EE-2025 Spring 2001

Lecture 12

Digital Filtering of Analog Signals

23-February-01

Info: Web-CT, Lab, HW

- Quiz #2 on 2-March (Friday)
  - Coverage: HW #3, #4, #5, #6, and #7
  - Solutions to #7 posted at 6pm on Friday
- Do we need a review session on March 1?
- MATLAB Help on Monday & Tuesday
  - 6 PM, VL-361

COURSE OBJECTIVE

- Students will be able to:
- Understand **mathematical** descriptions of signal processing **algorithms** and express those algorithms as computer **implementations** (MATLAB)

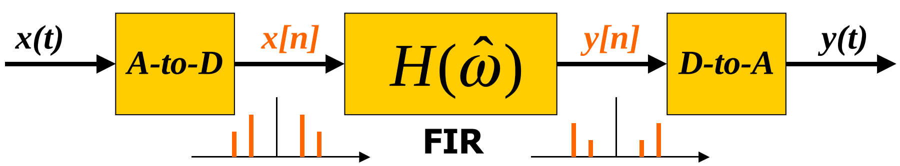
LECTURE

READING ASSIGNMENTS

- This Lecture:
  - Chapter 6, pp. 188-194
- Other Reading:
  - Recitation: Ch. 6, pp. 176-188
    - FREQUENCY RESPONSE EXAMPLES
  - Next Lecture: Chapter 7, start

LECTURE OBJECTIVES

- Two Domains: Time & Frequency
- Track the spectrum of x[n] thru an FIR Filter.
- **UNIFICATION**: How does Frequency Response affect x(t) to produce y(t) ?



PREVIOUS LECTURE REVIEW

- SINUSOIDAL INPUT SIGNAL
  - OUTPUT has SAME FREQUENCY
  - DIFFERENT Amplitude and Phase
- FREQUENCY RESPONSE of FIR
  - MAGNITUDE vs. Frequency
  - PHASE vs. Freq
  - PLOTTING:

