GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Spring 2001
Problem Set #3

Assigned: 19-January-01
Due Date: Week of 29-January-01

Quiz #1 will be held in lecture on Friday 2-February-01. It will cover material from Chapters 2 and 3, as represented in Problem Sets #1, #2, and #3.
Closed book, calculators permitted, and one hand-written formula sheet ($8\frac{1}{2}'' \times 11''$, **both sides)**

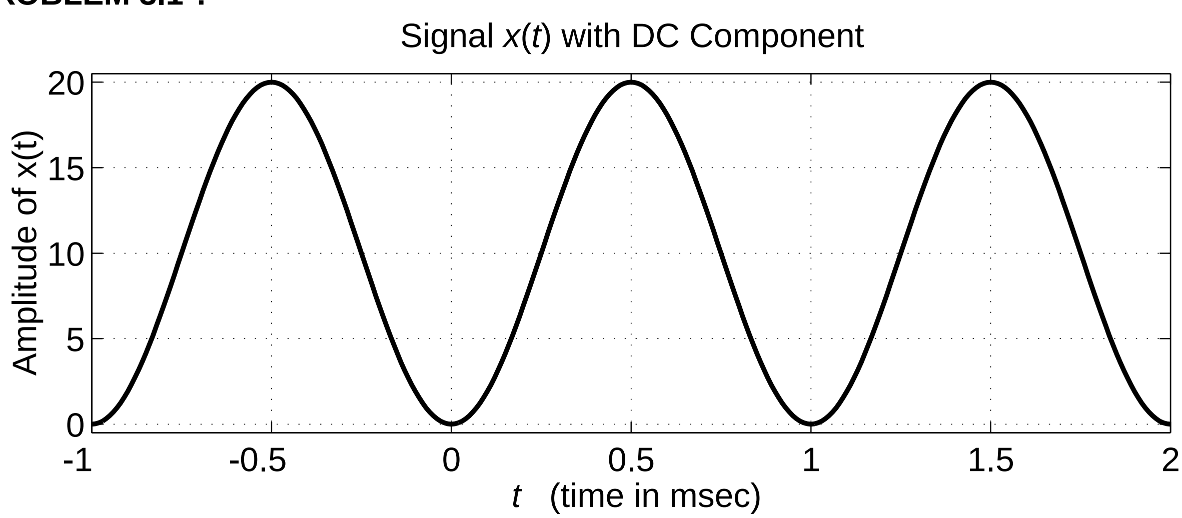
Reading: In *DSP First*, all of Chapter 3 on *Spectrum Representation,* especially pp. 48–73.

$\Longrightarrow$ **Please check the "Bulletin Board" often. All official course announcements are posted there.**

ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web. Some problems have solutions similar to those found on the CD-ROM.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.

PROBLEM 3.1*:



The above signal $x(t)$ consists of a DC component plus a cosine signal. The terminology *DC component* means a component that is constant versus time.

(a) What is the frequency of the DC component? What is the frequency of the cosine component?

(b) Write an equation for the signal $x(t)$. You should be able to determine numerical values for all the amplitudes, frequencies, and phases in your equation by inspection of the above graph.

(c) Expand the equation obtained in the previous part into a sum of positive and negative frequency complex exponential signals.

(d) Plot the two-sided spectrum of the signal $x(t)$. Show the complex amplitudes for each positive and negative frequency contained in $x(t)$.

PROBLEM 3.2:

Try working this problem after you have worked Problem 3.1. It should be easy.
Consider the signal

$$x(t) = 20[\sin(1000\pi t)]^2.$$

(a) Using the inverse Euler relation for the sine function, express $x(t)$ as a sum of complex exponential signals with positive and negative frequencies.

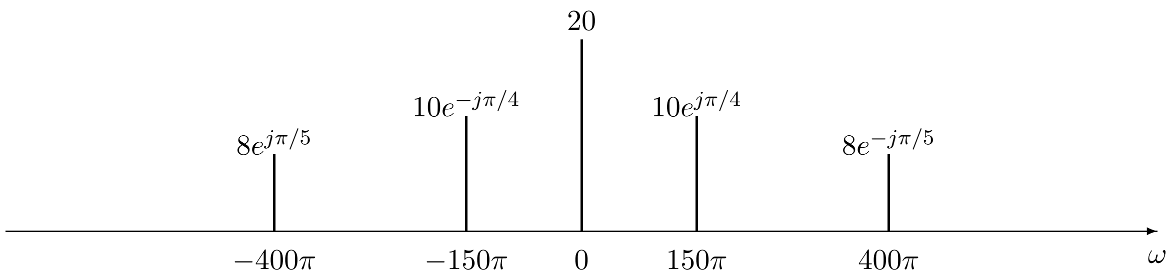
(b) Use your result in part (a) to express $x(t)$ in the form $x(t) = A_0 + A_1 \cos(\omega_0 t)$.

(c) Determine the period T_0 of $x(t)$ and sketch its waveform over the interval $-T_0 \le t \le 2T_0$. Carefully label the graph.

