EE-2025 Spring 2001

Lecture 26

Review: Digital Filtering,

Frequency Response,

& Sampling

27-April-01

LAST LAB This Week

- ALL Lab Reports due by Friday (TODAY)
  - 4:30-5pm in VanLeer-475, or directly to TA
    - In PERSON
- Course Evaluations during last week
    - Two for GT: Lecture & Recitation
- HW #13 posted today

Final Exam Info

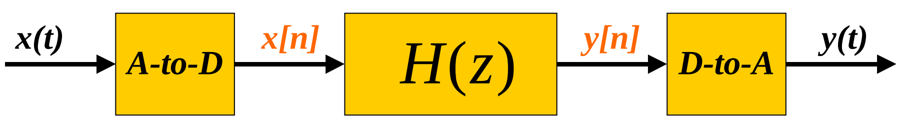
- Calendar: Final Exam
  - Period 15, Friday, May 4, 2:50pm
  - **ID check will be done at Final Exam**
- **CONFLICTS**
  - e.g., 3 exams in one day - see instructor of middle exam
- Reviews will be held on Weds and Thurs
  - 7pm in ECE Auditorium

FINAL EXAM

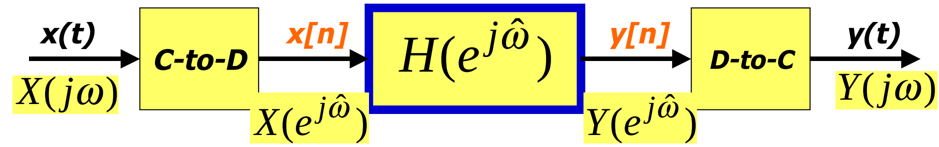
- FORMULA PAGES ?
  - Students bring **ONE** page **HAND-WRITTEN**
    - Tables 12.1 & 12.2 will be supplied with the exam.
- COVERAGE / EMPHASIS?
  - Fourier Transform
    - Sampling, Filtering & Spectrum
  - Digital Filters: IIR & FIR & H(z), frequency response
  - Hard problems from Quizzes #2, #3.
  - Homework & Old Quizzes & Finals

LECTURE OBJECTIVES

- THREE-DOMAIN APPROACH
  - EXHIBIT BANDPASS FILTERS
- RE-UNIFICATION:
  - How does Frequency Response affect x(t) to produce y(t) ?