MAG

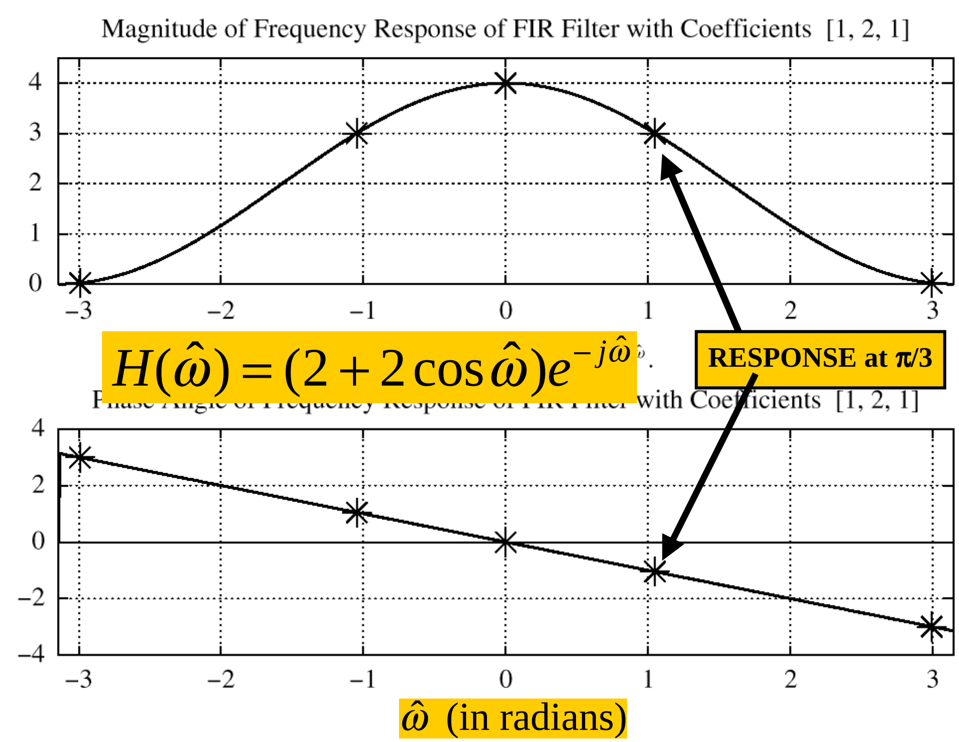
PHASE

$$H(\hat{\omega}) = |H(\hat{\omega})| e^{j\angle H(\hat{\omega})}$$

FREQ. RESPONSE PLOTS

- DENSE GRID **(ww)** from $-\pi$ to $+\pi$
  - `ww = -pi:(pi/100):pi;`
- `yy = freqz(bb,1,ww)`
  - VECTOR **bb** contains Filter Coefficients
  - DSP-First: `yy = freekz(bb,1,ww)`

$$\mathcal{H}(\hat{\omega}) = \sum_{k=0}^{M} b_k e^{-j\hat{\omega}k}$$



TIME & FREQ DOMAINS

- LTI: Linear & Time-Invariant
  - COMPLETELY CHARACTERIZED by:
    - IMPULSE RESPONSE h[n] (time domain)
    - FREQUENCY RESPONSE

$$x[n] \rightarrow \boxed{H(\hat{\omega}) = |H(\hat{\omega})| e^{j\angle H(\hat{\omega})}} \rightarrow y[n]$$

- Two DOMAINS: time & frequency
  - Go back and forth QUICKLY

TIME & FREQUENCY

$$y[n] = \sum_{k=0}^{M} b_k x[n-k] = \sum_{k=0}^{M} h[k] x[n-k]$$

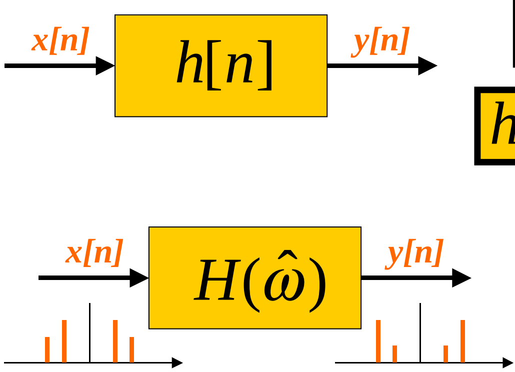
FIR DIFFERENCE EQUATION is the TIME-DOMAIN

$$H(\hat{\omega}) = \sum_{k=0}^{M} h[k] e^{-j\hat{\omega}k}$$

$$H(\hat{\omega}) = h[0]e^{-j0} + h[1]e^{-j\hat{\omega}} + h[2]e^{-j\hat{\omega}2} + h[3]e^{-j\hat{\omega}3} + \ldots$$

