EE-2025 Spring 2001

Lecture 20

Amplitude Modulation (AM)

2-April-01

Info: Web-CT, Lab, HW

- Calendar:
  - Quiz #3 is 6-April (this Friday)
  - One page hand-written notes
  - Calculator
  - Covers z-Transforms, CT signals/sys, Impulses, Convolution and Fourier Transform.
- Prob Set #10 is due this week
- Prob Set #11 is due next week
- Lab #11 not due until week of 6-April

Info: Final Exam, etc.

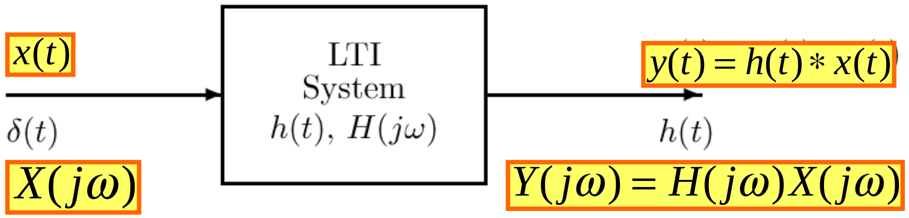
- Finals Week:
  - Period 15. Friday, May 4 at 2:50pm.
  - Don't plan to leave early for summer vacation!
- Last Day for turning in Lab Reports:
  - Friday, 27-April at 5pm.

LECTURE

LECTURE OBJECTIVES

- Review of FT properties
  - Convolution <--> multiplication
  - Frequency shifting
- Sinewave Amplitude Modulation
  - AM radio
- Frequency-division multiplexing
  - FDM
- Finish reading Chapter 12. Begin Ch. 13

Convolution Property



- Convolution in the time-domain

$$y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau$$

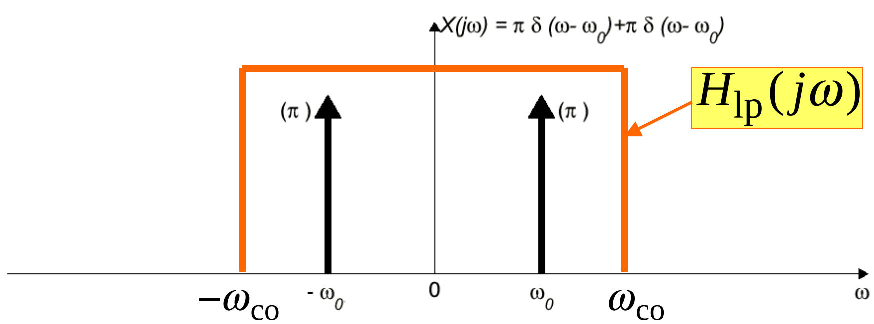
corresponds to **MULTIPLICATION** in the frequency-domain $Y(j\omega) = H(j\omega)X(j\omega)$

Cosine Input to LTI System

$$\begin{aligned} Y(j\omega) &= H(j\omega)X(j\omega) \\ &= H(j\omega)[\pi\delta(\omega-\omega_0) + \pi\delta(\omega+\omega_0)] \\ &= H(j\omega_0)\pi\delta(\omega-\omega_0) + H(-j\omega_0)\pi\delta(\omega+\omega_0) \end{aligned}$$

$$\begin{aligned} y(t) &= H(j\omega_0)\tfrac{1}{2}e^{j\omega_0 t} + H(-j\omega_0)\tfrac{1}{2}e^{-j\omega_0 t} \\ &= H(j\omega_0)\tfrac{1}{2}e^{j\omega_0 t} + H^*(j\omega_0)\tfrac{1}{2}e^{-j\omega_0 t} \\ &= |H(j\omega_0)|\cos(\omega_0 t + \angle H(j\omega_0)) \end{aligned}$$