(d) Plot the spectrum of $x(t)$.

PROBLEM 3.3*:

A real signal $x(t)$ has the following two-sided spectrum:



(a) Write an equation for $x(t)$ as a sum of cosines.

(b) Plot the spectrum of the signal $y(t) = 0.5x(t) + 6\cos(250\pi(t - 0.002))$.

PROBLEM 3.4*:

A signal composed of sinusoids is given by the equation

$$x(t) = 4\cos(50\pi t - \pi/4) - 2\cos(150\pi t).$$

(a) Sketch the spectrum of this signal indicating the complex amplitude of each frequency component. You do not have to make separate plots for real/imaginary parts or magnitude/phase. Just indicate the complex amplitude value at the appropriate frequency.

(b) Is $x(t)$ periodic? If so, what is the period? Which harmonics are present?

(c) Now consider a new signal $y(t) = x(t) + 6\cos(60\pi t + \pi/3)$. How is the spectrum changed? Is $y(t)$ periodic? If so, what is the period of $y(t)$?

(d) Finally, consider another new signal $w(t) = x(t) + \cos(50t)$. How is the spectrum changed? Is $w(t)$ periodic? If so, what is the period of $w(t)$? If not, why not?

PROBLEM 3.5*:

(Similar to *DSP First*, Chapter 3, Problem 8, page 80.)
We have seen that musical tones can be modeled mathematically by sinusoidal signals. If you read music or play the piano you know well that the piano keyboard is divided into octaves, with the tones in each octave being twice the frequency of the corresponding tones in the next lower octave. To calibrate the frequency scale, the reference tone is the A above middle-C, which is usually called A440 since its frequency is 440 Hz. Each octave contains 12 tones, and the ratio between the frequencies of successive tones is constant. Thus, the ratio must be $2^{1/12}$. Since middle C is 9 tones below A440, its frequency is approximately $(440)2^{-9/12} \approx 262$ Hz. The names of the tones (notes) of the octave starting with middle-C and ending with high-C are:

note name	C	$C^\sharp$	D	$E^\flat$	E	F	$F^\sharp$	G	$G^\sharp$	$A(440)$	$B^\flat$	B	C
note number	40	41	42	43	44	45	46	47	48	49	50	51	52
frequency										440			

(a) Make a table of the frequencies of the tones of the octave beginning with middle-C assuming that A above middle C is tuned to 440 Hz.

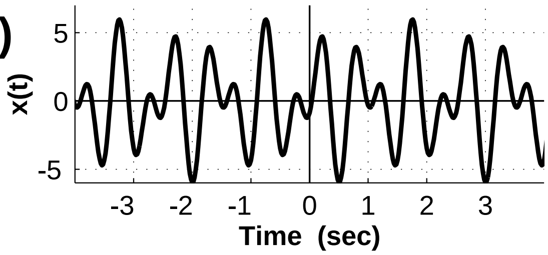
(b) The above notes on a piano are numbered 40 through 52. If n denotes the note number, and f denotes the frequency of the corresponding tone, give a formula for the frequency of the tone as a function of the note number.

(c) A *chord* is a combination of musical notes sounded simultaneously. A *triad* is a three note chord. The D Major chord is composed of the tones of $D\ \ F^\sharp\ \ A$ sounded simultaneously. From the set of corresponding frequencies determined in part (a), make a sketch of the essential features of the spectrum of the D Major chord assuming that each note is realized by a pure sinusoidal tone. (You do not have to specify the complex phasors precisely.)

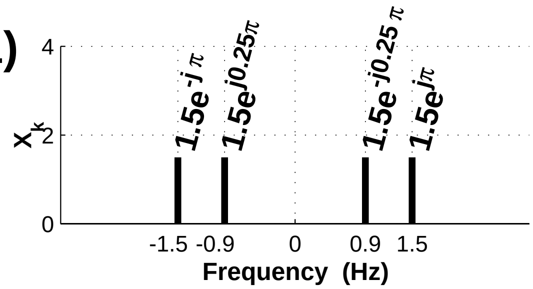
PROBLEM 3.6*:

The following plots show waveforms on the left and spectra on the right. Hand in a table matching the waveform letter with its corresponding spectrum number.

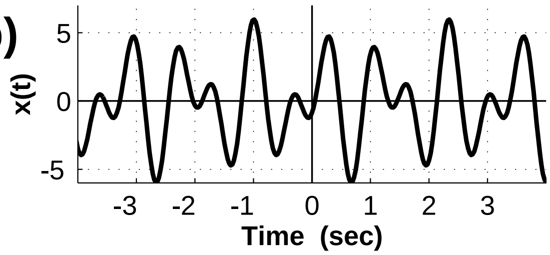
(a)



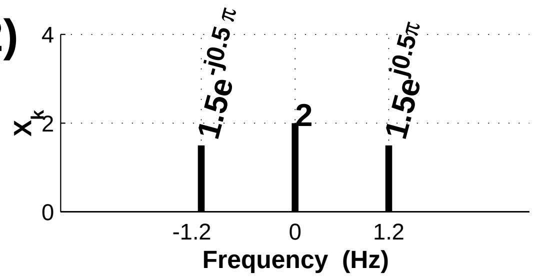
(1)



