EE-2025 Spring-2001

Lecture 25

3-Domains for IIR

23-April-01

Info: Web-CT, Lab, HW

- Final Exam, 4-May @ 2:50 pm
  - Calculator, 1 page Handwritten Notes
  - Review: Weds and Thurs, 2 & 3-May @ **7 pm**
- HW #13 due last day (in Lecture)
- Lab #12 due this week in lab
- Labs DEADLINE - All Labs must be done
  - ALL Lab Reports due by 27-April, no later than Friday at 5 pm.

READING ASSIGNMENTS

- This Lecture:
  - Chapter 8, pp. 279-300
- Other Reading:
  - Recitation: Ch. 8, pp. 261-272
    - POLES & ZEROS
  - Next Lecture: Chapter 8, all

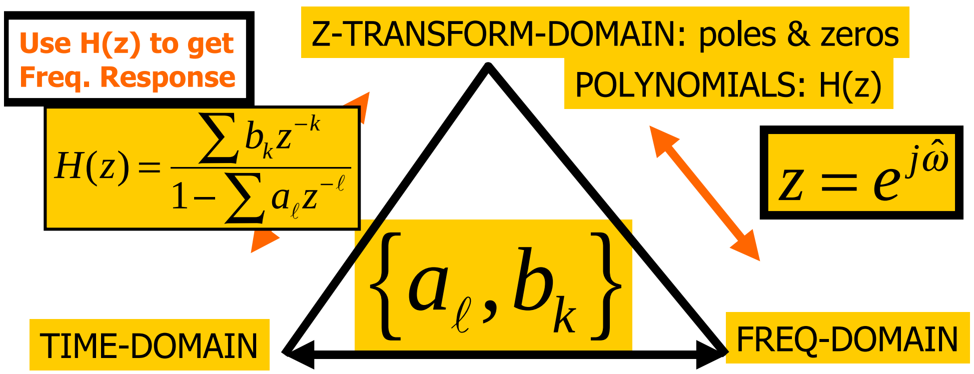
LECTURE OBJECTIVES

- SECOND-ORDER IIR FILTERS
  - TWO FEEDBACK TERMS

$$y[n] = a_1 y[n-1] + a_2 y[n-2] + \sum_{k=0}^{2} b_k x[n-k]$$

- THREE-DOMAIN APPROACH
  - BPFs have POLES NEAR THE UNIT CIRCLE

THREE DOMAINS



$$y[n]=a_1 y[n-1]+\sum_{k=0}^{M} b_k x[n-k]$$

Z-TRANSFORM TABLES

SHORT TABLE OF z-TRANSFORMS

	$x[n]$	$\Longleftrightarrow$	$X(z)$
1.	$ax_1[n]+bx_2[n]$	$\Longleftrightarrow$	$aX_1(z)+bX_2(z)$
2.	$x[n-n_0]$	$\Longleftrightarrow$	$z^{-n_0}X(z)$
3.	$y[n]=x[n]*h[n]$	$\Longleftrightarrow$	$Y(z)=H(z)X(z)$
4.	$\delta[n]$	$\Longleftrightarrow$	1
5.	$\delta[n-n_0]$	$\Longleftrightarrow$	z^{-n_0}
6.	$a^n u[n]$	$\Longleftrightarrow$	$\dfrac{1}{1-az^{-1}}$

SECOND-ORDER FILTERS

- Two FEEDBACK TERMS

$$y[n]=a_1 y[n-1]+a_2 y[n-2]$$
$$+b_0 x[n]+b_1 x[n-1]+b_2 x[n-2]$$

$$H(z)=\frac{b_0+b_1 z^{-1}+b_2 z^{-2}}{1-a_1 z^{-1}-a_2 z^{-2}}$$

MORE POLES

- Denominator is QUADRATIC
  - 2 Poles: REAL
  - or COMPLEX CONJUGATES

$$\frac{a_1 \pm \sqrt{a_1^2+4a_2}}{2}$$

$$H(z)=\frac{b_0+b_1 z^{-1}+b_2 z^{-2}}{1-a_1 z^{-1}-a_2 z^{-2}}=\frac{b_0 z^2+b_1 z+b_2}{z^2-a_1 z-a_2}$$

PROPERTY OF REAL POLYNOMIALS

A polynomial of degree N has N roots. If all the coefficients of the polynomial are real, the roots either must be real, or must occur in complex conjugate pairs.

