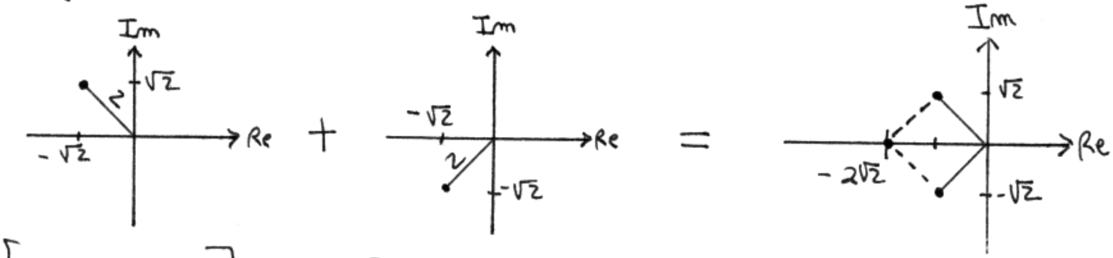
2.1

a) $x_a(t) = 2\cos\left(40\pi t + \frac{3}{4}\pi\right) + 2\cos\left(40\pi t - \frac{3}{4}\pi\right)$

$$2\angle\tfrac{3}{4}\pi \qquad + \qquad 2\angle -\tfrac{3}{4}\pi$$



$$\left[-\sqrt{2}+j\sqrt{2}\right] + \left[-\sqrt{2}-j\sqrt{2}\right] = -2\sqrt{2}+j0$$

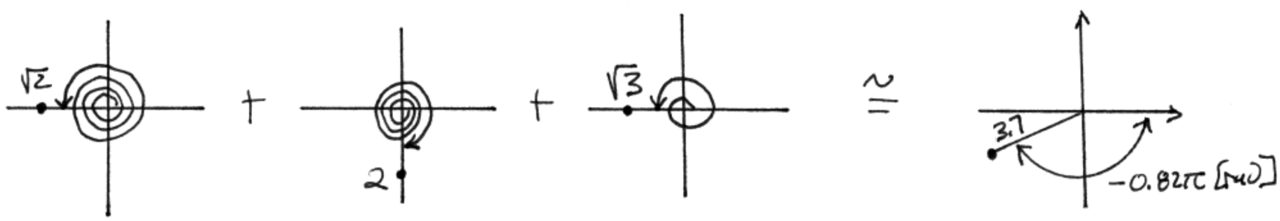
+ or −

$$= 2\sqrt{2}\angle\pi \text{ or } 2\sqrt{2}\angle -\pi$$

$x_a(t) = 2\sqrt{2}\cos\left(40\pi t + \pi\right)$

b) $x_b(t) = \sqrt{2}\cos\left(200\pi t + 21\pi\right) + 2\cos\left(200\pi t - 24.5\pi\right) + \sqrt{3}\cos\left(200\pi t + 3\pi\right)$

$$\left[\sqrt{2}\angle 21\pi = \sqrt{2}\angle\pi\right] + \left[2\angle -24.5\pi = 2\angle -\tfrac{\pi}{2}\right] + \left[\sqrt{3}\angle 3\pi = \sqrt{3}\angle\pi\right]$$



INITIAL GUESS: SOMEWHERE IN QUADRANT 3 $\frac{2|1}{3|4}$

$$\Sigma = \left[-\sqrt{2}+j0\right] + \left[0-j2\right] + \left[-\sqrt{3}+j0\right]$$