Ideal Lowpass Filter

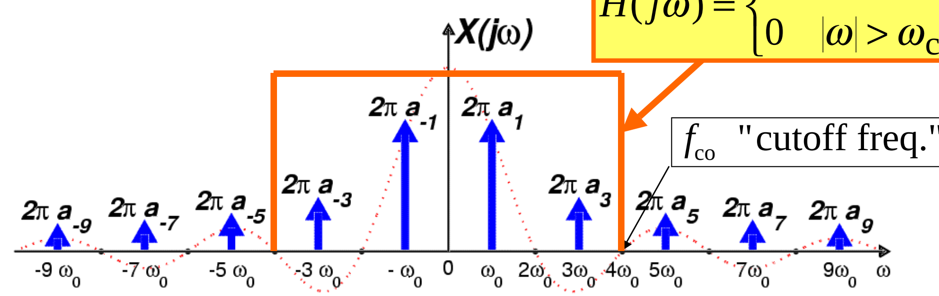


$$y(t) = x(t) \quad \text{if } \omega_0 < \omega_{\mathrm{co}}$$

$$y(t) = 0 \quad \text{if } \omega_0 > \omega_{\mathrm{co}}$$

Ideal Lowpass Filter

$$H(j\omega) = \begin{cases} 1 & |\omega| < \omega_{\mathrm{co}} \\ 0 & |\omega| > \omega_{\mathrm{co}} \end{cases}$$



$$y(t) = \frac{4}{\pi}\sin(50\pi t) + \frac{4}{3\pi}\sin(150\pi t)$$

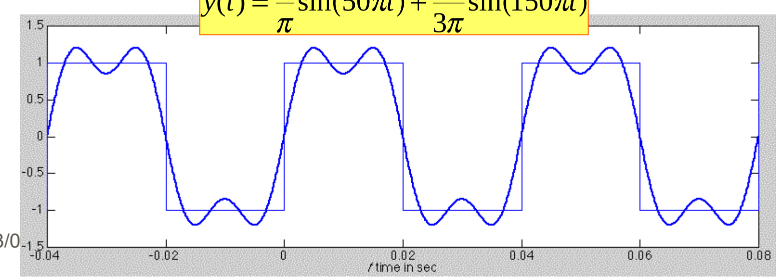


Table of Easy FT Properties

Linearity Property

$$ax_1(t)+bx_2(t) \Leftrightarrow aX_1(j\omega)+bX_2(j\omega)$$

Delay Property

$$x(t-t_d) \Leftrightarrow e^{-j\omega t_d} X(j\omega)$$

Frequency Shifting

$$x(t)e^{j\omega_0 t} \Leftrightarrow X(j(\omega-\omega_0))$$

Scaling

$$x(at) \Leftrightarrow \frac{1}{|a|} X(j(\tfrac{\omega}{a}))$$

Table of FT Properties

$$x(t)*h(t) \Leftrightarrow H(j\omega)X(j\omega)$$

$$x(t)p(t) \Leftrightarrow \frac{1}{2\pi} X(j\omega)*P(j\omega)$$

$$x(t)e^{j\omega_0 t} \Leftrightarrow X(j(\omega-\omega_0))$$

Differentiation Property

$$\frac{dx(t)}{dt} \Leftrightarrow (j\omega)X(j\omega)$$

Frequency Shifting Property

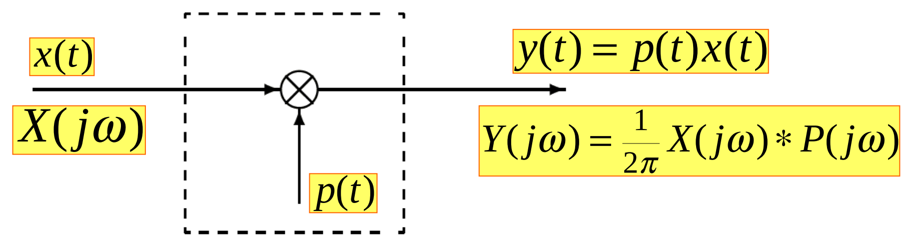
$$x(t)e^{j\omega_0 t} \Leftrightarrow X(j(\omega-\omega_0))$$

$$\int_{-\infty}^{\infty} e^{-j\omega_0 t} x(t)e^{-j\omega t}dt = \int_{-\infty}^{\infty} x(t)e^{-j(\omega-\omega_0)t}dt$$

$$= X(j(\omega-\omega_0))$$

$$y(t)=\frac{\sin 7t}{\pi t}e^{j\omega_0 t} \Leftrightarrow Y(j\omega)=\begin{cases}1 & \omega_0-7<\omega<\omega_0+7 \\ 0 & elsewhere\end{cases}$$