DT Filtering of CT Signals

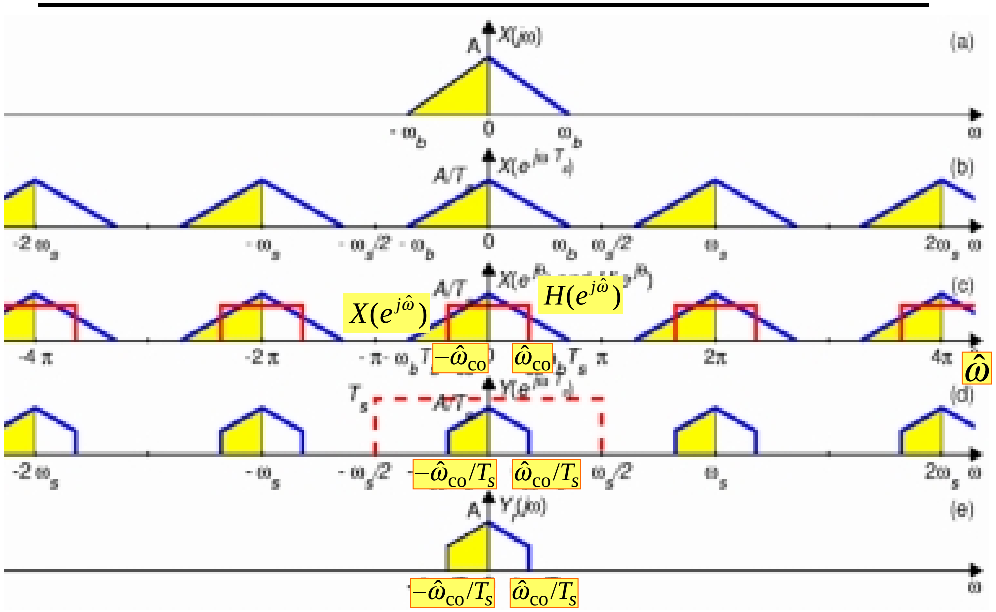


If no aliasing occurs in sampling x(t), then it follows that

$$Y(j\omega) = H_{\text{eff}}(j\omega)X(j\omega)$$

$$H_{\text{eff}}(j\omega) = \begin{cases} H(e^{j\omega T_s}) & |\omega| < \frac{1}{2}\omega_s \\ 0 & |\omega| > \frac{1}{2}\omega_s \end{cases}$$

Illustration of DT Filtering of a CT Signal



EFFECTIVE Freq. Response

- Assume NO Aliasing, then
  - ANALOG FREQ <--> DIGITAL FREQ

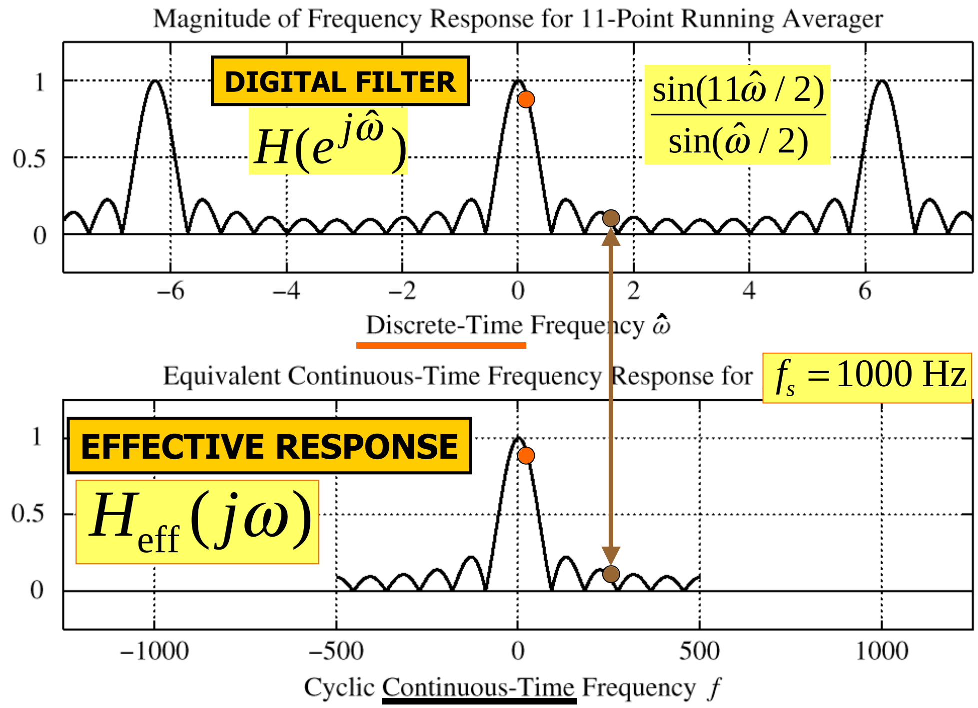
$$\hat{\omega} = \omega T_s = \frac{\omega}{f_s}$$

