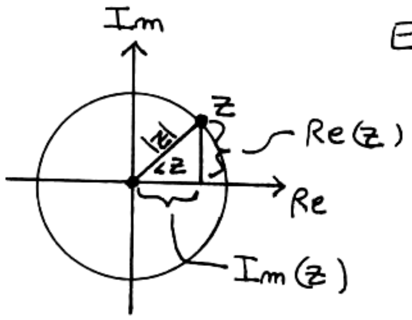


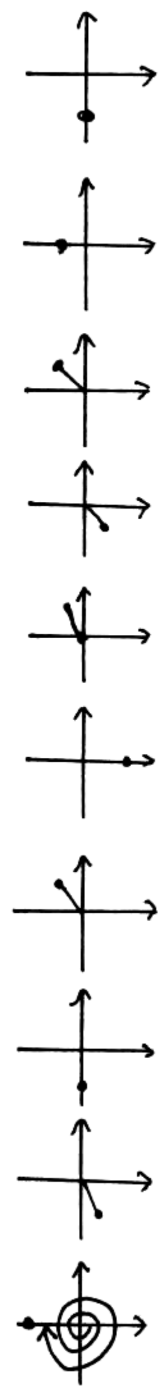
EULER: $z = |z| e^{\pm j \angle z} = |z| \cos(\angle z) \pm j\, |z| \sin(\angle z)$

$j = \sqrt{-1} = i$

EE DEPT → j

MATH DEPT → i

QUICK PLOTS



(1.1)

VECTOR FORM: (Rectangular) CARTESIAN, POLAR; EULER

a) $0 - j10 = (0, -10) = 10\angle -\frac{\pi}{2} = 10 e^{-j\frac{\pi}{2}}$

b) $-5 + j0 = (-5, 0) = 5\angle \pi = 5 \angle -\pi = 5 e^{j\pi}$

(Yes, It could be $-5\angle 0$, but try to use a magnitude ≥ 0)

c) $-10 + j10 = (-10, +10) = \sqrt{(-10)^2 + (+10)^2}\,\angle -\frac{\pi}{4} = 10\sqrt{2}\angle +\frac{\pi}{4}3 = 10\sqrt{2}\, e^{+j\frac{\pi}{4}3}$

d) $2 - j2 = (2, -2) = 2\sqrt{2}\angle -\frac{\pi}{4} = 2\sqrt{2}\, e^{-j\pi/4}$

e) $-1 + j\sqrt{3} = (-1, \sqrt{3}) = \sqrt{-1^2 + \sqrt{3}^2}\,\angle \frac{2}{3}\pi = 2\angle \frac{2}{3}\pi = 2 e^{j\frac{2\pi}{3}}$

f) $5 + j0 = (5, 0) = 5\angle 0 = 5 e^{j0} = 5$

(1.2)

a) $-\frac{3}{\sqrt{2}} + j\frac{3}{\sqrt{2}} = \left(-\frac{3}{\sqrt{2}}, \frac{3}{\sqrt{2}}\right) = 3\angle \frac{3}{4}\pi = 3 e^{j\frac{3}{4}\pi}$

b) $0 - j10 = (0, -10) = 10\angle -\frac{\pi}{2} = 10 e^{-j\frac{\pi}{2}}$

c) $2 - j2\sqrt{3} = (2, -2\sqrt{3}) = 4\angle -\frac{\pi}{3} = 4 e^{-j\frac{\pi}{3}}$

d) $-10 = (-10, 0) = 10\angle \pi = 10\angle 11\pi = 10 e^{-j11\pi}$

Standard trick question

$\angle -100001\pi = \angle -\pi = \angle 1{,}000{,}000{,}001\pi = \angle \pi$

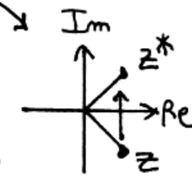
CONFIRM ALL USING MATLAB AS A CALCULATOR AND ZPRINT() TO SHOW POLAR + RECT FORMS

