

Lecture 15

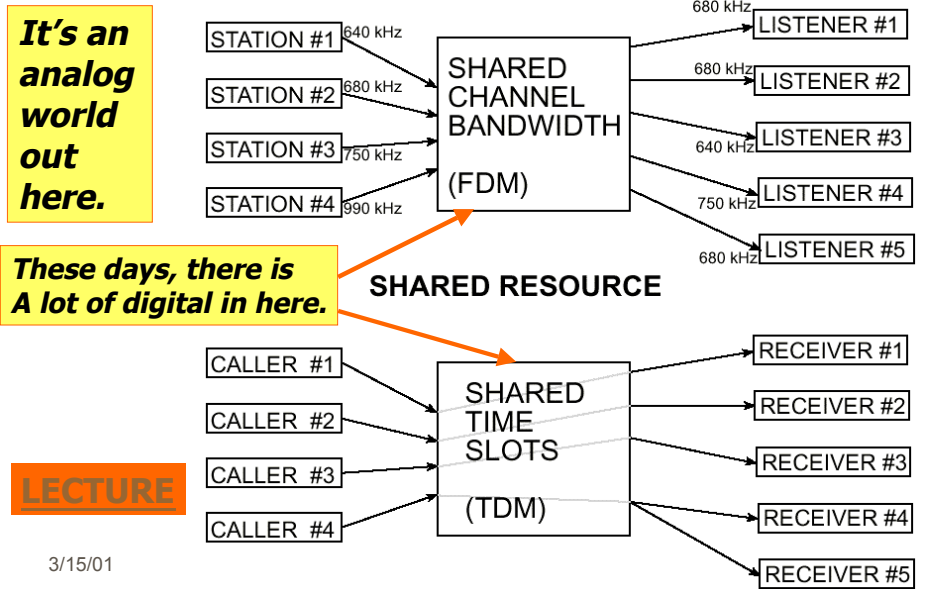
Continuous-Time Signals and Systems

16-March-2001

Info: Web-CT, Lab, HW

- Quiz #2: Average = 80.6, Median = 84
 - Quiz #3 will be 6-April (Friday)
- Get NEW CHAPTERS
 - PDF ("New Chapters") or Bookstore
- Prob Set #8 is due Next week
- Lab #7 is due next week in lab
- Lab #9 will be done in lab next week
- Lab #8 and Prob Set #9 due week of 26-March

The way communication systems work



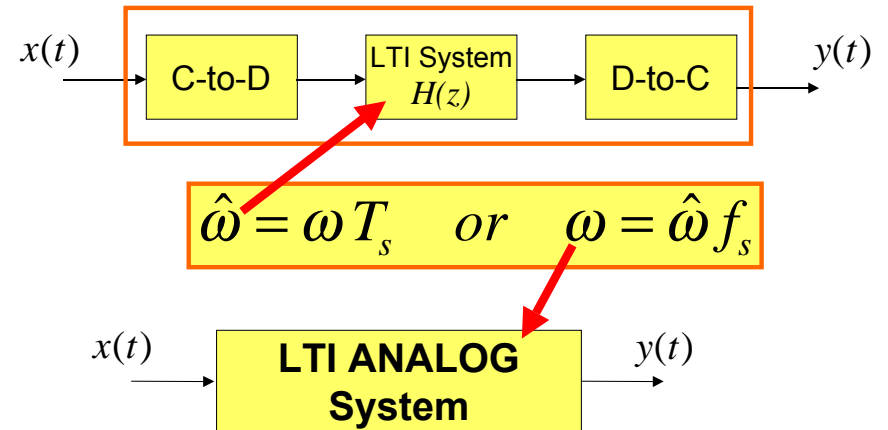
READING ASSIGNMENTS

- This Lecture:
 - Chapter 10, pp. 1000-1022
- Other Reading:
 - Recitation: Ch. 10, pp. 1022-1029
 - Next Lecture: Chapter 10, all

LECTURE OBJECTIVES

- Bye bye to D-T Systems for a while
- The UNIT IMPULSE signal
 - Definition
 - Properties
- Continuous-time signals and systems
 - Example systems
 - Review: **L**inearity and **T**ime-**I**nvariance
 - Convolution integral: **impulse** response