TWO COMPLEX POLES

- Find Impulse Response ?
  - Can OSCILLATE vs. n
  - "RESONANCE"

$$\left(p_k\right)^n = \left(re^{j\theta}\right)^n = r^n e^{jn\theta}$$

- Find **FREQUENCY RESPONSE**
  - Depends on Pole Location
  - Close to the Unit Circle?
    - Make **BANDPASS FILTER**

$$\text{pole} = re^{j\theta}$$

$$r \to 1\,?$$

2nd ORDER EXAMPLE

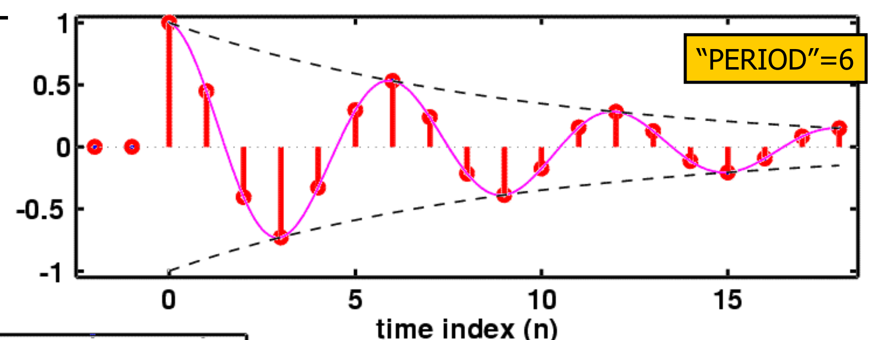
$$h[n] = 0.9^n \cos(\tfrac{\pi}{3}n)u[n] = 0.9^n\,\tfrac{1}{2}\left(e^{j\pi n/3} + e^{-j\pi n/3}\right)u[n]$$

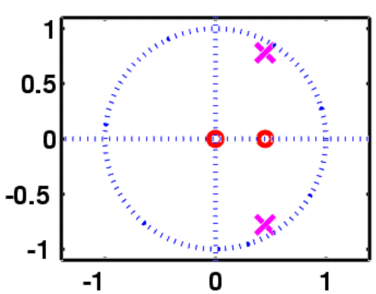
$$H(z) = \frac{0.5}{1-0.9e^{j\pi/3}z^{-1}} + \frac{0.5}{1-0.9e^{-j\pi/3}z^{-1}}$$

$$H(z) = \frac{1-0.9\cos(\tfrac{\pi}{3})z^{-1}}{(1-0.9e^{j\pi/3}z^{-1})(1-0.9e^{-j\pi/3}z^{-1})}$$

$$H(z) = \frac{1-0.45z^{-1}}{1-0.9z^{-1}+0.81z^{-2}}$$

h[n]: Decays & Oscillates





$$h[n] = (0.9)^n \cos(\tfrac{\pi}{3}n)u[n]$$

$$\frac{1-0.45z^{-1}}{1-0.9z^{-1}+0.81z^{-2}}$$

2nd ORDER Z-transform PAIR

GENERAL ENTRY for z-Transform TABLE

$$h[n] = r^n \cos(\theta n)u[n]$$

$$H(z) = \frac{1-r\cos\theta z^{-1}}{1-2r\cos\theta z^{-1}+r^2z^{-2}}$$

$$h[n] = Ar^n \cos(\theta n+\varphi)u[n]$$

$$H(z) = A\frac{\cos\varphi - r\cos(\theta-\varphi)z^{-1}}{1-2r\cos\theta z^{-1}+r^2z^{-2}}$$