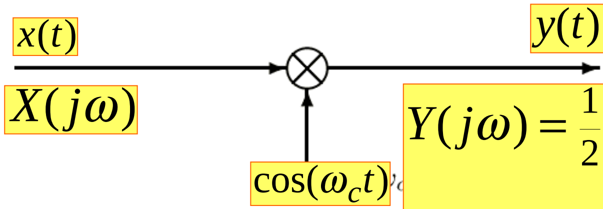
Signal Multiplier (Modulator)



- Multiplication in the time-domain corresponds to convolution in the frequency-domain.

$$Y(j\omega)=\frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\theta)P(j(\omega-\theta))d\theta$$

Amplitude Modulator



$y(t) = x(t)\cos(\omega_c t)$

$$Y(j\omega) = \frac{1}{2}X(j(\omega-\omega_c)) + \frac{1}{2}X(j(\omega+\omega_c))$$

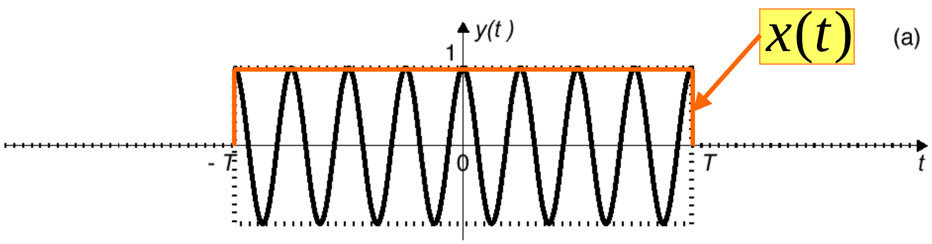
- $x(t)$ modulates the amplitude of the cosine wave. The result in the frequency-domain is two shifted copies of $X(j\omega)$.

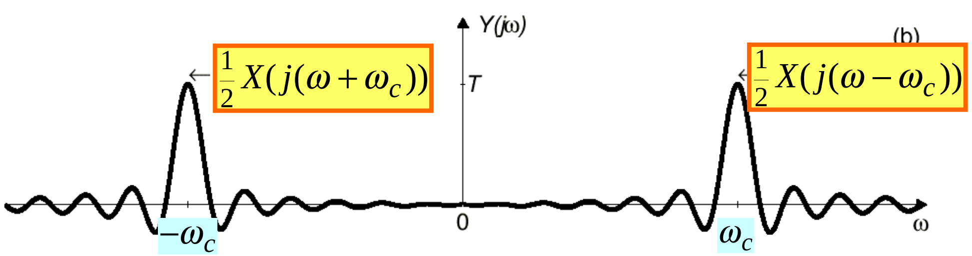
$$y(t) = x(t)\cos(\omega_c t) \Leftrightarrow Y(j\omega) = \frac{1}{2}X(j(\omega-\omega_c)) + \frac{1}{2}X(j(\omega+\omega_c))$$

$$x(t) = \begin{cases} 1 & |t| < T \\ 0 & |t| > T \end{cases} \Leftrightarrow X(j\omega) = 2\frac{\sin(\omega T)}{(\omega)}$$

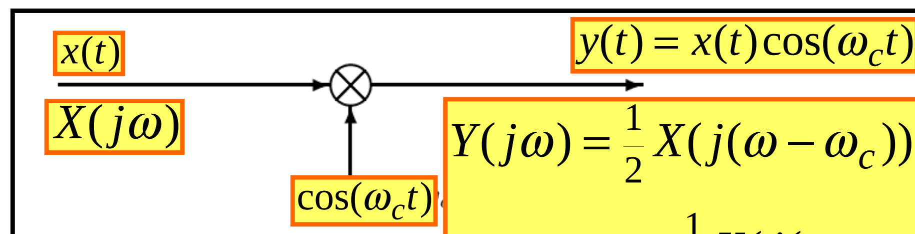
$$y(t) = x(t)\cos(\omega_c t) \Leftrightarrow Y(j\omega) = \frac{\sin((\omega-\omega_c))}{(\omega-\omega_c)} + \frac{\sin((\omega+\omega_c))}{(\omega+\omega_c)}$$

$$y(t) = x(t)\cos(\omega_c t) \Leftrightarrow Y(j\omega) = \frac{1}{2}X(j(\omega-\omega_c)) + \frac{1}{2}X(j(\omega+\omega_c))$$