- So, we can plot:
- Scaled Freq. Axis

DIGITAL FILTER

$$H(e^{j\omega T_s}) \text{ vs. } \omega$$

ANALOG FREQUENCY



THREE DOMAINS

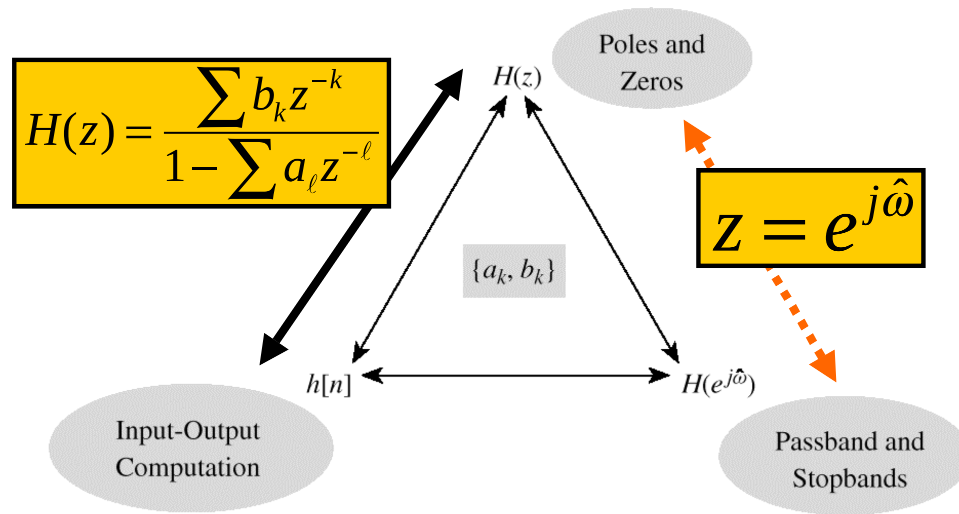
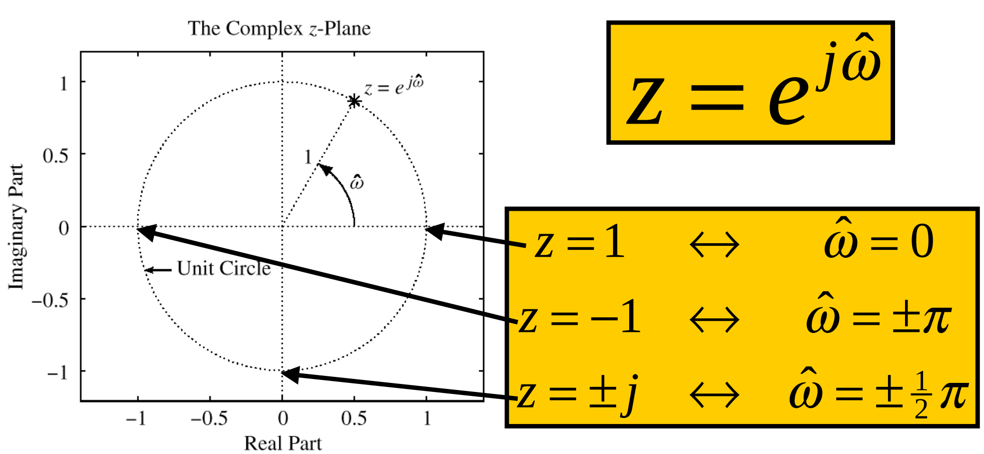


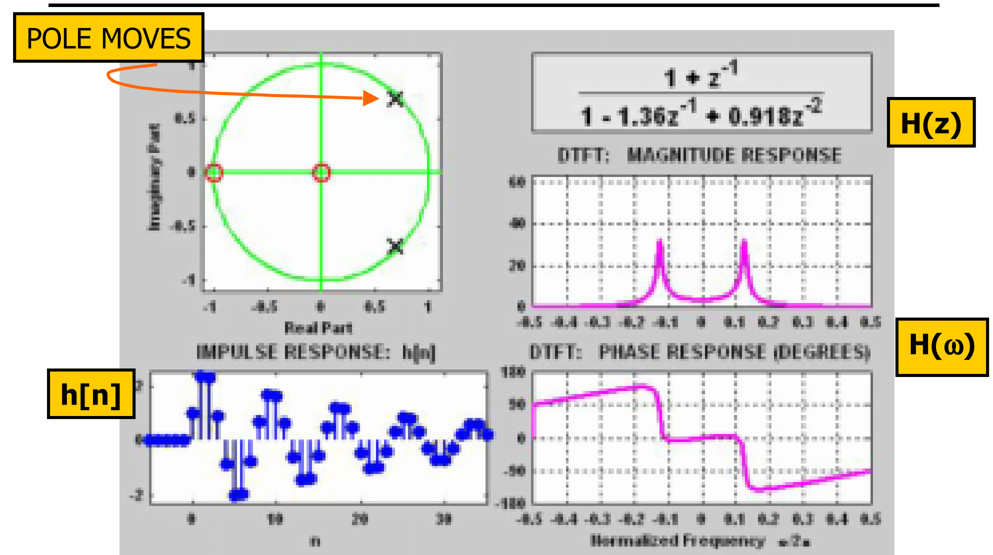
Figure 8.13 Relationship among the n-, z-, and $\hat{\omega}$-domains. The filter coefficients $\{a_k, b_k\}$ play a central role.