(1.3) $z_1 = 2 - j2 = 2\sqrt{2}\angle -\frac{\pi}{4}$ 

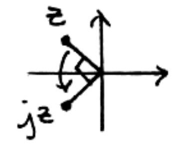
$z_2 = -\frac{3}{\sqrt{2}} + j\frac{3}{\sqrt{2}} = 3\angle \frac{3}{4}\pi$ 

a) $z_1^* = 2 + j2 = 2\sqrt{2}e^{j\pi/4} = 2\sqrt{2}\angle\frac{\pi}{4}$

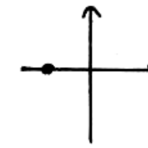
conjugation



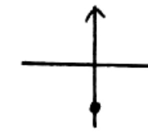
b) $-\frac{3}{\sqrt{2}} - j\frac{3}{\sqrt{2}} = 3\angle -\frac{3\pi}{4} = 3e^{-j\frac{3\pi}{4}} = \left(jz_2 = 1e^{\pm j\pi} \cdot 3e^{j\frac{3}{4}\pi}\right)$



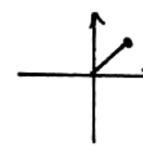
c) $-\frac{3}{4}\sqrt{2} + j0 = \frac{3\sqrt{2}}{4}\angle\pi = \frac{3}{4}\sqrt{2}e^{j\pi} = \dfrac{3e^{+j3\pi/4} = z_2}{2\sqrt{2}e^{-j\pi/4} = z_1}$



d) $0 - j9 = 9e^{j\frac{6}{4}\pi}\left(= 9e^{-j\frac{2}{4}\pi}\right) = 3^2 e^{(j\frac{3}{4}\pi)\cdot 2} = z_2^2$



e) $\frac{1}{4} + j\frac{1}{4} = \frac{\sqrt{2}}{4}\angle\frac{\pi}{4} = \frac{\sqrt{2}}{4}e^{j\frac{\pi}{4}} = \dfrac{1}{2\sqrt{2}e^{-j\pi/4} = z_1} = z_1^{-1}$

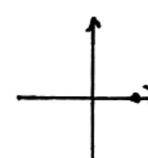


f) $0 + j6\sqrt{2} = 6\sqrt{2}e^{-j\frac{\pi}{2}} = (2\cdot 3)e^{j\left[-\frac{\pi}{4} + \frac{3}{4}\pi\right]} = z_1 \cdot z_2$

g) $z_1 + z_2 = (2 - j2) + \left(3e^{-j\frac{3}{4}\pi} = -\frac{3}{\sqrt{2}} - j\frac{3}{\sqrt{2}}\right) \simeq -0.121 - j4.12 \simeq 4.12\angle -1.6$

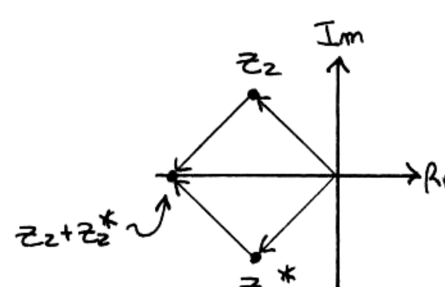
$\simeq 4.12e^{-j1.6} \simeq 4.12e^{-j\pi 0.509}$

h) $9 = 3^2 = |z_2|^2 = z_2 \cdot z_2^*$



i) $z_2 + z_2^* = \left[Re(z_2) + j\cancel{Im(z_2)}\right] + \left[Re(z_2) - \cancel{Im(z_2)}\right]$

$= 2\,Re(z_2) = 2\left(-\frac{3}{\sqrt{2}}\right) = -\frac{6}{\sqrt{2}}$



(1.4) FROM EULER: $Ae^{\pm j\theta} = A\cos(\theta) \pm jA\sin(\theta) = Re(Ae^{\pm j\theta}) \pm j\, Im(Ae^{\pm j\theta})$

a) $z = Ae^{-j\frac{\pi}{3}}$

$z^* = A e^{+j\frac{\pi}{3}}$

$Re(z^*) = Re\left(A e^{+j\frac{\pi}{3}}\right) = A\cos(\theta = \frac{\pi}{3}) = A\cos\left(\frac{\pi}{3}\right) = \boxed{\frac{A}{2}}$

b) $z - z^* = [Re(\cancel{z}) + Im(z)] - [\cancel{Re(z)} - Im(z)] = 2\,Im(z) = 2A\sin\left(-\frac{\pi}{3}\right)$

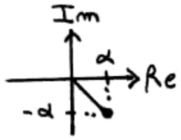
FOR $z = Ae^{-j\frac{\pi}{3}}$ $\phi = -\frac{\pi}{3}$

$= 2A\sin\left(-\frac{\pi}{3}\right) = -2A\frac{\sqrt{3}}{2} = \boxed{-\sqrt{3}A}$

c) $z = 10e^{j\phi}$

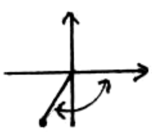
$jz = e^{j\frac{\pi}{2}} \cdot 10e^{j\phi} = 10e^{j\left(\frac{\pi}{2}+\phi\right)} =$