$$= -\left(\sqrt{2}+\sqrt{3}\right) + j(-2)$$

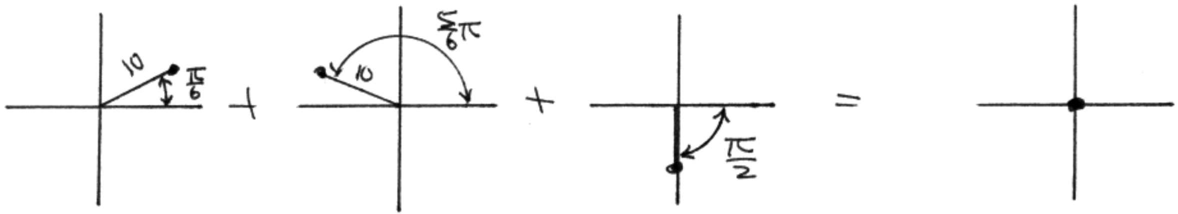
$$\cong 3.728\angle -0.82\pi$$

$x_b(t) \cong 3.728\cos\left(200\pi t - 0.82\pi\right)$

2.1

c) $x_c(t) = 10\cos\left(60\pi t + \frac{\pi}{6}\right) + 10\cos\left(60\pi t + \frac{\pi 5}{6}\right) + 10\cos\left(60\pi t - \frac{\pi}{2}\right)$

$$10\angle\frac{\pi}{6} \quad + \quad 10\angle\frac{5}{6}\pi \quad + \quad 10\angle -\frac{\pi}{2}$$



EACH VECTOR IS $\frac{1}{3}$ OF A CIRCLE ($120°$ or $\frac{2\pi}{3}$ [rad]) FROM IT'S NEIGHBUR AND EQUAL IN MAGNITUDE.

$\Rightarrow$ SO THE SUM = ZERO

$x_c(t) = 0$

2.2 $x(t) = 7\cos(100\pi t - \frac{3}{4}\pi) + 5\cos(100\pi t + \frac{\pi}{2})$

a)

$$= Re[z(t)]$$

$$= Re[(7e^{-j\frac{3}{4}\pi} + 5e^{j\frac{\pi}{2}})e^{j100\pi t}]$$

$$\cong Re[\underbrace{4.95e^{j\pi}}_{\text{complex amplitude}} \underbrace{e^{j100\pi t}}_{\text{time varying component}}]$$

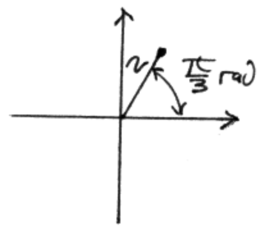
$X \cong 4.95\angle\pi$ $\qquad$ $\omega = 100\pi$ [rad/sec]

b) $Re[(1+j\sqrt{3})e^{j20\pi t}]$

$\hookrightarrow \omega = 20\pi$ [rad/sec]

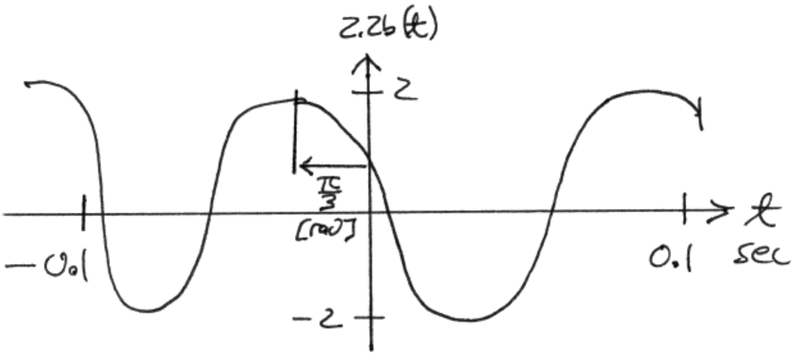
$f = 10$ [cycle/sec = Hz]

$\Rightarrow T = \frac{1}{f} = 0.1$ [sec/cycle]



$Re[2e^{j\frac{\pi}{3}}e^{j20\pi t}]$

$2\cos(20\pi t + \frac{\pi}{3})$ $\qquad$ $[-0.1 < t < 0.1 \text{ sec}] \Rightarrow 2$ Periods



(2.3)

$$[1\angle 4\pi = 1\angle 0] = M_1\angle\Phi_1 + M_2\angle\Phi_2 \qquad \boxed{\text{eqn 1}}$$

$$3\sqrt{2}\angle -\tfrac{3}{4}\pi = M_1\angle\Phi_1 - M_2\angle\Phi_2 \qquad \boxed{\text{eqn 2}}$$