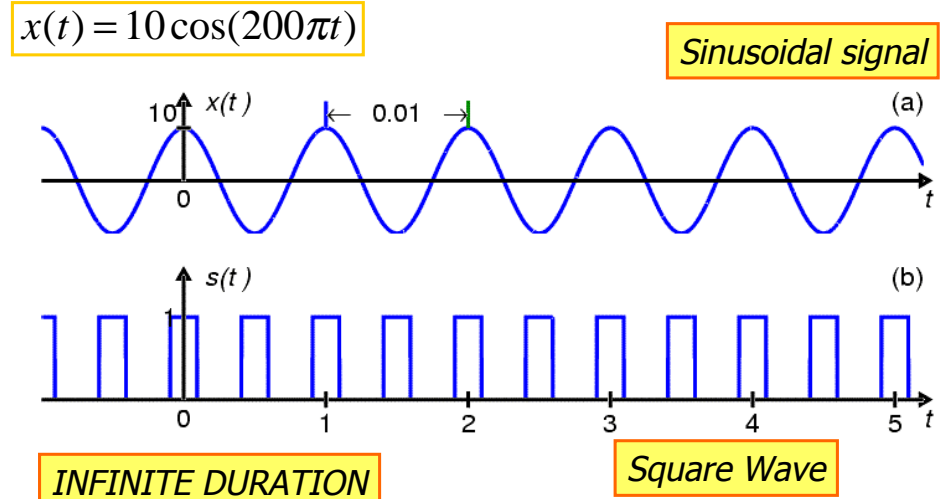
D-T Filtering of C-T Signals



ANALOG SIGNALS $x(t)$

- INFINITE LENGTH
 - SINUSOIDS: $(t = \text{time in secs})$
 - PERIODIC SIGNALS
 - ONE-SIDED, e.g., for $t > 0$
 - UNIT STEP: $u(t)$
 - FINITE LENGTH
 - SQUARE PULSE
 - IMPULSE SIGNAL: $\delta(t)$
- DISCRETE-TIME: $x[n]$ is list of numbers

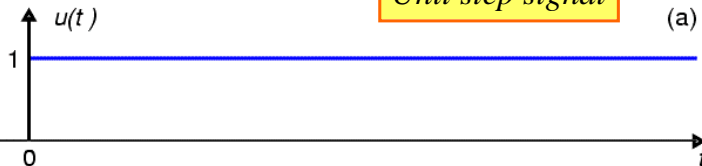
CT Signals: PERIODIC



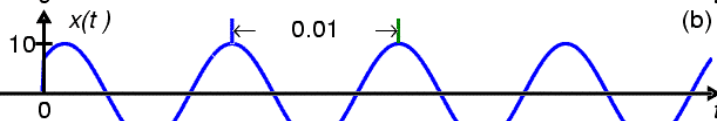
CT Signals: ONE-SIDED

$$u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$$

Unit step signal

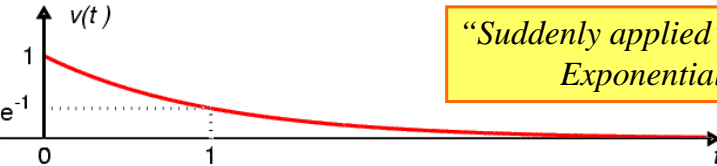


One-Sided Sinusoid



$$v(t) = e^{-t} u(t)$$

"Suddenly applied" Exponential

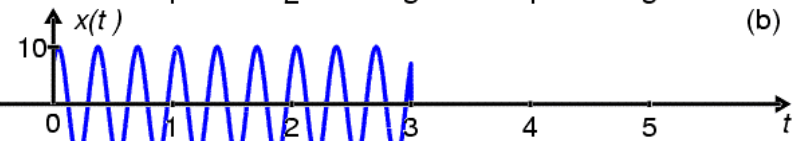
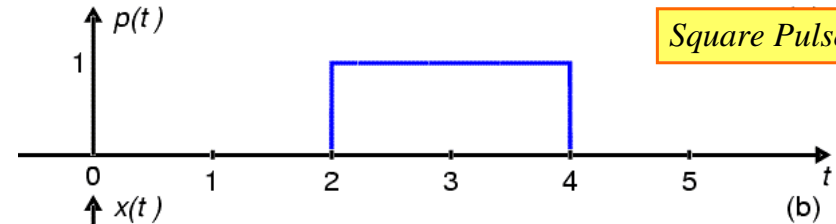