UNIT CIRCLE

- MAPPING BETWEEN z and $\hat{\omega}$



3 DOMAINS MOVIE: IIR



DIGITAL FILTER DESIGN

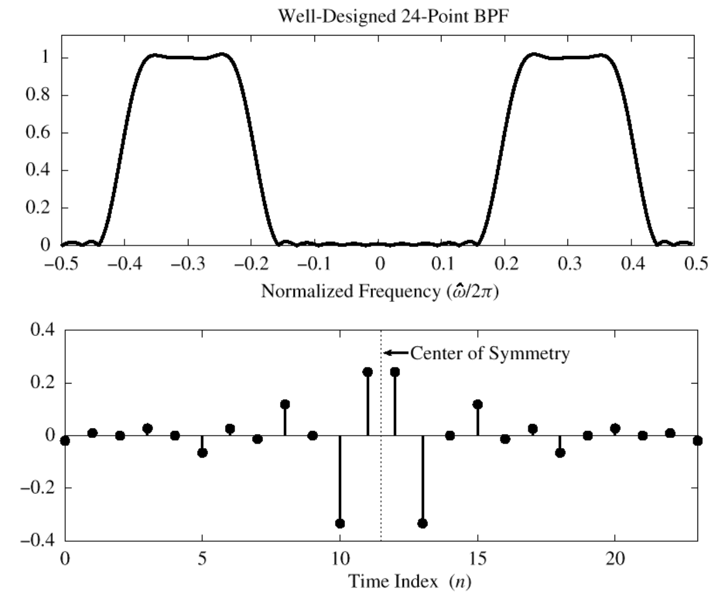
- Find the COEFFICIENTS to satisfy
  - PASSBAND & STOPBAND specifications
- FIR FILTERS
  - High Order: Many ZEROS. e.g., L=100
- IIR FILTERS
  - Poles & Zeros: 8–10 poles for a good filter
    - Implementation tricky with finite-precision

REALISTIC FIR BANDPASS

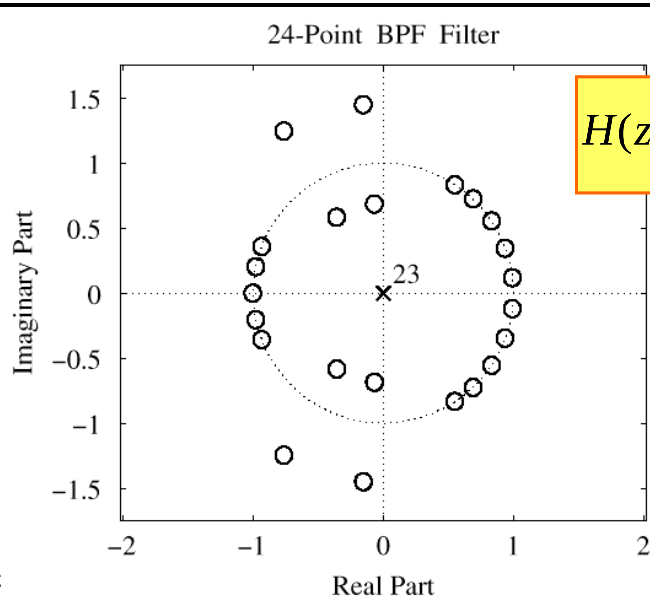
$$H(e^{j\hat{\omega}}) = \sum_{n=0}^{23} h[n]e^{-j\hat{\omega}n}$$

- FIR
- L = 24
- M=23
- 23 zeros

How would you make the gain = 2?



