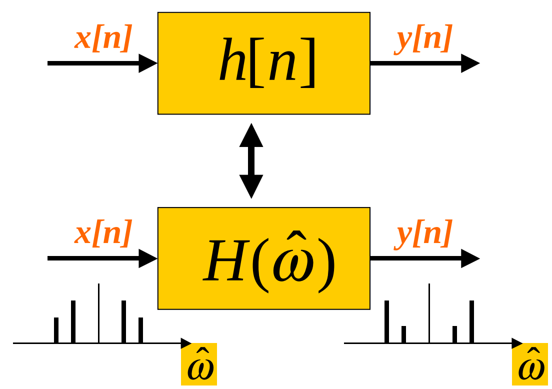
$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

$$H(\hat{\omega}) = \sum_{k=0}^{M} b_k e^{-j\hat{\omega}k} = \sum_{k=0}^{M} h[k] e^{-j\hat{\omega}k}$$

IMPULSE RESPONSE

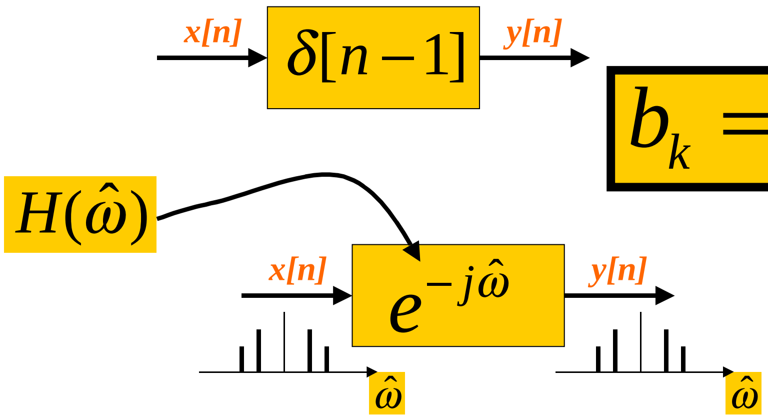
BLOCK DIAGRAMS

- Equivalent Representations



UNIT-DELAY SYSTEM

Find $h[n]$ and $H(\hat{\omega})$ for $y[n] = x[n-1]$

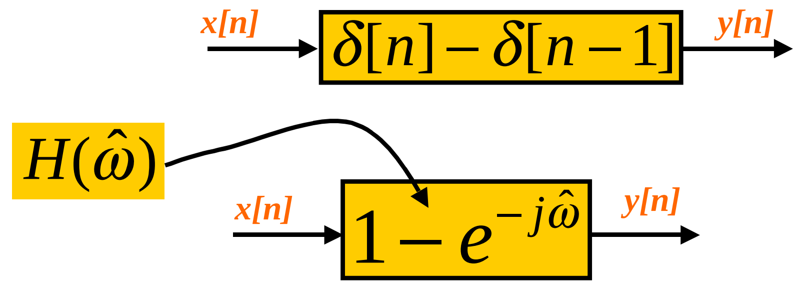


$$b_k = \{0,1\}$$

FIRST DIFFERENCE SYSTEM

Find $h[n]$ and $H(\hat{\omega})$ for the Diff. Eqn:

$$y[n] = x[n] - x[n-1]$$



CASCADE SYSTEMS

- Does the order of S_1 & S_2 matter?
  - NO, LTI SYSTEMS can be rearranged !!!
  - WHAT ARE THE FILTER COEFFS? $\{b_k\}$
  - WHAT is the overall FREQUENCY RESPONSE ?

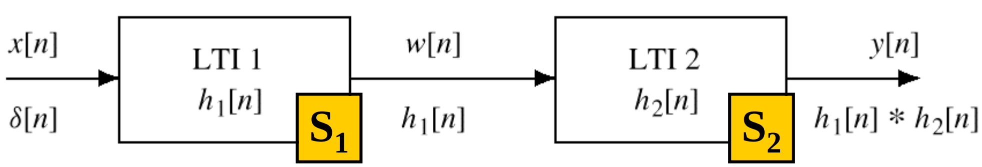
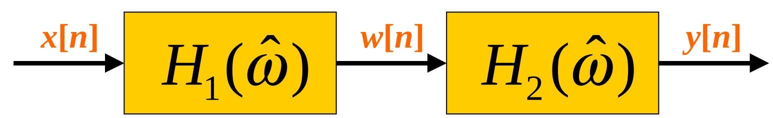


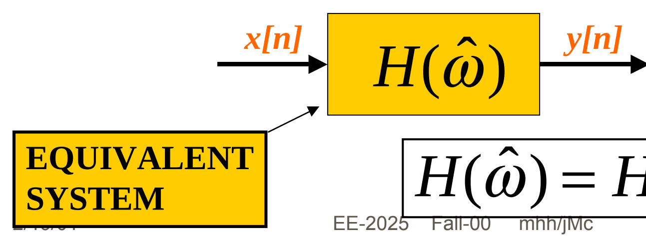
Figure 5.19 A Cascade of Two LTI Systems.

CASCADE EQUIVALENT



$$w[n] = H_1(\hat{\omega})Ae^{j\phi}e^{j\hat{\omega}n}$$

$$y[n] = H_2(\hat{\omega})H_1(\hat{\omega})Ae^{j\phi}e^{j\hat{\omega}n}$$



$$H(\hat{\omega}) = H_1(\hat{\omega})H_2(\hat{\omega})$$