$Im(jz) = 10\sin\left(\frac{\pi}{2}+\phi\right) = \boxed{10\cos\phi}$

d) $z = \alpha - j\alpha =$  $= \sqrt{\alpha^2+\alpha^2}\angle -\frac{\pi}{4} = \alpha\sqrt{2}\angle -\frac{\pi}{4} = \alpha\sqrt{2}\, e^{-j\frac{\pi}{4}}$

e) $\dfrac{|z| = \cancel{A}}{z = \cancel{A}e^{-j\pi/3}} = e^{+j\pi/3}$

(1.5)

a) $e^{-j\frac{2}{3}\pi}$ 

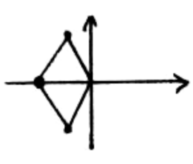
$e^{j\frac{2}{3}\pi}$ 

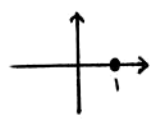
$+$

$\Sigma = z + z^* = 2\,Re(z)$

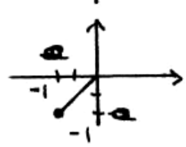
$= 2Re\left(e^{j\frac{2}{3}\pi}\right)$

$= 2\cdot\left(-\frac{1}{2}\right) = -1$



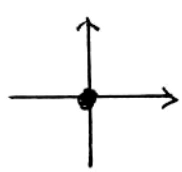
1 

$e^{j\frac{\pi}{2}}$ 

$\sqrt{2}\, e^{-j\pi\frac{3}{4}}$ 

$+$

$\Sigma = 0$



Problem 1.5: MATLAB statements to execute complex arithmetic

1.5 a)

```
» za=exp(-j*2*pi/3) + exp(j*2*pi/3)

za =
   -1.0000
```

$= \boxed{-1+j0}$

```
» abs(za)

ans =
    1.0000

» angle(za)

ans =
    3.1416
```

$1 \angle \pi$

```
» angle(za)/pi

ans =
     1
```

1.5 b)

```
» zb = 1 + exp(j*pi/2) + sqrt(2)*exp(-j*3*pi/4)

zb =
          0 -2.2204e-016i

» abs(zb)

ans =
  2.2204e-016
```

almost zero

```
» angle(zb)

ans =
   -1.5708

» angle(zb)/pi

ans =
   -0.5000

»
```

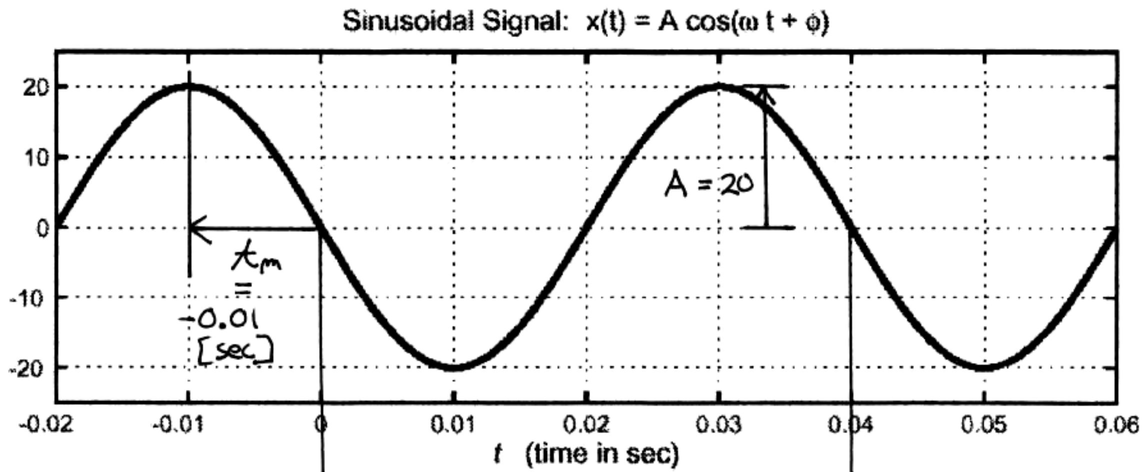
← Angle does not matter if magnitude $\cong 0$

PROBLEM 1.6*:

The waveform in the following figure can be expressed as

$$x(t) = A\cos[\omega_0(t - t_m)] = A\cos(\omega_0 t + \phi) = A\cos(2\pi f_0 t + \phi)$$

From the waveform, determine A, ω_0, f_0, t_m, and ϕ. Choose the value of ϕ such that $-\pi < \phi \leq \pi$.