FIR BPF: 23 ZEROS

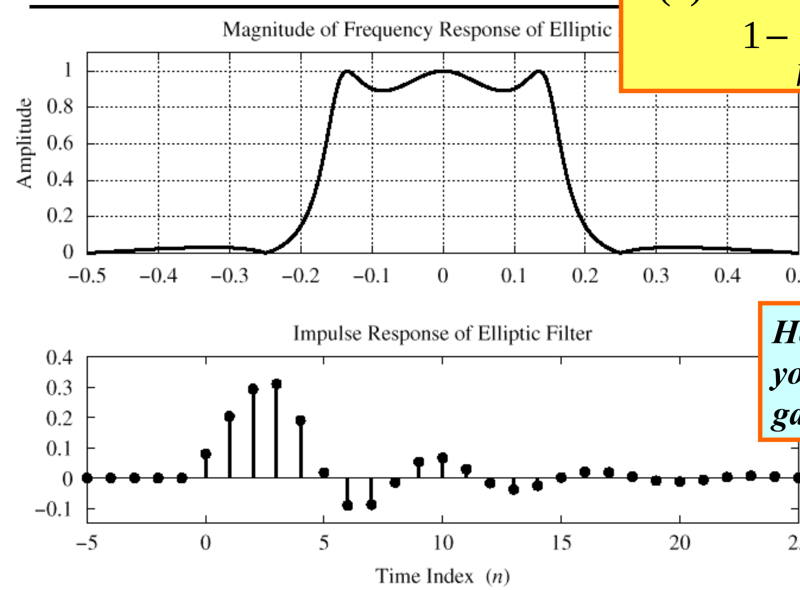


$$H(z) = \sum_{n=0}^{23} h[n]z^{-n}$$

IIR Elliptic LPF

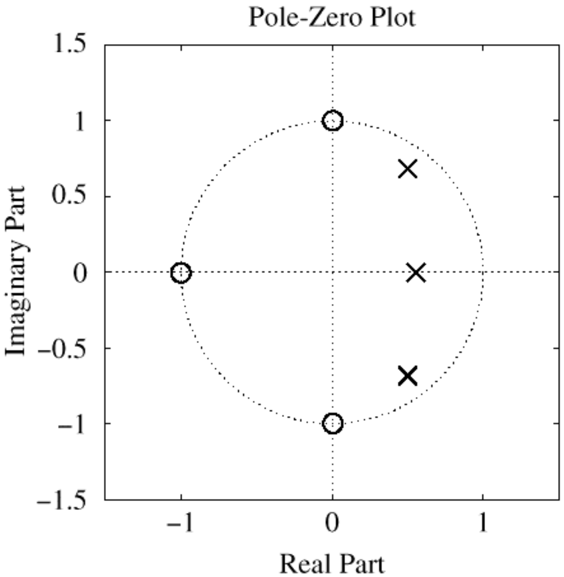
$$H(z) = \frac{\sum_{k=0}^{3} b_k z^{-k}}{1 - \sum_{k=1}^{3} a_k z^{-k}}$$

3 POLES



How would you make the gain = 2?