Ex: DELAY by 2 SYSTEM

Find $h[n]$ and $H(\hat{\omega})$ for $y[n] = x[n-2]$

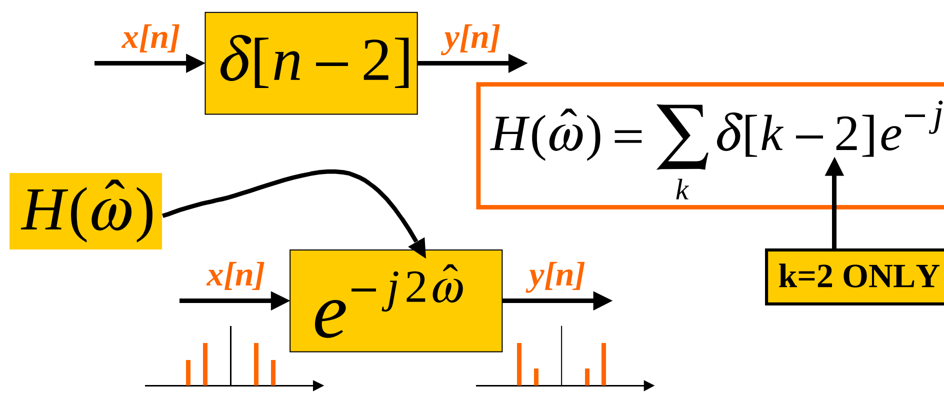


$$b_k = \{0,0,1\}$$

$$h[n] = \delta[n-2]$$

DELAY by 2 SYSTEM

Find $h[n]$ and $H(\hat{\omega})$ for $y[n] = x[n-2]$



$$H(\hat{\omega}) = \sum_{k} \delta[k-2] e^{-j\hat{\omega}k}$$

GENERAL DELAY PROPERTY

Find $h[n]$ and $H(\hat{\omega})$ for $y[n] = x[n - n_d]$

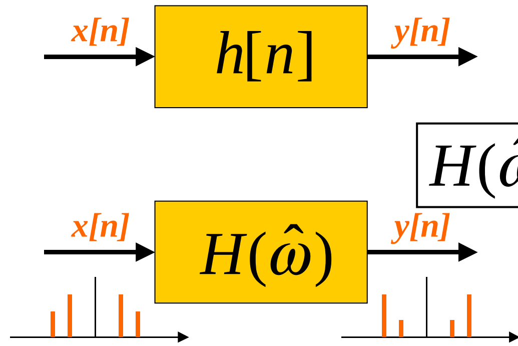
$h[n] = \delta[n - n_d]$ $\Rightarrow x[n] * \delta[n - n_d] = x[n - n_d]$

$$H(\hat{\omega}) = \sum_k \delta[k - n_d] e^{-j\hat{\omega}k} = e^{-j\hat{\omega}n_d}$$

ONLY ONE non-ZERO TERM for k at k = n_d

FREQ DOMAIN --> TIME ??

- START with $H(\hat{\omega})$ and find $h[n]$ or b_k



$h[n] = ?$

$H(\hat{\omega}) = 7e^{-j2\hat{\omega}} \cos(\hat{\omega})$

FREQ DOMAIN --> TIME

$H(\hat{\omega}) = 7e^{-j2\hat{\omega}} \cos(\hat{\omega})$

EULER's Formula

$$= 7e^{-j2\hat{\omega}}(0.5e^{j\hat{\omega}} + 0.5\,e^{-j\hat{\omega}})$$

$$= (3.5e^{-j\hat{\omega}} + 3.5e^{-j3\hat{\omega}})$$

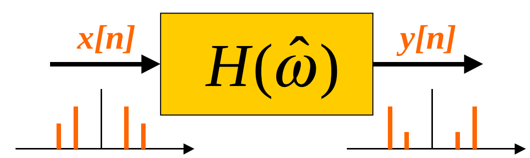
$$h[n] = 3.5\delta[n-1] + 3.5\delta[n-3]$$

$$b_k = \{0,\ 3.5,\ 0, 3.5\}$$

$$y[n] = 3.5x[n-1] + 3.5x[n-3]$$

Frequency-Domain Example

Find $y[n]$ when $H(\hat{\omega})$ is known & $x[n] = 2e^{j\pi/4}e^{j(\pi/3)n}$



$$H(\hat{\omega}) = 7\cos(\hat{\omega})\, e^{-j\hat{\omega}2}$$

EXAMPLE 6.2 (answer)

Find $y[n]$ when $x[n]=2e^{j\pi/4}e^{j(\pi/3)n}$

Answer: Eval $H(\hat{\omega})$ at $\hat{\omega}=\pi/3$.

$$H(\hat{\omega})=7\cos(\hat{\omega})e^{-j\hat{\omega}2}$$

$$H(\hat{\omega})=3.5e^{-j2\pi/3} \qquad @\ \hat{\omega}=\pi/3$$

$$y[n]=\left(3.5e^{-j2\pi/3}\right)\times 2e^{j\pi/4}e^{j(\pi/3)n}$$

$$y[n]=7e^{-j5\pi/12}e^{j(\pi/3)n}$$

SINUSOID thru FIR

$$x[n] = X_0 + \sum_{k=1}^{N}\left(\frac{X_k}{2}e^{j\hat{\omega}_k n} + \frac{X_k^*}{2}e^{-j\hat{\omega}_k n}\right)$$