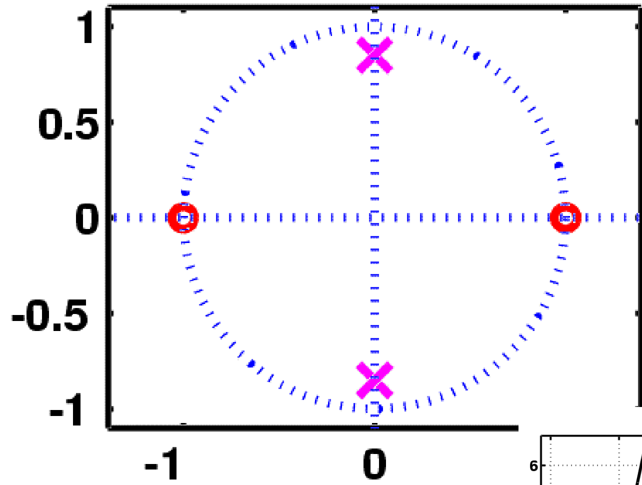
2nd ORDER EX: n-Domain

$$\frac{1-0.45z^{-1}}{1-0.9z^{-1}+0.81z^{-2}}$$

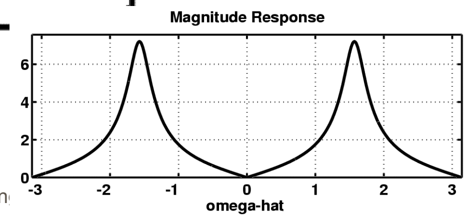
$$y[n]=0.9y[n-1]-0.81y[n-2]+x[n]-0.45x[n-1]$$

```
aa = [ 1, -0.9, 0.81 ];
bb = [ 1, -0.45 ];
nn = -2:19;
hh = filter( bb, aa, (nn==0) );
HH = freqz( bb, aa, [-pi,pi/100:pi] );
```