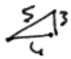
$\boxed{\text{eqn 1} + \text{eqn 2}}$

$$\underbrace{1\angle 0 + 3\sqrt{2}\angle -\tfrac{3}{4}\pi}_{\cong\, 3.606\angle -0.687\pi} = 2M_1\angle\Phi_1$$

$$M_1 \cong 1.803$$

$$\Phi_1 \cong -0.687\pi$$

$\boxed{\text{eqn 1} - \text{eqn 2}}$



$$\underbrace{1\angle 0 - 3\sqrt{2}\angle -\tfrac{3}{4}\pi}_{5\angle(0.205\pi = 36.9^\circ)} = 2M_2\angle\Phi_2$$

$$M_2 = 2.5$$

$$\Phi_2 \cong 0.205\pi$$

UNIQUENESS: Φ_1, Φ_2: EITHER CAN WORK WITH ANY ADDED INTEGER MULTIPLES OF 2π

M_1, M_2: IF WE ARE WILLING TO ACCEPT NEGATIVE VALUES FOR M_1, M_2 (WHICH WE DON'T), THEN OFFSET THE CORRESPONDING ANGLE BY π

In MATLAB, the backslash operator "\" will solve simultaneous equations. In this problem

$$\begin{bmatrix} 1e^{j4\pi} \\ 3\sqrt{2}e^{-j3\pi/4} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$

So you can do the following in MATLAB

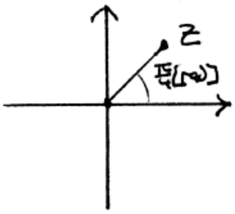
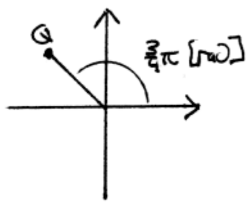
```
z = [1,1;1,-1] \ [1; 3*sqrt(2)*exp(-j*3*pi/4)]
```

2.4

a) $$\frac{\partial z(t)}{\partial t} = \frac{\partial\left(e^{j\frac{\pi}{4}} e^{j2\pi t}\right)}{\partial t} = e^{j\frac{\pi}{4}} \frac{\partial e^{j2\pi t}}{\partial t}$$

$$= e^{j\frac{\pi}{4}} \left(j = e^{j\frac{\pi}{2}}\right) 2\pi\, e^{j2\pi t}$$

$$= \left(Q = 2\pi e^{j\frac{3}{4}\pi}\right) e^{j2\pi t}$$

b) $Z =$  $Q =$ 

$\Delta = 90^\circ$

c) $$\frac{\partial \operatorname{Re}[z(t)]}{\partial t} = \frac{\partial}{\partial t} \cos\left(2\pi t + \frac{\pi}{4}\right)$$

$$= -2\pi \sin\left(2\pi t + \frac{\pi}{4}\right)$$

$$= -2\pi \cos\left(2\pi t + \frac{\pi}{4} - \frac{\pi}{2}\right)$$

$$= 2\pi \cos\left(\underbrace{2\pi t + \frac{\pi}{4}}_{\text{initial cosine arguement}} - \frac{\pi}{2} + \pi\right)$$

$-\frac{\pi}{2}$: sine to cosine

$+\pi$: change magnitude sign

$$= 2\pi \cos\left(2\pi t + \frac{3}{4}\pi\right)$$

$$= \operatorname{Re}\left[\frac{\partial z(t)}{\partial t}\right] \Rightarrow \text{They are the same}$$

d) Note: $\omega = 2\pi$ [rad/sec] $\Rightarrow f = 1$ [cycle/sec] $\Rightarrow T = 1$ sec $\Rightarrow \int_{-\frac{1}{2}}^{\frac{1}{2}} \partial t$ IS ONE CYCLE

$$\int_{-\frac{1}{2}}^{\frac{1}{2}} z(t)\,\partial t = \int_{-\frac{1}{2}}^{\frac{1}{2}} e^{j\frac{\pi}{4}} e^{j2\pi t}\,\partial t = e^{j\pi/4} \int e^{j2\pi t}\,\partial t$$

$$= \frac{e^{j\frac{\pi}{4}}}{\left(j = e^{j\frac{\pi}{2}}\right) 2\pi} \left[e^{j2\pi t}\Big|_{-\frac{1}{2}}^{\frac{1}{2}} = -1 - (-1) = 0 \right]$$

$$= 0$$

MAKES SENSE:
+ AND − SINUSOID PORTIONS CANCEL

2.4

e)

$$z(t) = e^{j\frac{\pi}{4}} e^{j2\pi t}$$

$$z(t-t_0) = e^{j\frac{\pi}{4}} e^{j2\pi(t-t_0)}$$

$$= \underbrace{e^{j\pi(\frac{1}{4}-2t_0)}}_{W = e^{j\pi(\frac{1}{4}-2t_0)}} e^{j2\pi t}$$

$$W(t_0 = 0.125 = \tfrac{1}{8}\,\text{sec}) = e^{j\pi(\frac{1}{4}-2\cdot\frac{1}{8})} = 1$$

