GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Spring 2001
Problem Set #2

Assigned: 13-Jan-01
Due Date: Week of 22-Jan-01

Reading: In *DSP First*, all of Chapter 2 on *Sinusoids*. Then start reading in Chapter 3: *Spectrum Representation*, especially pp. 48–61.

⟹ Please check the "Bulletin Board" often. All official course announcements are posted there.

ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web. CD-ROM.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.

PROBLEM 2.1*:

Each of the following signals may be simplified, and expressed as a single sinusoid of the form: $A\cos(\omega t + \phi)$. For each signal, draw a vector diagram of the complex amplitudes (phasors), and use vector addition to estimate the amplitude A and phase ϕ of the sinusoid. Then use the phasor addition theorem to find the exact values for A and ϕ.

(a) $x_a(t) = 2\cos(40\pi t + 3\pi/4) + 2\cos(40\pi t - 3\pi/4)$

(b) $x_b(t) = \sqrt{2}\cos(200\pi t + 21\pi) + 2\cos(200\pi t - 24.5\pi) + \sqrt{3}\cos(200\pi t + 3\pi)$

(c) $x_c(t) = 10\cos(60\pi t + \pi/6) + 10\cos(60\pi t + 5\pi/6) + 10\cos(60\pi t - \pi/2)$

PROBLEM 2.2*:

Define $x(t)$ as

$$x(t) = 7\cos(100\pi t - 3\pi/4) + 5\cos(100\pi(t + 0.005)).$$

(a) Find a complex-valued signal $z(t) = Xe^{j\omega_0 t}$ such that $x(t) = \Re e\{z(t)\}$. Simplify $z(t)$ as much as possible, so that you can identify its complex amplitude. Give the numerical values of X and ω_0.

(b) Make a plot of $\Re e\{(1 + j\sqrt{3})e^{j20\pi t}\}$ over the range $-0.1 \le t \le 0.1$ secs. How many periods are included in the plot?

PROBLEM 2.3*:

Solve the following simultaneous equations by using a method based on complex amplitudes. Show how to convert the sinusoidal equations into complex-number equations. If we assume that the amplitudes are positive, will the answers for M_1 and M_2 be unique? How about ψ_1 and ψ_2; are there other answers for the phases?

$$\begin{aligned} \cos(1.5\pi t + 4\pi) &= M_1 \cos(1.5\pi t + \psi_1) + M_2 \cos(1.5\pi t + \psi_2) \\ 3\sqrt{2}\cos(1.5\pi t - 3\pi/4) &= M_1 \cos(1.5\pi t + \psi_1) - M_2 \cos(1.5\pi t + \psi_2) \end{aligned}$$

PROBLEM 2.4*:

Complex exponentials obey the expected rules of algebra when doing operations such as integrals, derivatives, and time-shifts. Consider the complex signal $z(t) = Ze^{j2\pi t}$ where $Z = e^{j\pi/4}$.

(a) Show that the first derivative of $z(t)$ with respect to time can be represented as a new complex exponential $Qe^{j2\pi t}$, i.e., $\frac{d}{dt}z(t) = Qe^{j2\pi t}$. Determine the value for the complex amplitude Q.

(b) Plot both Z and Q in the complex plane. How much greater (or smaller) is the angle of Q than the angle of Z?

(c) Compare $\Re e\{\frac{d}{dt}z(t)\}$ to $\frac{d}{dt}\Re e\{z(t)\}$ for the given signal $z(t)$. Do you think that this would be true for any complex exponential signal?

(d) Evaluate the definite integral of $z(t)$ over the range $-0.5 \le t \le 0.5$:

$$\int_{-0.5}^{0.5} z(t)dt = ?$$

Note that integrating a complex quantity follows the expected rules of algebra: you could integrate the real and imaginary parts separately, but you can also *use the integration formula for an exponential* directly on $z(t)$.

(e) Show that the time-shifted version of $z(t)$ can be represented as a new complex exponential $We^{j2\pi t}$, i.e., $z(t - t_d) = We^{j2\pi t}$. Determine the value for the complex amplitude W when $t_d = 0.125$ secs.

PROBLEM 2.5*:
Consider the signal

$$x(t) = 5 + 5\cos(3000\pi t - \pi/6) + 4\cos(8000\pi t + \pi/2).$$

This signal has three sinusoidal components (including the "DC" component, whose frequency is equal to zero).

(a) Express the signal $x(t)$ as a sum of five complex exponential components using the *inverse Euler* relationship

$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}.$$

(b) Which positive and negative frequencies (in Hz) are present in this signal?

(c) For each frequency identified in part (b), identify the complex amplitude of the corresponding complex exponential component. Give your answer as a table containing frequency in one column and complex amplitude in a second column.

PROBLEM 2.6:
Define $x(t)$ as

$$x(t) = 5\sqrt{2}\cos(20\pi t + \pi/4) + A\cos(20\pi t + \phi) \quad (1)$$

where A is a *positive* number. In addition, assume that $x(t)$ has a phase of zero, so that it may be written as

$$x(t) = B\cos(20\pi t), \quad (2)$$

where B is a *positive* number.

(a) What relationship must exist between A and ϕ in order for $x(t)$ to have zero phase as indicated in Eq. 2?

(b) If $B = 10$, what are the values for A and ϕ?

(c) Now assume that B is unspecified. Find the values for A, B, and ϕ so that the value of A is *minimized*. Draw a plot of the complex amplitudes to prove using a geometrical argument that you have found the minimum for A. *Hint: Recall the geometrical "theorem" that tells you how to find the shortest distance between a line and a point that is not on the line (have you heard the term "projection"?).*