$$= X_0 + \sum_{k=1}^{N}|X_k|\cos(\hat{\omega}_k n + \angle X_k)$$

MULTIPLY MAGS

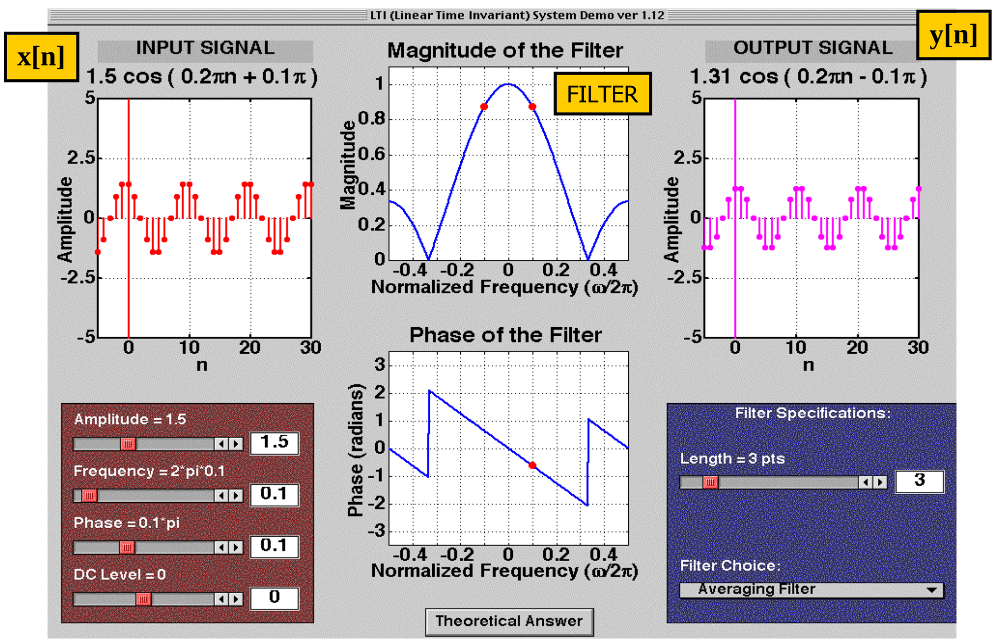
ADD PHASES

if $\mathcal{H}(-\hat{\omega}) = \mathcal{H}^*(\hat{\omega})$, the corresponding output is

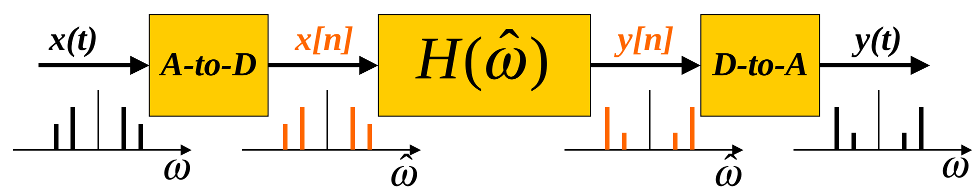
$$y[n] = \mathcal{H}(0)X_0 + \sum_{k=1}^{N}\left(\mathcal{H}(\hat{\omega}_k)\frac{X_k}{2}e^{j\hat{\omega}_k n} + \mathcal{H}(-\hat{\omega}_k)\frac{X_k^*}{2}e^{-j\hat{\omega}_k n}\right)$$

$$= \mathcal{H}(0)X_0 + \sum_{k=1}^{N}|\mathcal{H}(\hat{\omega}_k)||X_k|\cos\left(\hat{\omega}_k n + \angle X_k + \angle\mathcal{H}(\hat{\omega}_k)\right)$$

LTI Demo with Sinusoids

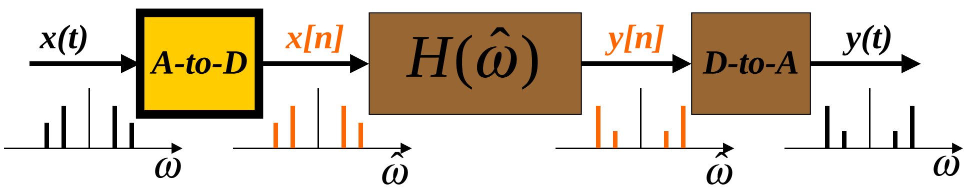


DIGITAL "FILTERING"