Complex POLE-ZERO PLOT

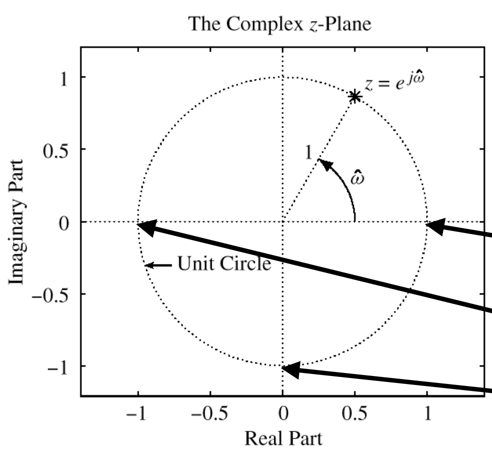


$$\frac{1-z^{-2}}{1+0.7225z^{-2}}$$



UNIT CIRCLE

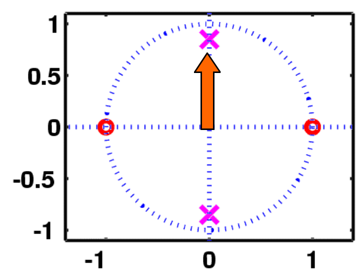
- MAPPING BETWEEN z and $\hat{\omega}$



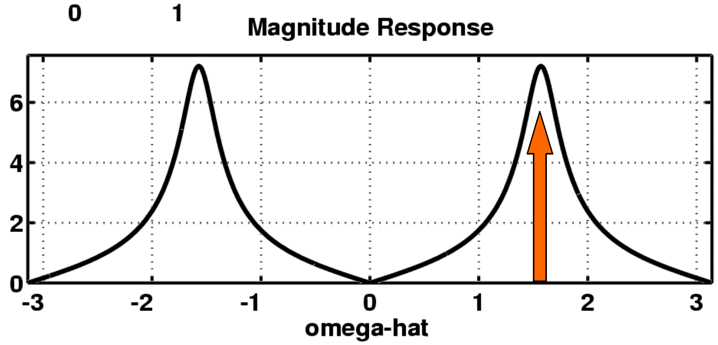
$$z=e^{j\hat{\omega}}$$

$z=1$	$\leftrightarrow$	$\hat{\omega}=0$
$z=-1$	$\leftrightarrow$	$\hat{\omega}=\pm\pi$
$z=\pm j$	$\leftrightarrow$	$\hat{\omega}=\pm\frac{1}{2}\pi$

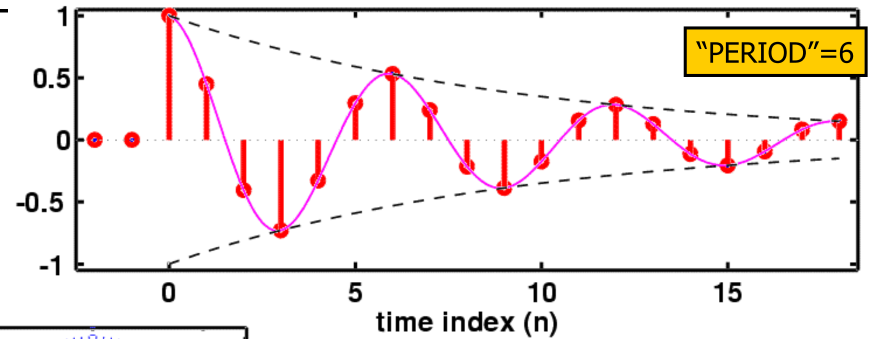
FREQUENCY RESPONSE from POLE-ZERO PLOT

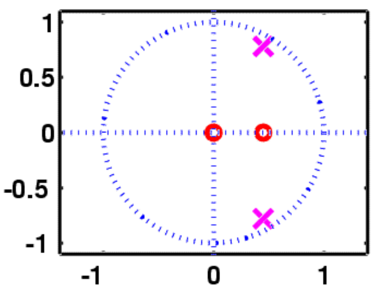


$$H(e^{j\hat{\omega}})=\frac{1-e^{-j2\hat{\omega}}}{1+0.7225e^{-j2\hat{\omega}}}$$