DSBAM Modulator

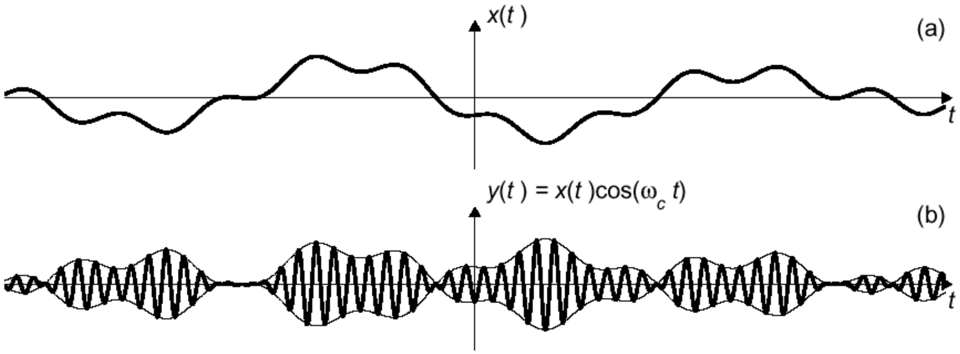


$$Y(j\omega) = \frac{1}{2}X(j(\omega-\omega_c)) + \frac{1}{2}X(j(\omega+\omega_c))$$

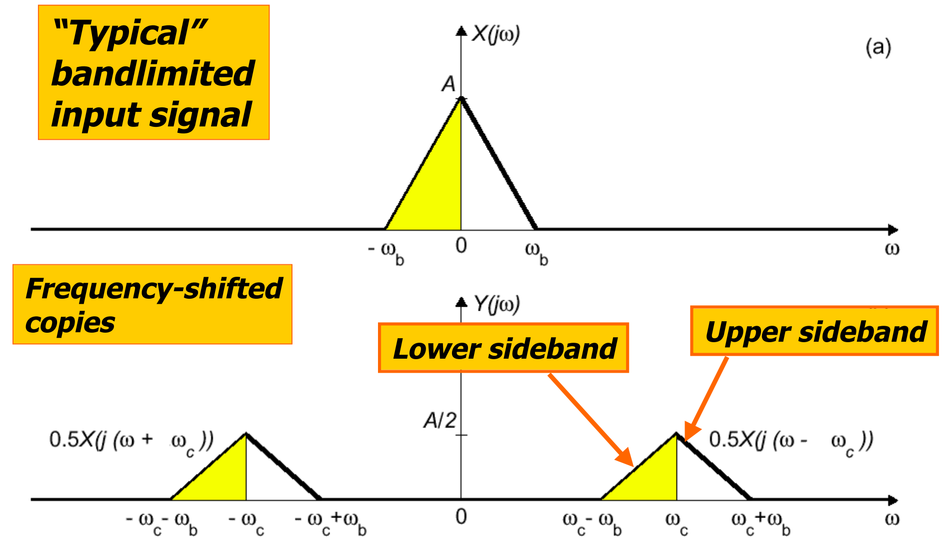
- If $X(j\omega)=0$ for $|\omega|>\omega_b$ and $\omega_c>\omega_b$, the result in the frequency-domain is two shifted and scaled exact copies of $X(j\omega)$.
- *$Y(j\omega)$ has "sidebands" around ω_c*