- ω SPECTRUM of x(t) (SUM of SINUSOIDS)
- SPECTRUM of x[n]
  - Is ALIASING a PROBLEM ?
- $\hat{\omega}$ SPECTRUM y[n] (FIR Gain or Nulls)
- ω Then, OUTPUT y(t) = SUM of SINUSOIDS

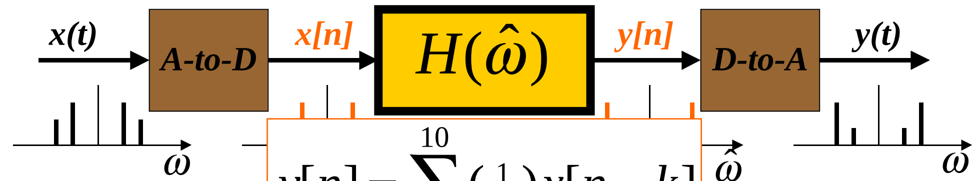
FREQUENCY SCALING



- TIME SAMPLING: $t = nT_s$
- FREQUENCY SCALING. $\hat{\omega} = \omega T_s = \frac{\omega}{f_s}$

If no aliasing, then $-\pi < \hat{\omega} < \pi$

11-pt AVERAGER Example



$$y[n] = \sum_{k=0}^{10} (\tfrac{1}{11}) x[n-k]$$

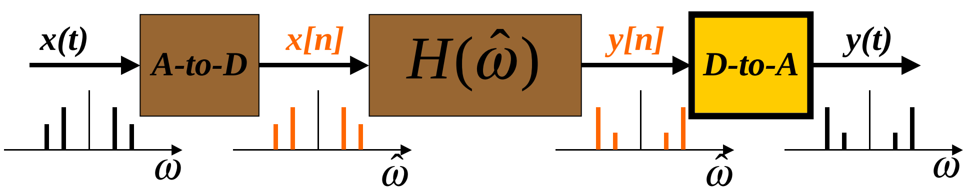
- 250 Hz
- 25 Hz

$$H(\hat{\omega}) = \frac{\sin(\frac{11}{2}\hat{\omega})}{11\sin(\frac{1}{2}\hat{\omega})} e^{-j5\hat{\omega}}$$

?

$$x(t) = \cos(2\pi(25)t) + \cos(2\pi(250)t - \tfrac{1}{2}\pi)$$

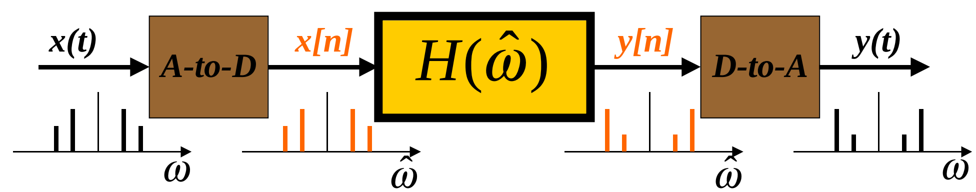
D-A FREQUENCY SCALING



- TIME SAMPLING: $t = nT_s \Rightarrow n \leftarrow t\, f_s$
- RECONSTRUCT up to $0.5f_s$
  - FREQUENCY SCALING