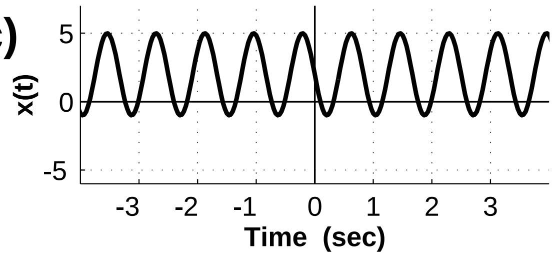
(b)



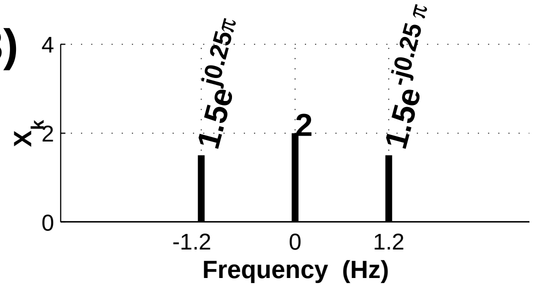
(2)



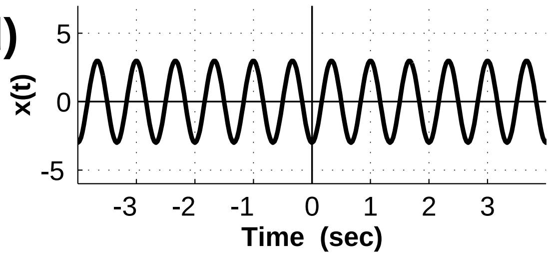
(c)



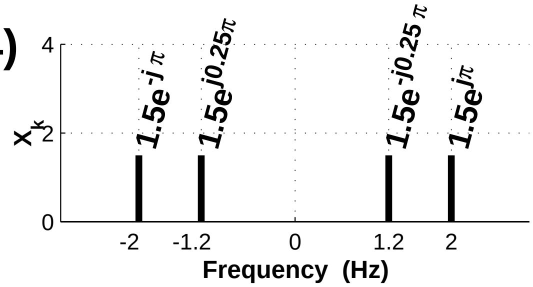
(3)



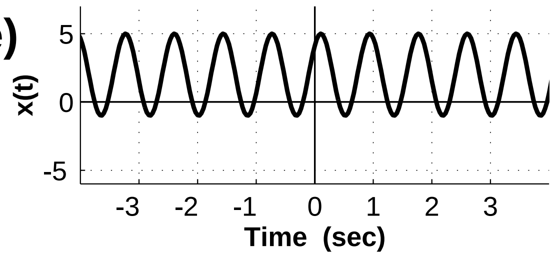
(d)



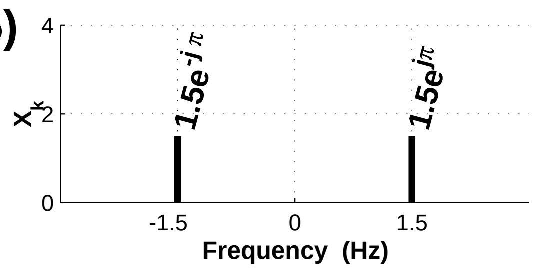
(4)



(e)



(5)



PROBLEM 3.7:

A linear-FM "chirp" signal is one that sweeps in frequency from $\omega_1 = 2\pi f_1$ to $\omega_2 = 2\pi f_2$ as time goes from $t = 0$ to $t = T_2$. We can define the *instantaneous frequency* of the chirp as the derivative of the phase of the sinusoid:

$$x(t) = A\cos(\alpha t^2 + \beta t + \phi) \tag{1}$$

where the cosine function operates on a time-varying argument

$$\psi(t) = \alpha t^2 + \beta t + \phi$$

The derivative of the argument $\psi(t)$ is the *instantaneous frequency* which is also the audible frequency heard from the chirp *if the chirping frequency does not change too rapidly.*

$$\omega_i(t) = \frac{d}{dt}\psi(t) \qquad \text{radians/sec} \tag{2}$$

There are examples on the CD-ROM in the Chapter 3 demos.

(a) For the linear-FM "chirp" in (1), determine formulas for the beginning instantaneous frequency (ω_1) and the ending instantaneous frequency (ω_2) in terms of α, β and T_2. For this problem, assume that the starting time of the "chirp" is $t = 0$.

(b) For the "chirp" signal

$$x(t) = \Re e\left\{e^{j2\pi(30t^2 - 30t)}\right\}$$

derive a formula for the *instantaneous* frequency versus time. Should your answer for the frequency be a positive number?

(c) For the signal in part (b), make a plot of the *instantaneous* frequency (in Hz) versus time over the range $0 \le t \le 1$ sec.