DSBAM Waveform

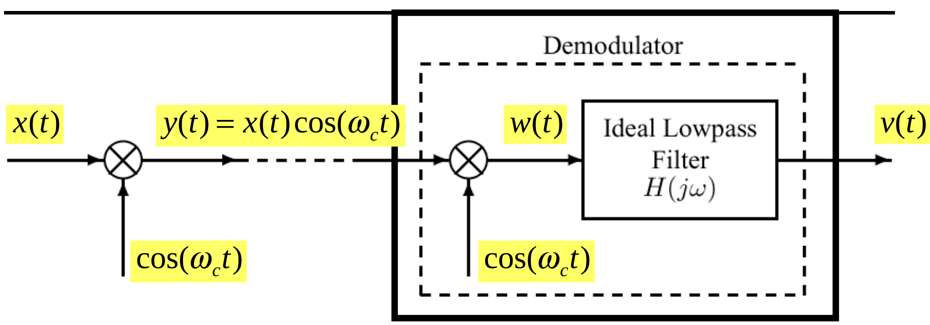
- In the time-domain, the envelope of sinewave peaks follows |*x(t)*|



Double Sideband AM (DSBAM)



DSBAM Demodulator

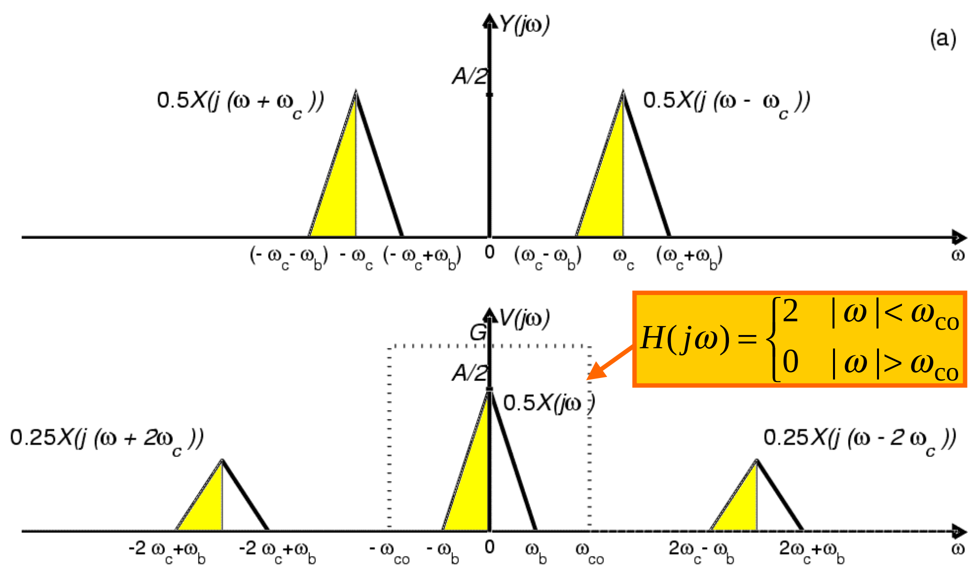


$$w(t) = x(t)[\cos(\omega_c t)]^2 = \frac{1}{2}x(t) + \frac{1}{2}x(t)\cos(2\omega_c t)$$

$$W(j\omega) = \frac{1}{2}X(j\omega) + \frac{1}{4}X(j(\omega - 2\omega_c)) + \frac{1}{4}X(j(\omega + 2\omega_c))$$