PERIOD = T = 0.04 [sec] / [cycle]

$$f_0 = \frac{1}{T} = 25 \left[\frac{\text{cycle}}{\text{sec}} = \text{Hz}\right]$$

$$\omega_0 = 2\pi\left[\frac{\text{rad}}{\text{cycle}}\right] \cdot \left(f_0 = 25\left[\frac{\text{cycle}}{\text{sec}}\right]\right) = 50\pi\left[\frac{\text{rad}}{\text{sec}}\right]$$

$$t_m = -0.01\ [\text{sec}]$$

$$x(t) = 20\cos\left[2\pi 25(t - (-0.01))\right]$$

$$= 20\cos\left[2\pi 25 t + \frac{\pi}{2}\right]$$

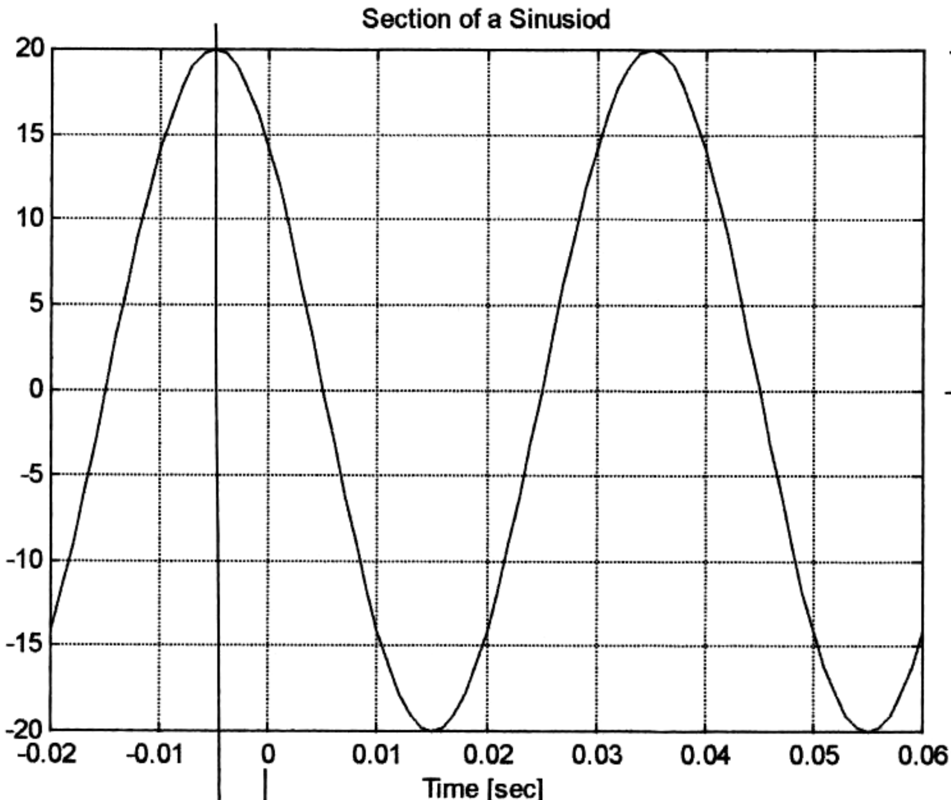
ϕ

Problem 1.7:

```
» dt=0.001;
» tt=-0.02:dt:0.06;
» Fo=25;
» Z=10*sqrt(2)*(1+j);
» xx=real(Z*exp(2*j*pi*Fo*tt));
» plot(tt,xx), grid
» title('Section of a Sinusiod'), xlabel('Time [sec]')
»
```

$T = 1/F_o = 1/25 = 0.04$ sec

Amplitude = abs(Z) = 20
Phase (assuming a cosine) = (angular fraction of peak distance from tt=0) = [radians]