$$\omega = \hat{\omega} f_s$$

TRACK the FREQUENCIES



250 Hz	0.5π	$H(0.5\pi)$	0.5π	250 Hz
25 Hz	$.05\pi$	$H(0.05\pi)$	$.05\pi$	25 Hz

Fs = 1000 Hz

NO new freqs

Work out the equations

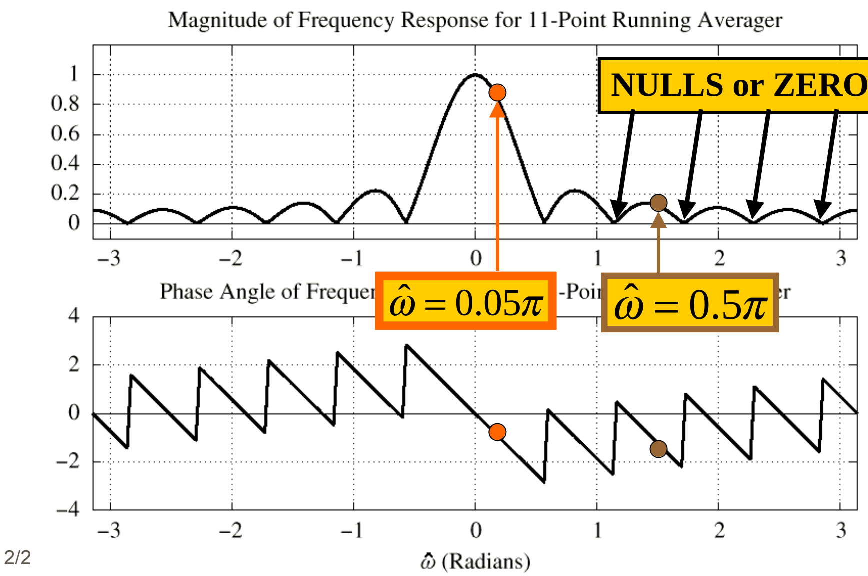
$$x(t) = \cos(2\pi(25)t) + \cos(2\pi(250)t - \tfrac{1}{2}\pi)$$

$$x[n] = \cos(0.05\pi n) + \cos(0.5\pi n - \tfrac{1}{2}\pi)$$

$$y[n] = |H(.05\pi)|\cos(0.05\pi n + \angle H(.05\pi)) + |H(.5\pi)|\cos(0.5\pi n + \angle H(.5\pi) - \tfrac{1}{2}\pi)$$

$$y(t) = |H(.05\pi)|\cos(0.05\pi 1000t + \angle H(.05\pi)) + |H(.5\pi)|\cos(0.5\pi 1000t + \angle H(.5\pi) - \tfrac{1}{2}\pi)$$

11-pt AVERAGER



EVALUATE Freq. Response

$$H(\hat\omega) = \frac{\sin(\hat\omega 11/2)}{11\sin(\hat\omega/2)}e^{-j\hat\omega 5}$$

At $\hat\omega = 0.5\pi$

$$H(\hat\omega) = \frac{\sin((0.5\pi)11/2)}{11\sin(0.5\pi/2)}e^{-j(0.5\pi)5}$$

$$= \frac{\sin(2.75\pi)}{11\sin(0.25\pi)}e^{-j2.5\pi}$$

$$= 0.0909e^{-j0.5\pi}$$

EVALUATE Freq. Response

$$x(t) = \cos(2\pi(25)t) + \sin(2\pi(250)t)$$

evaluating at 25 and 250 Hz.

$f_s = 1000$

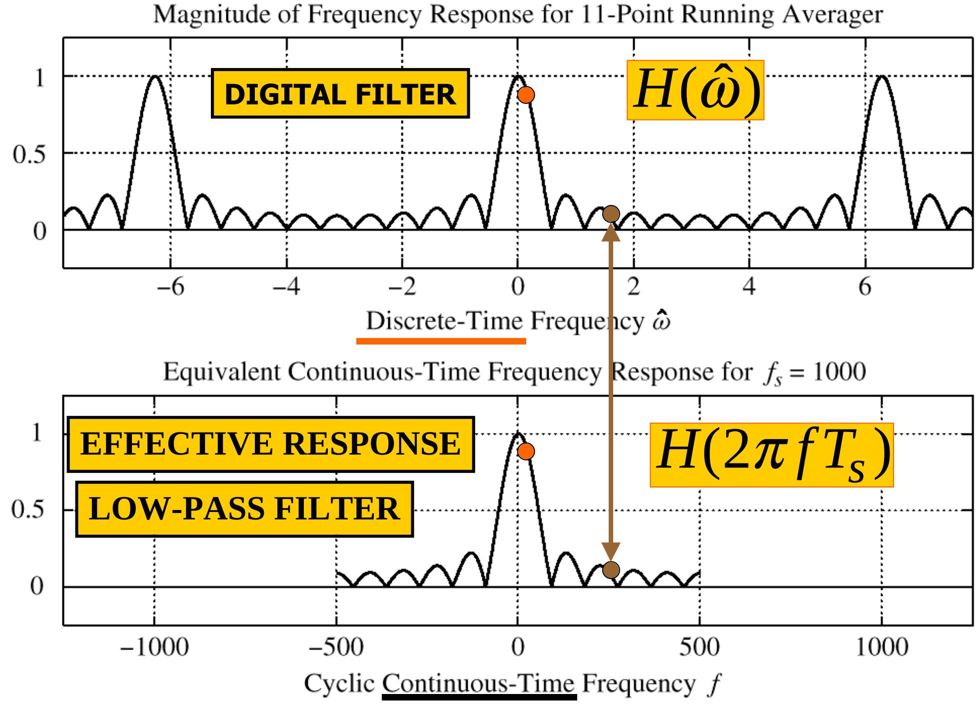
$$\mathcal{H}(2\pi(25)/1000) = \frac{\sin(\pi(25)(11)/1000)}{11\sin(\pi(25)/1000)}e^{-j2\pi(25)(5)/1000} = 0.8811e^{-j\pi/4}$$