(2.5)

$$5 + 5\cos\left(3000\pi t - \frac{\pi}{6}\right) + 4\cos\left(8000\pi t + \frac{\pi}{2}\right) = x(t)$$

$$2e^{-j\frac{\pi}{2}}e^{-j8000\pi t} + \frac{5}{2}e^{+j\frac{\pi}{6}}e^{-j3000\pi t} + 5 + \frac{5}{2}e^{-j\frac{\pi}{6}}e^{j3000\pi t} + 2e^{j\frac{\pi}{2}}e^{j8000\pi t} = x(t)$$

0 Hz "DC"

1500 Hz

4000 Hz

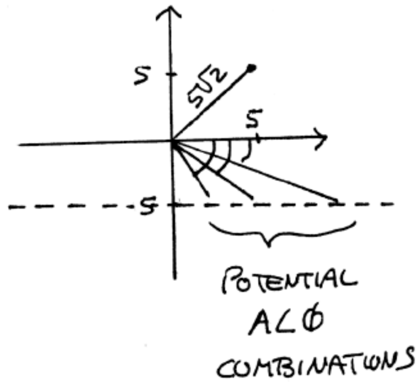
COMPLEX AMPLITUDE		$2e^{-j\frac{\pi}{2}} = 2\angle -\frac{\pi}{2}$	$\frac{5}{2}e^{j\frac{\pi}{6}} = \frac{5}{2}\angle\frac{\pi}{6}$	$5 = 5\angle 0$	$\frac{5}{2}e^{-j\frac{\pi}{6}} = \frac{5}{2}\angle -\frac{\pi}{6}$	$2e^{j\frac{\pi}{2}} = 2\angle\frac{\pi}{2}$
f	cycles/sec	-4000	-1500	$0 =$ "DC"	1500	4000
ω	rad/sec	-8000π	-3000π	$0 =$ "DC"	3000π	8000π

AMPLITUDES ARE COMPLEX CONJUGATES

2.6

IF $\angle x(t) = 0$ THEN $\mathrm{IM}(X) = 0$

a) $$B\angle 0 = 5\sqrt{2}\angle \pi/4 + A\angle \phi$$



THE IMAG PART OF THE A TERM MUST CANCEL THAT OF THE $5\sqrt{2}\angle\frac{\pi}{4}$ TERM

SO, $A \sin(\phi) = -5$

b) IF $B = 10$

$$10\angle 0 = 5\sqrt{2}\angle\frac{\pi}{4} + A\angle\phi$$

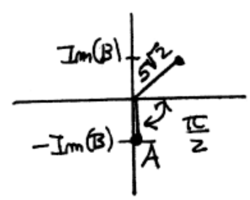
$$10 = \mathrm{Real}(5\sqrt{2}\angle\tfrac{\pi}{4}) + \mathrm{Real}(A\angle\phi)$$

$$10 = 5 + A\cos(\phi)$$

$5 = A\cos(\phi)$ AND ϕ IN QUADRANT 4 $\quad \begin{array}{c|c} 2 & 1 \\ \hline 3 & 4 \end{array}$

FOR EXAMPLE: $A = 5\sqrt{2}$ AND $\phi = -\frac{\pi}{4}$

c) A is minimized when $\phi = -\frac{\pi}{2} \Rightarrow A\sin(-\frac{\pi}{2}) = -5$ CANCELING THE IMAG PORTION OF THE $5\sqrt{2}\angle\frac{\pi}{4}$ TERM



GEOMETRIC PROOF: PLOT (for $\forall\phi$): $\frac{5}{\sin(\phi)} = A$

B = don't care