POLES & ZEROS of IIR

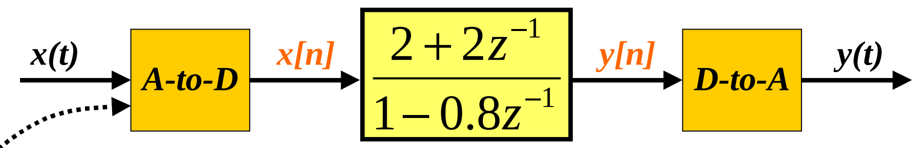


$$H(z) = \frac{\sum_{k=0}^{3} b_k z^{-k}}{1 - \sum_{k=1}^{3} a_k z^{-k}}$$

3 POLES

Example

- Given:



- Find the output, y(t)
  - When $x(t) = \cos(1250\pi t)$

$$f_s = 5000\,\text{Hz}$$

After sampling, the INSIDE problem becomes:

- Given: $H(z) = \dfrac{2+2z^{-1}}{1-0.8z^{-1}}$
- Find the output, y[n]
  - When $x[n] = \cos(0.25\pi n)$
  - Because

$$\omega T_s = 1250\pi / 5000 = 0.25\pi < 0.5\pi$$

NO Aliasing

D-T SINUSOIDAL RESPONSE

- x[n] = SINUSOID => y[n] is SINUSOID
- Get MAGNITUDE & PHASE from H(z)

$$\text{if } \ x[n] = e^{j\hat{\omega}n} \ \text{ then}$$

$$y[n] = H(e^{j\hat{\omega}})e^{j\hat{\omega}n}$$

$$\text{where } \ H(e^{j\hat{\omega}}) = H(z)\big|_{z=e^{j\hat{\omega}}}$$

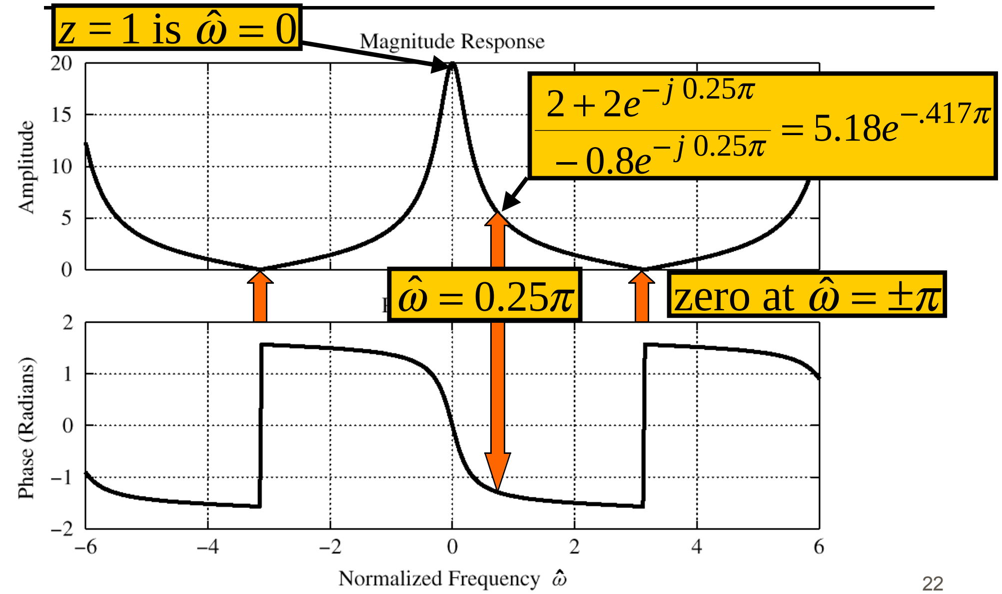
INSIDE ANSWER

- Given: $H(z) = \dfrac{2+2z^{-1}}{1-0.8z^{-1}}$
- Find output, y[n], when $x[n] = \cos(0.25\pi n)$
  - Evaluate at $z = e^{j0.25\pi}$

$$H(e^{j0.25\pi}) = \frac{2+2(\frac{\sqrt{2}}{2}+j\frac{\sqrt{2}}{2})}{1-0.8(\frac{\sqrt{2}}{2}+j\frac{\sqrt{2}}{2})} = 5.18e^{-j0.417\pi}$$