MAG SCALE

PHASE CHANGE

$$\mathcal{H}(2\pi(250)/1000) = \frac{\sin(\pi(250)(11)/1000)}{11\sin(\pi(250)/1000)}e^{-j2\pi(250)(5)/1000} = 0.0909e^{-j\pi/2}$$

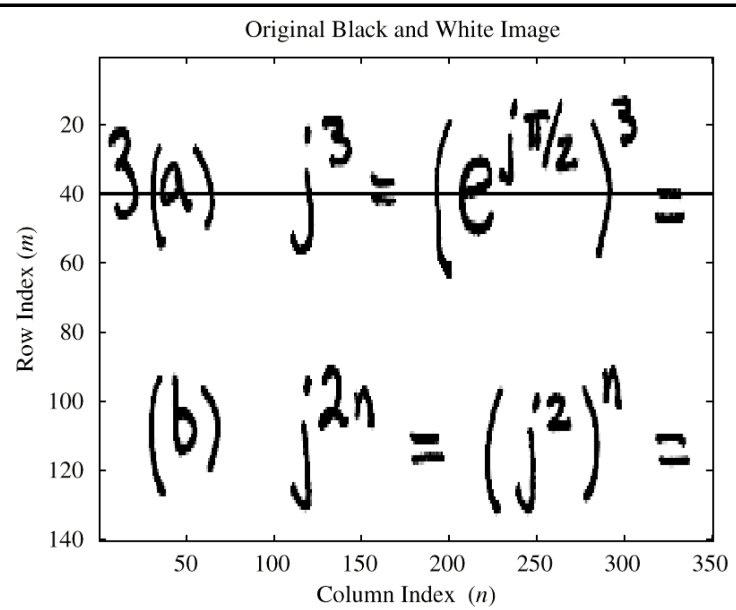
$$y(t) = 0.8811\cos(2\pi(25)t - \pi/4) + 0.0909\sin(2\pi(250)t - \pi/2)$$



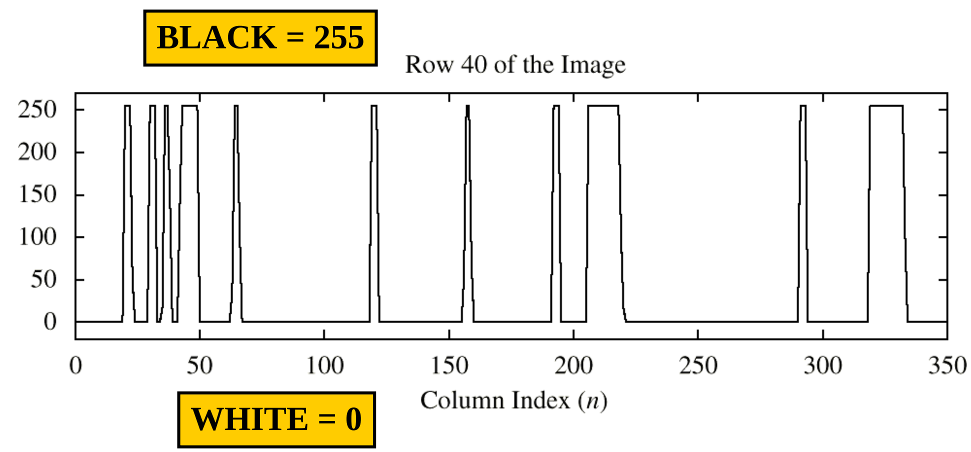
FILTER TYPES

- LOW-PASS FILTER (LPF)
  - BLURRING
  - ATTENUATES HIGH FREQUENCIES
- HIGH-PASS FILTER (HPF)
  - SHARPENING for IMAGES
  - BOOSTS THE HIGHS
  - REMOVES DC
- BAND-PASS FILTER (BPF)