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CT Signals: FINITE LENGTH

$$p(t) = u(t-2) - u(t-4)$$

Square Pulse signal



Sinusoid multiplied by a square pulse

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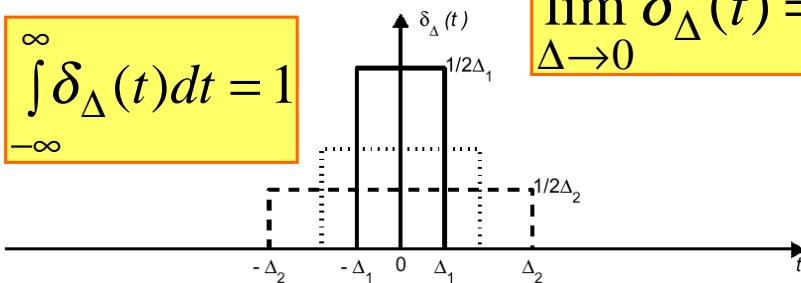
11

What is an Impulse?

- A signal that is concentrated at one point.

$$\int_{-\infty}^{\infty} \delta_{\Delta}(t) dt = 1$$

$$\lim_{\Delta \rightarrow 0} \delta_{\Delta}(t) = \delta(t)$$



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12

Defining the Impulse

- Assume the properties apply to the limit:

$$\lim_{\Delta \rightarrow 0} \delta_{\Delta}(t) = \delta(t)$$

- One "INTUITIVE" definition is:

$$\delta(t) = 0, \quad t \neq 0 \quad \text{Concentrated at } t=0$$

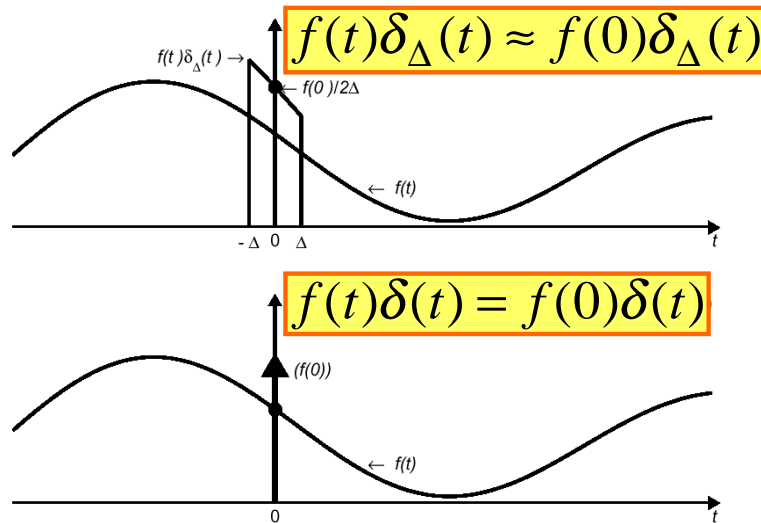
$$\int_{-\infty}^{\infty} \delta(\tau) d\tau = 1 \quad \text{Unit area}$$

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13

Sampling Property



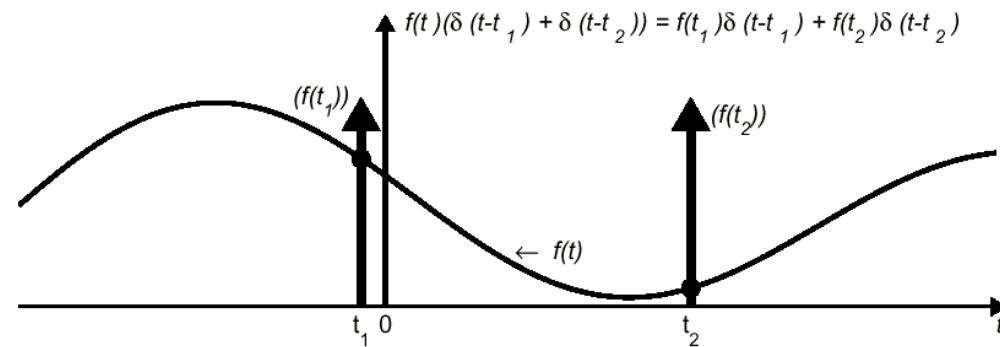
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14