$$V(j\omega) = H(j\omega)W(j\omega)$$

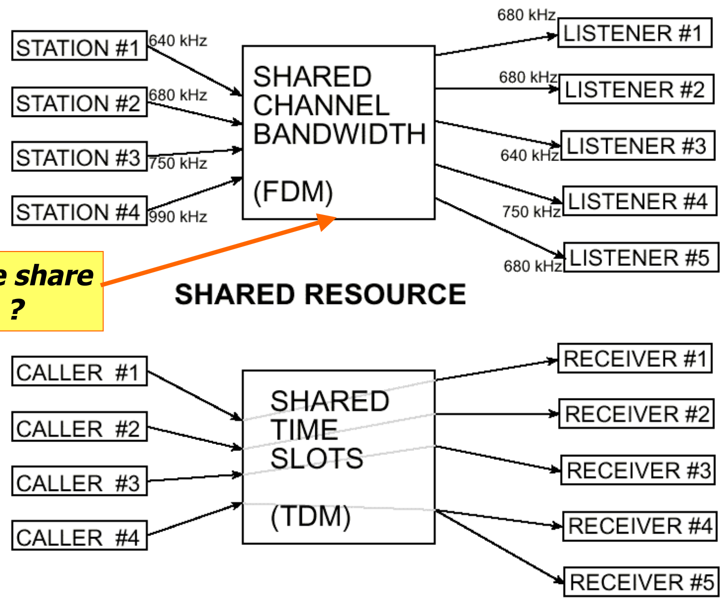
DSBAM Demodulation



$$H(j\omega) = \begin{cases} 2 & |\omega| < \omega_{co} \\ 0 & |\omega| > \omega_{co} \end{cases}$$

$$V(j\omega) = H(j\omega)W(j\omega) = X(j\omega) \;\; \text{if} \;\; \omega_b < \omega_{co} < 2\omega_c - \omega_b$$

The way communication systems work

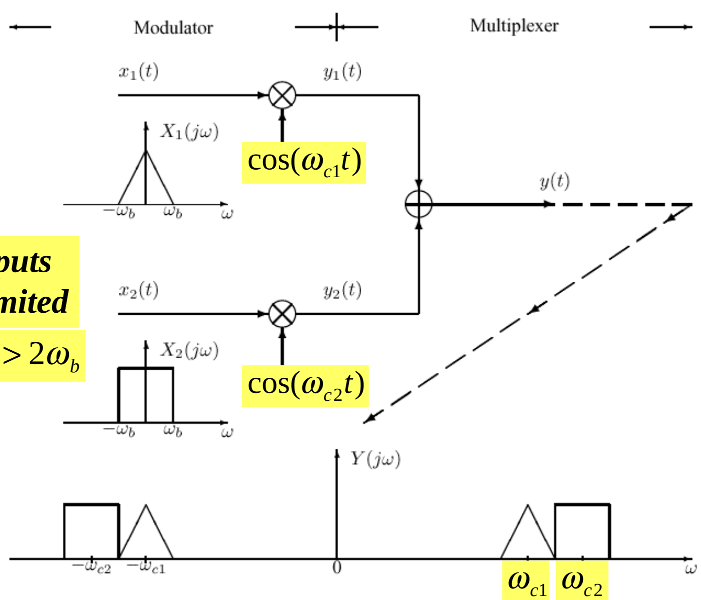


How do we share bandwidth ?

Frequency-Division Multiplexing (FDM)

- Shifting spectrum of signal to higher frequency:
  - Permits transmission of low-frequency signals with high-frequency EM waves
  - By allocating a frequency band to each signal multiple bandlimited signals can share the same channel
  - AM radio: 530-1620 kHz (10 kHz bands)
  - FM radio: 88.1-107.9 MHz (200 kHz bands)