A = Amplitude = 20

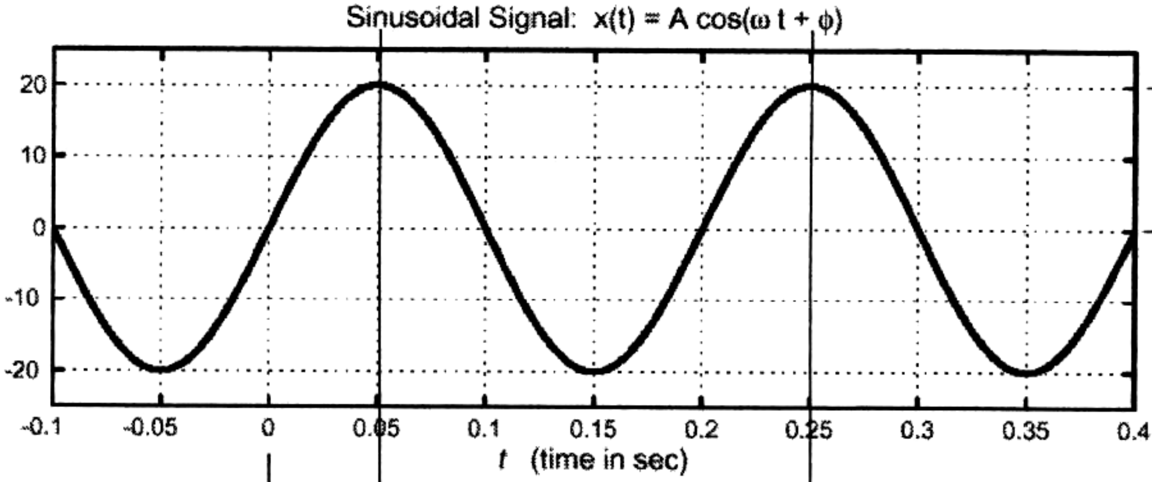
$t_m = -0.005$ [sec]

$\Phi = -t_m \cdot 2\pi\,(f = 25)$
$= -(-0.005) \cdot 2 \cdot \pi \cdot 25$)
$= \frac{\pi}{4}$ [rad]

$\boxed{\text{our sinusoid}(t) = 20\cos\left(2\pi\, 25t + \frac{\pi}{4}\right)}$

Problem 1.8

(a)



$A = 20$

t_m

$t_m = 0.05$ [sec]

$\frac{1}{f_0} = T = \text{PERIOD} = 0.2$ [sec]

2π [rad]

$\rightarrow f_0 = \frac{1}{T} = \frac{1}{0.2\,\text{sec}} = 5\,\text{Hz}$

$\omega_0 = 2\pi \frac{\text{rad}}{\text{cycle}} \cdot f_0 \left[\frac{\text{cycle}}{\text{sec}} = \text{Hz}\right]$

$= 2\pi\, 5 = 10\pi \left[\frac{\text{rad}}{\text{sec}}\right]$

$\phi = -t_m\, 2\pi f_0$

$= -0.05 \cdot 2 \cdot \pi \cdot 5$

$= -\frac{\pi}{2}$ [rad]

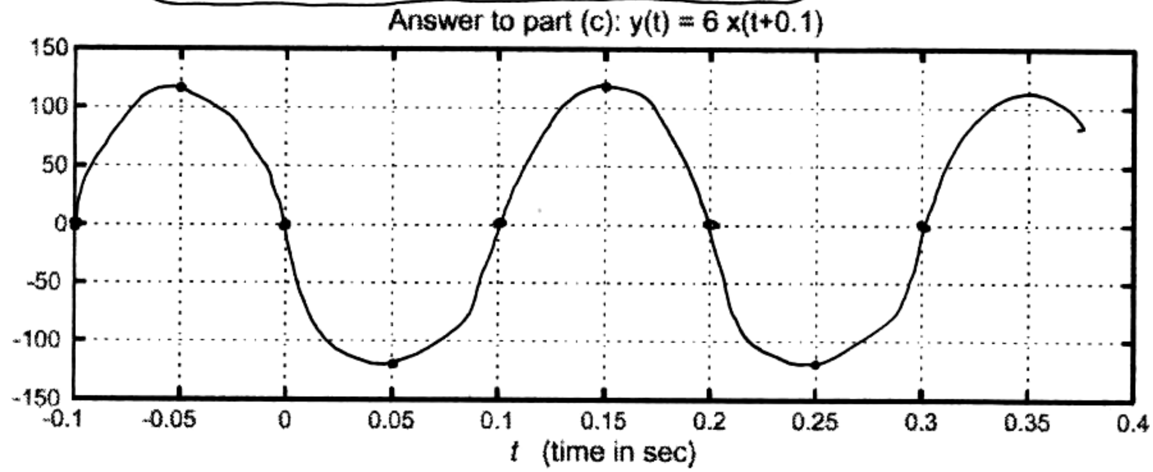
$x(t) = 20\cos\left(2\pi 5t - \frac{\pi}{2}\right)$