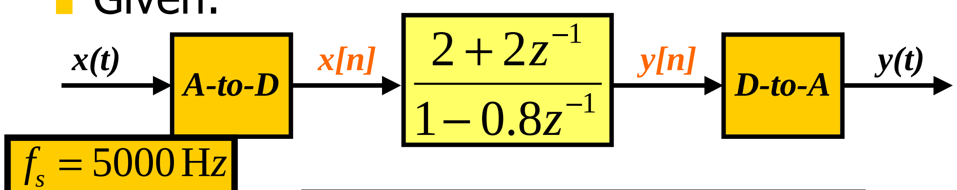
$$y[n] = 5.182\cos(0.25\pi n - 0.417\pi)$$

Evaluate FREQ. RESPONSE



FINAL ANSWER

- Given:

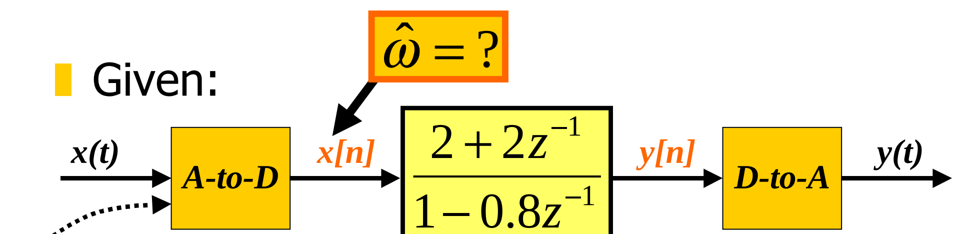


- When $x(t) = \cos(1250\pi t)$
- The output is

$$y(t) = 5.18\cos(1250\pi t - 0.417\pi)$$

ANOTHER INPUT FREQ

- Given:



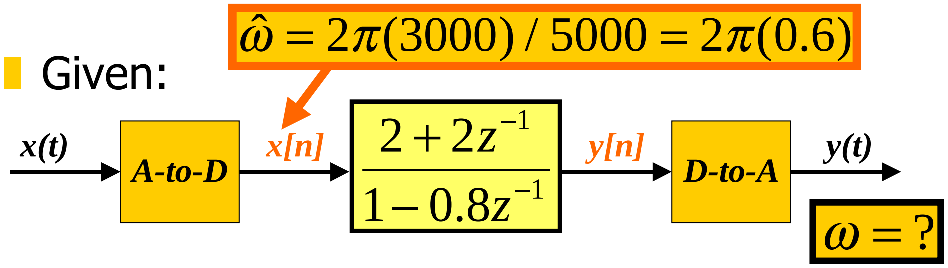
- Find the output, y(t)
  - When $x(t) = \cos(2\pi(3000)t)$

$f_s = 5000\,\text{Hz}$

$\hat{\omega} = ?$

2nd PROBLEM

- Given:



- When $x(t) = \cos(2\pi(3000)t)$

$f_s = 5000\text{ Hz} \rightarrow \hat{\omega} = 1.2\pi \rightarrow y(t) = ?$

EXAMPLE-2

- Given: $H(z) = \frac{2+2z^{-1}}{1-0.8z^{-1}}$
- Find the Impulse Response, h[n]
- I.e., find the output, y[n]
  - When $x[n] = \delta[n]$

Z-TRANSFORM TABLES

SHORT TABLE OF z-TRANSFORMS

	$x[n]$	$\Longleftrightarrow$	$X(z)$
1.	$ax_1[n] + bx_2[n]$	$\Longleftrightarrow$	$aX_1(z) + bX_2(z)$
2.	$x[n-n_0]$	$\Longleftrightarrow$	$z^{-n_0}X(z)$
3.	$y[n] = x[n] * h[n]$	$\Longleftrightarrow$	$Y(z) = H(z)X(z)$
4.	$\delta[n]$	$\Longleftrightarrow$	1
5.	$\delta[n-n_0]$	$\Longleftrightarrow$	z^{-n_0}
6.	$a^n u[n]$	$\Longleftrightarrow$	$\frac{1}{1-az^{-1}}$