h[n]: Decays & Oscillates

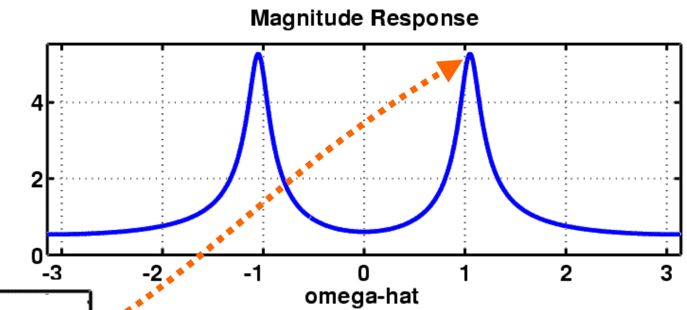


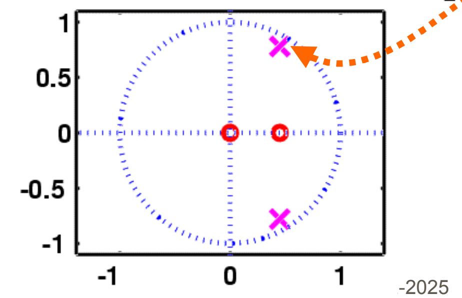


$$h[n] = (0.9)^n \cos(\tfrac{\pi}{3} n) u[n]$$

$$\frac{1 - 0.45z^{-1}}{1 - 0.9z^{-1} + 0.81z^{-2}}$$

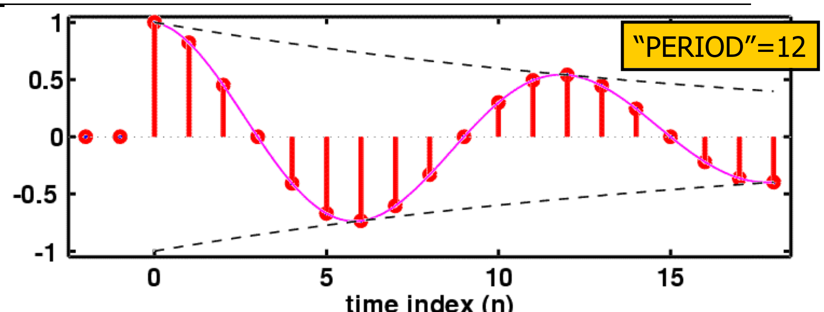
Complex POLE-ZERO PLOT

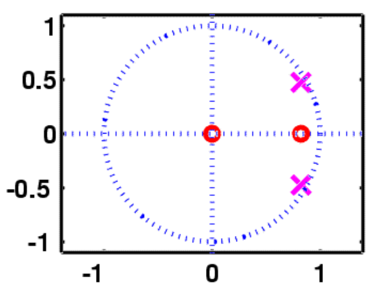




$$\frac{1 - 0.45z^{-1}}{1 - 0.9z^{-1} + 0.81z^{-2}}$$

h[n]: Decays & Oscillates

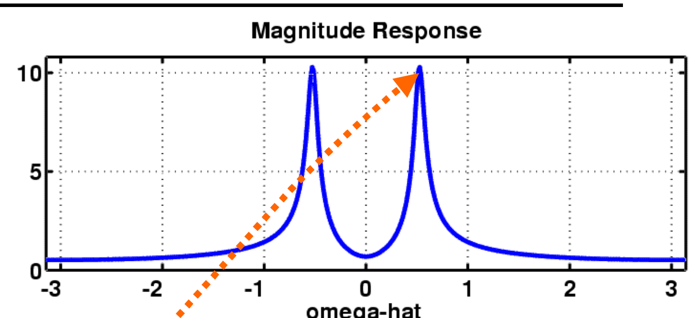


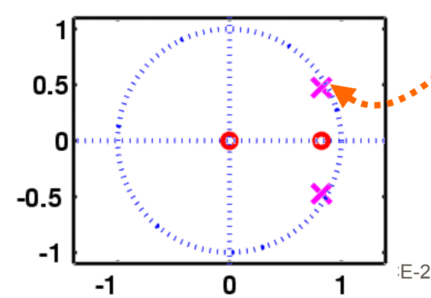


$$h[n] = (0.95)^n \cos(\tfrac{\pi}{6} n) u[n]$$

$$\frac{1 - 0.8227z^{-1}}{1 - 1.6454z^{-1} + 0.9025z^{-2}}$$

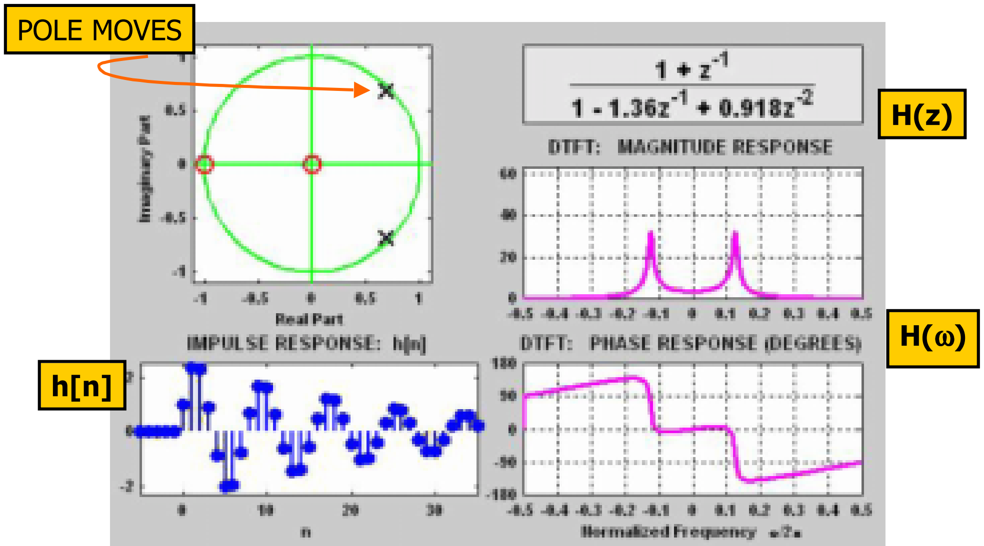
Complex POLE-ZERO PLOT





$$\frac{1 - 0.8227z^{-1}}{1 - 1.6454z^{-1} + 0.9025z^{-2}}$$

3 DOMAINS MOVIE: IIR



THREE INPUTS

- Given: $\rightarrow H(z) = \dfrac{5}{1 + 0.8z^{-1}} \rightarrow$
- Find the output, y[n]
  - When

$$x[n] = \cos(0.2\pi n)$$

$$x[n] = u[n]$$

$$x[n] = \cos(0.2\pi n)u[n]$$

SINUSOID ANSWER

- Given: $H(z) = \dfrac{5}{1 + 0.8z^{-1}}$
- The input: $x[n] = \cos(0.2\pi n)$
- Then y[n] $y[n] = M\cos(0.2\pi n + \psi)$

$$H(e^{j0.25\pi}) = \frac{5}{1 + 0.8e^{-j0.2\pi}} = 2.919e^{j0.089\pi}$$