(c)

$y(t) = 6\, x(t+0.1) = 6 \cdot 20\cos\left(2\pi\, 5(t+0.1) - \frac{\pi}{2}\right)$

$= 120\cos\left(2\pi 5t + \pi - \frac{\pi}{2}\right)$

$y(t) = 120\cos\left(2\pi 5t + \pi/2\right)$



1.8 b

$$x(t) = 20\cos\left(2\pi 5t - \frac{\pi}{2}\right)$$

FROM EULER:

$$Ae^{\pm j\theta} = A\cos(\theta) \pm jA\sin(\theta)$$
$$= Re\left(Ae^{\pm j\theta}\right) \pm j\,Im\left(Ae^{\pm j\theta}\right)$$

$$x(t) = Re\left[20\, e^{j[2\pi 5t - \frac{\pi}{2}]}\right]$$
$$= Re\left[20\, e^{-j\frac{\pi}{2}}\, e^{j2\pi 5t}\right]$$
$$= Re\Big[\underbrace{z}_{\text{Constant Complex Amplitude}}\;\underbrace{e^{j2\pi 5t}}_{\text{time varying}}\Big]$$

$$z = 20\, e^{-j\frac{\pi}{2}}$$

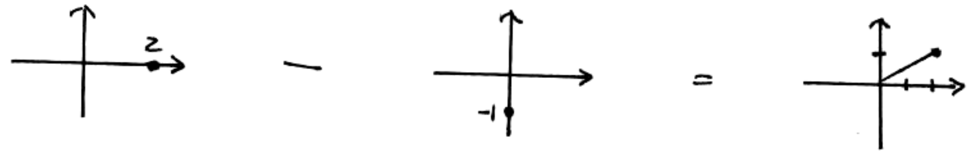
(1.9) $x_a(t) = 2\cos(333\pi t) - \sin(333\pi t)$

a) $\quad = 2\cos(333\pi t) - \cos\left(333\pi t - \frac{\pi}{2}\right)$

SAME FREQUENCY SO ADD COMPLEX AMPLITUDES

$$2\angle 0 \quad - \quad 1\angle -\tfrac{\pi}{2} \quad = \sqrt{5}\angle 0.464\,[\text{rad}]$$

$$(2+j0) \quad - \quad (-1j) \quad \Rightarrow \quad 2+j1$$



$\Sigma = \sqrt{5}\angle 0.464$

$$x_a(t) = \sqrt{5}\cos(333\pi t + 0.464)$$

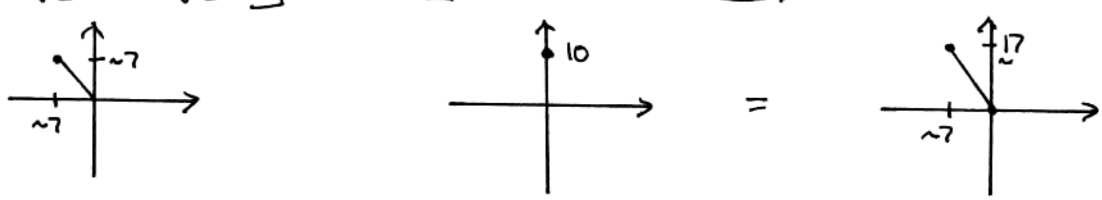
---

b) $x_b(t) = 10\cos(245t + 3\pi/4) + 10\cos(245t + \pi/2)$

COMPLEX AMPLITUDES

$$10\angle 3\pi/4 \quad + \quad 10\angle \pi/2 \quad \cong \quad 18.5\angle 1.96\,[\text{rad}]$$

$$\left[-\frac{10}{\sqrt{2}} + j\frac{10}{\sqrt{2}}\right] + [0 + j10] \cong -7.07 + j17.07$$



$$x_b(t) = 18.5\cos(245t + 1.96)$$

---

c) COMPLEX AMPLITUDES

$$(1\angle 17\pi = 1\angle\pi) + \sqrt{2}\angle\frac{\pi}{4} + \sqrt{2}\angle -\frac{\pi}{4} = 1\angle 0$$

NOTE

$$\Rightarrow x_c(t) = 1\cos(41t + 0) = \cos(41t)$$

ADD COMPLEX AMPLITUDES WITH MATLAB OR YOUR CALCULATOR'S COMPLEX NUMBER FUNCTIONS