General Sampling Property

$$f(t)\delta(t - t_0) = f(t_0)\delta(t - t_0)$$



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15

Properties of the Impulse

$$\delta(t - t_0) = 0, \quad t \neq t_0$$

Concentrated at one time

$$\int_{-\infty}^{\infty} \delta(t - t_0) dt = 1$$

Unit area

$$f(t)\delta(t - t_0) = f(t_0)\delta(t - t_0)$$

Sampling Property

$$\int_{-\infty}^{\infty} f(t)\delta(t - t_0) dt = f(t_0)$$

Extract one value of f(t)

$$\frac{du(t)}{dt} = \delta(t)$$

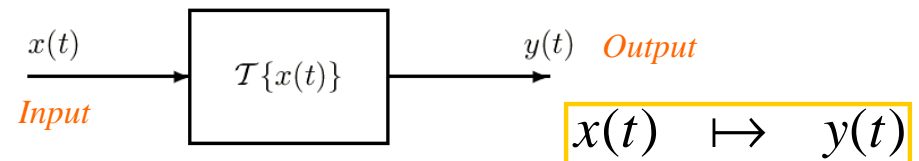
Derivative of unit step

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16

Continuous-Time Systems



Examples:

Delay $y(t) = x(t - t_d)$

Modulator $y(t) = [A + x(t)]\cos\omega_c t$

Integrator $y(t) = \int_{-\infty}^t x(\tau) d\tau$

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17

CT BUILDING BLOCKS

- INTEGRATOR (CIRCUITS)
- DIFFERENTIATOR
- DELAY by t_0
- MODULATOR (e.g., AM Radio)
- MULTIPLIER & ADDER

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18

Ideal Delay:

- Mathematical Definition:

$$y(t) = x(t - t_d)$$

- To find the IMPULSE RESPONSE, $h(t)$, let $x(t)$ be an impulse, so

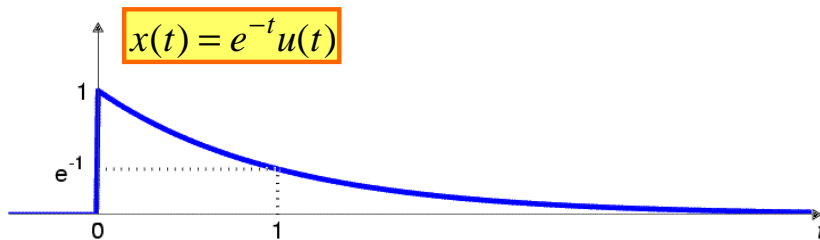
$$h(t) = \delta(t - t_d)$$

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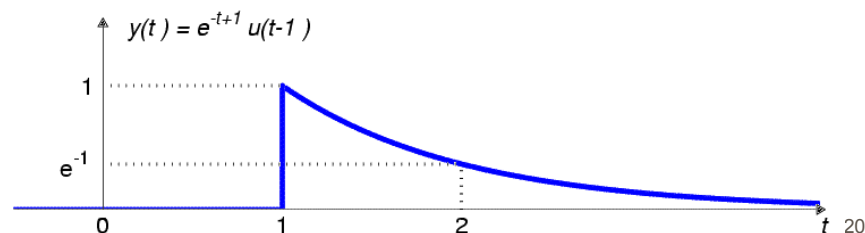
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19

Output of Ideal Delay of 1 sec



$$y(t) = x(t - 1) = e^{-(t-1)}u(t-1)$$



3/1

20

Integrator:

- Mathematical Definition:

$$y(t) = \int_{-\infty}^t x(\tau) d\tau$$

Running Integral

- To find the IMPULSE RESPONSE, $h(t)$, let $x(t)$ be an impulse, so

$$h(t) = \int_{-\infty}^t \delta(\tau) d\tau = u(t)$$

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21

Integrator:

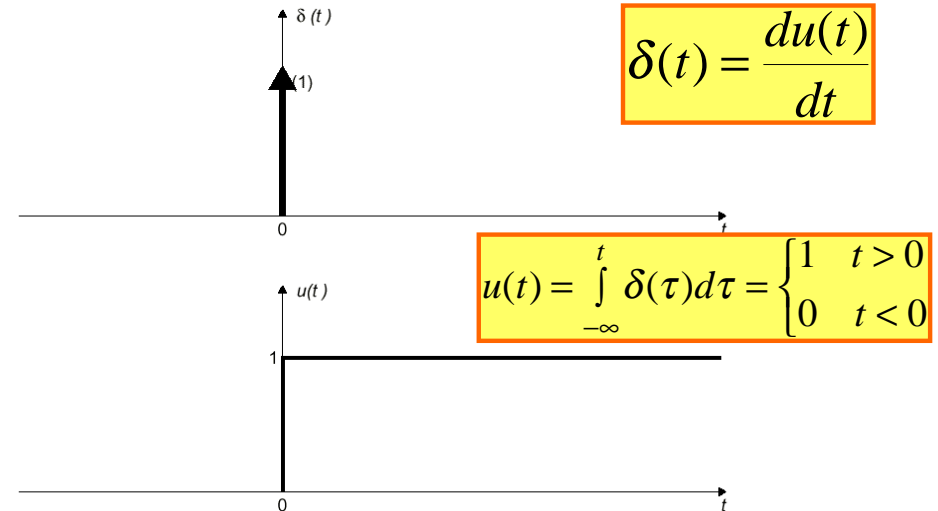
$$y(t) = \int_{-\infty}^t x(\tau) d\tau$$

- Integrate the impulse

$$\int_{-\infty}^t \delta(\tau) d\tau = u(t)$$

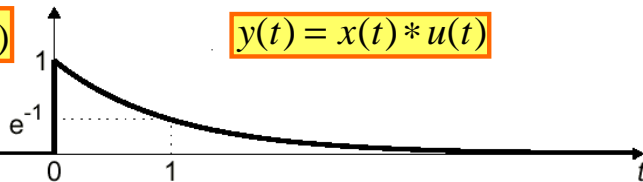
- IF $t < 0$, we get zero
- IF $t > 0$, we get one
 - Thus we have $h(t) = u(t)$ for the integrator

Graphical Representation



Output of Integrator

$$x(t) = e^{-t}u(t)$$



$$y(t) = x(t) * u(t)$$

$$y(t) = \int_{-\infty}^t e^{-\tau}u(\tau) d\tau = \begin{cases} 0 & t < 0 \\ \int_0^t e^{-\tau} d\tau & t \geq 0 \end{cases}$$
$$= (1 - e^{-t})u(t)$$

Differentiator:

- Mathematical Definition:

$$y(t) = \frac{dx(t)}{dt}$$

- To find $h(t)$, let $x(t)$ be an impulse, so

$$h(t) = \frac{d\delta(t)}{dt} = \delta^{(1)}(t)$$

Doublet

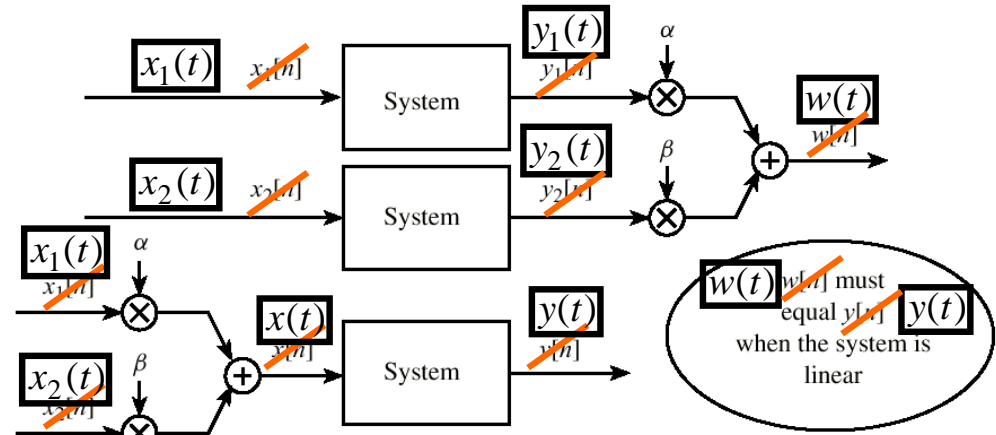
Differentiator Output: $y(t) = \frac{dx(t)}{dt}$

$$x(t) = e^{-t}u(t)$$

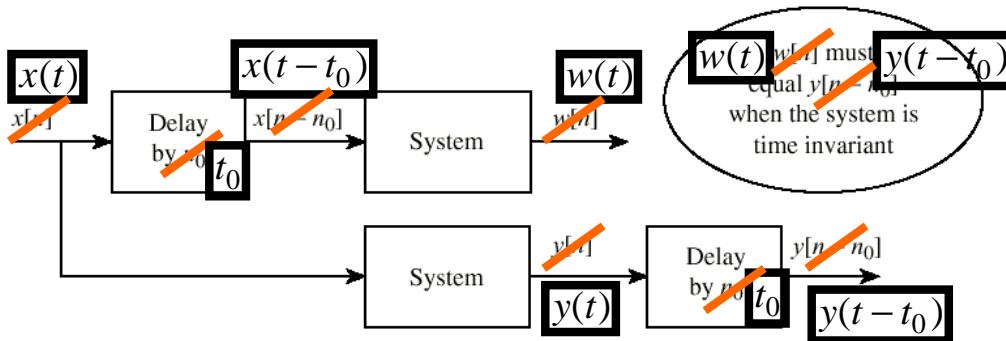


$$\begin{aligned} y(t) &= \frac{d}{dt}(e^{-t}u(t)) = \frac{d}{dt}(e^{-t})u(t) + e^{-t} \frac{d}{dt}(u(t)) \\ &= -e^{-t}u(t) + e^{-t}\delta(t) \\ &= -e^{-t}u(t) + e^{-0}\delta(t) = -e^{-t}u(t) + \delta(t) \end{aligned}$$

Testing for Linearity



Testing Time-Invariance



Ideal Delay: $y(t) = x(t - t_d)$

Linear

$$ax_1(t - t_d) + bx_2(t - t_d) = ay_1(t) + by_2(t)$$

and Time-Invariant

$$\begin{aligned} w(t) &= x((t - t_0) - t_d) \\ y(t - t_0) &= x((t - t_0) - t_d) \end{aligned}$$

Integrator:

$$y(t) = \int_{-\infty}^t x(\tau) d\tau$$

- Linear

$$\int_{-\infty}^t [ax_1(\tau) + bx_2(\tau)] d\tau = ay_1(t) + by_2(t)$$

- And Time-Invariant

$$w(t) = \int_{-\infty}^t x(\tau - t_0) d\tau \quad \text{let } \sigma = \tau - t_0$$

$$\Rightarrow w(t) = \int_{-\infty}^{t-t_0} x(\sigma) d\sigma = y(t - t_0)$$

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30

Modulator:

$$y(t) = [A + x(t)] \cos \omega_c t$$

- Not** linear--obvious because

$$A + ax_1(t) + bx_2(t) \neq$$

$$[A + ax_1(t)] + [A + bx_2(t)]$$

- Not** time-invariant

$$w(t) = [A + x(t - t_0)] \cos \omega_c t \neq y(t - t_0)$$

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31

Linear and Time-Invariant (LTI) Systems

- If a continuous-time system is both linear and time-invariant, then the output $y(t)$ is related to the input $x(t)$ by a **convolution integral**

$$y(t) = \int_{-\infty}^{\infty} x(\tau) h(t - \tau) d\tau = x(t) * h(t)$$

system.

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32