Step Response

$$Y(z) = H(z)X(z) = \left(\frac{5}{1 + .8z^{-1}}\right)\left(\frac{1}{1 - z^{-1}}\right)$$

$$Y(z) = \frac{A}{1 + .8z^{-1}} + \frac{B}{1 - z^{-1}} = \frac{(A + B) + (.8B - A)z^{-1}}{\left(1 + .8z^{-1}\right)\left(1 - z^{-1}\right)}$$

$$\Rightarrow \quad (A + B) = 5 \quad \text{and} \quad (.8B - A) = 0$$

$$Y(z) = \frac{A}{1 + .8z^{-1}} + \frac{B}{1 - z^{-1}}$$

Step Response

$$Y(z)=\frac{\frac{20}{9}}{1+.8z^{-1}}+\frac{\frac{25}{9}}{1-z^{-1}}$$

$$y[n]=\frac{20}{9}(-.8)^n u[n]+\frac{25}{9}u[n]$$

$$y[n]\to\frac{25}{9}\quad \text{as} \quad n\to\infty$$

Stability

- Nec. & suff. condition: $\sum_{n=-\infty}^{\infty}|h[n]|<\infty$

$$h[n]=b(a)^n u[n]\Leftrightarrow H(z)=\frac{b}{1-az^{-1}}$$

$$\sum_{n=0}^{\infty}|b||a|^n<\infty \text{ if } |a|<1\Rightarrow$$

Pole must be Inside unit circle

SINUSOID starting at n=0

- We'll look at an example in MATLAB
  - $\cos(0.2\pi n)$
  - Pole at –0.8, so a^n is $(-0.8)^n$
- There are two components:
  - TRANSIENT
    - Start-up region just after n=0; **$(-0.8)^n$**
  - STEADY-STATE
    - Eventually, y[n] looks sinusoidal.
    - **Magnitude & Phase from Frequency Response**

Transient & Steady State