Find Inverse z-Transform

- Given:

$$H(z) = \frac{2+2z^{-1}}{1-0.8z^{-1}} = \frac{2}{1-0.8z^{-1}} + \frac{2z^{-1}}{1-0.8z^{-1}}$$

- Find the Impulse Response, h[n]
  - Use the DELAY PROPERTY

$$h[n] = 2(0.8)^n u[n] + 2(0.8)^{n-1} u[n-1]$$

EXAMPLE-3

- Given: $H(z) = \dfrac{2+2z^{-1}}{1-0.8z^{-1}}$
- Find the output, y[n]
  - When $x[n] = (-1)^n u[n]$

Find Inverse z-Transform

- $Y(z) = H(z)X(z)$
  $= \dfrac{2+2z^{-1}}{1-0.8z^{-1}} \dfrac{1}{1-z^{-1}} = \dfrac{2}{1-0.8z^{-1}}$
- Therefore:

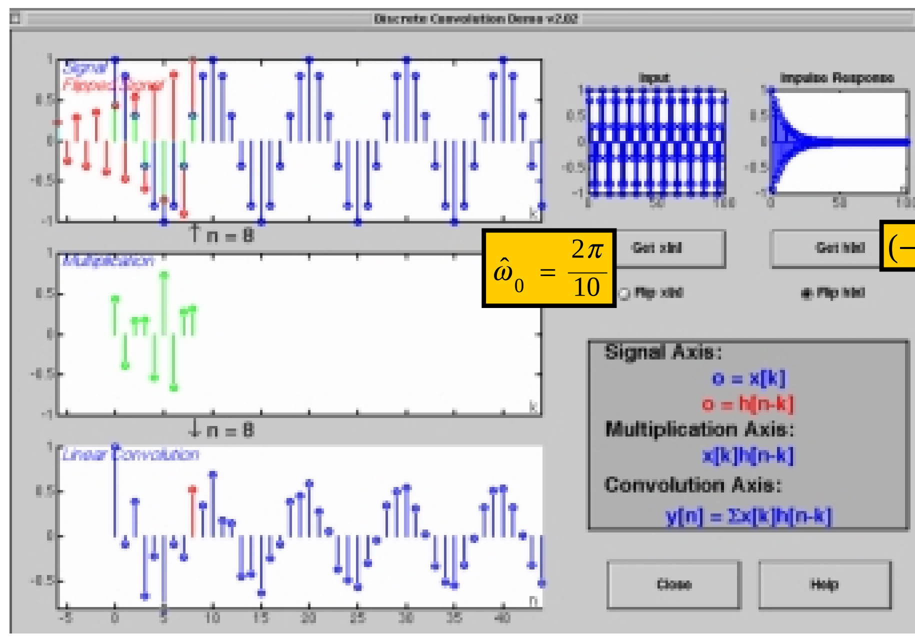
$$y[n] = 2(0.8)^n u[n]$$

THREE INPUTS

- Given: $\longrightarrow H(z) = \dfrac{5}{1+0.8z^{-1}} \longrightarrow$
- Find the output, y[n]
  - When $x[n] = \cos(0.2\pi n)$

$$x[n] = \cos(0.2\pi n)u[n]$$

Transient & Steady State



$\hat{\omega}_0 = \dfrac{2\pi}{10}$

$(-0.85)^n$

Mathematical Elegance

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega$$

Fourier S
(Inverse

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

Fourier Analysis
(Forward Transform)

$$\text{Time-domain} \Leftrightarrow \text{Frequency-domain}$$
$$x(t) \Leftrightarrow X(j\omega)$$

IMPORTANT CONCEPTS

- ALL Signals have Frequency Content
  - Sum of Sinusoids
  - Complex Exponentials
  - Impulses, Square Pulses
- FILTERS alter the Frequency Content
  - Image Processing Example: Blur
  - Linear Time-Invariant Processing
- 3 Domains for Analysis