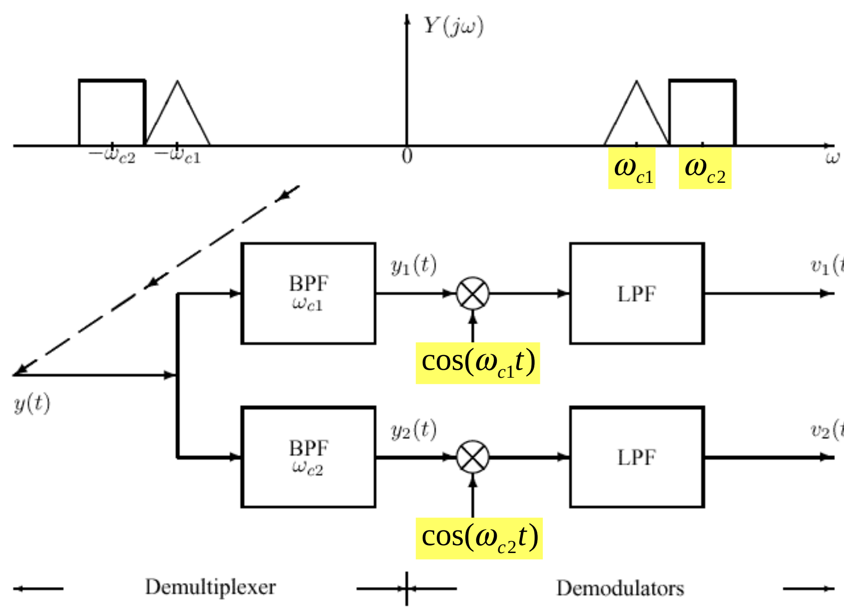
FDM Block Diagram



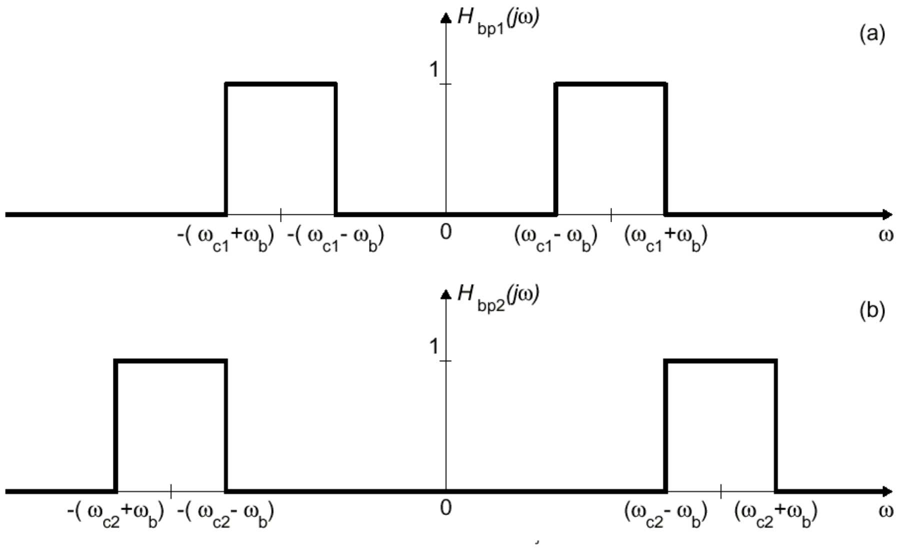
Spectrum of inputs must be bandlimited

Need $|\omega_{c2} - \omega_{c1}| > 2\omega_b$

Frequency-Division De-Mux

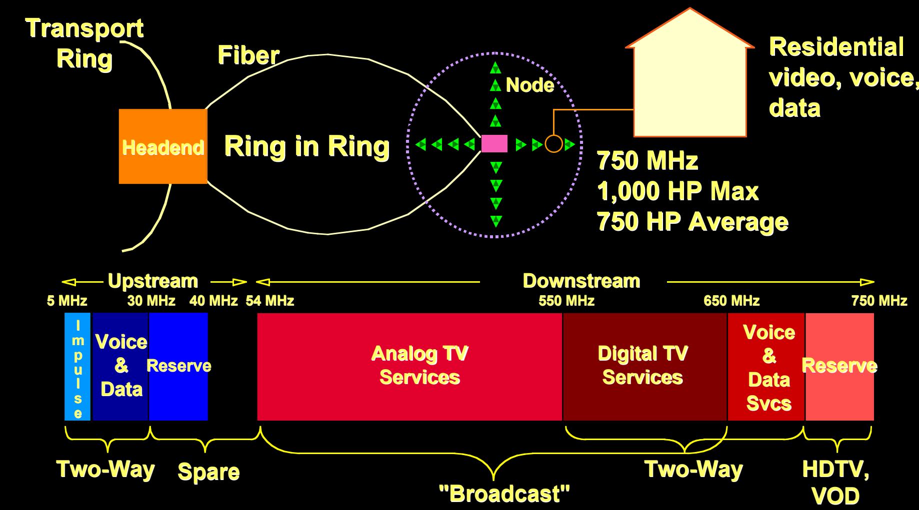


Bandpass Filters for De-Mux



From a Network Perspective,,,

Cox Cable Strategy: Broadband



HFC Full Service Network in Residential Areas