B & W IMAGE

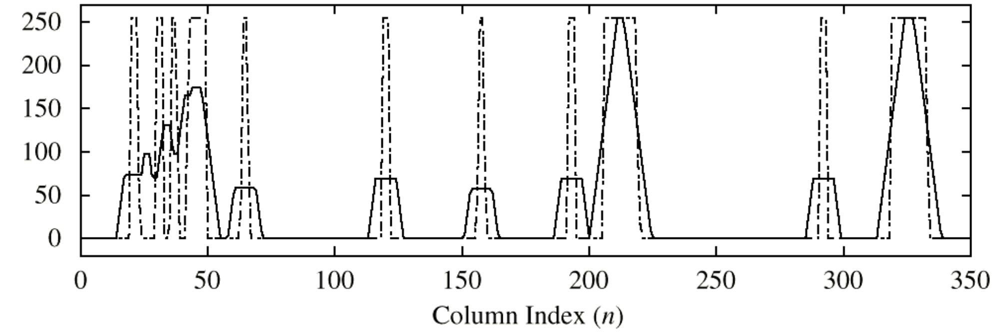


ROW of B&W IMAGE



FILTERED ROW of IMAGE

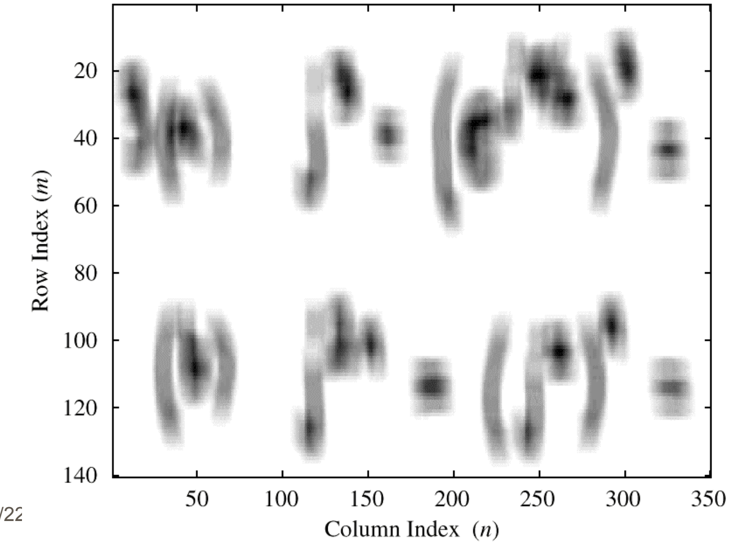
11-Point Averaging: 5-Sample Delay Equalization



ADJUSTED DELAY by 5 samples

FILTERED B&W IMAGE

Row and Column Filtered Image

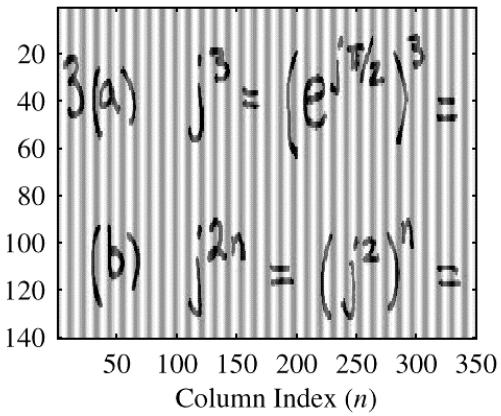


LPF: BLUR

B&W IMAGE with COSINE

FILTERED: 11-pt AVG

Homework plus Cosine



Remove Cosine Stripe with Averaging Fi