GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Spring 2001
Problem Set #6

Assigned: 9-Feb-01
Due Date: Week of 19-Feb-01

There will be a lab quiz at the beginning of Lab #6 (20-23-Feb).

Quiz #2 on 2-March (Friday).

Reading: In *DSP First*, Chapter 5 on *FIR Filters* and Chapter 6 on *Frequency Response.*

⟹ **Please check the "Bulletin Board" often. All official course announcements are posted there.**

ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web. Some problems have solutions similar to those found on the CD-ROM.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.

PROBLEM 6.1:

A signal composed of sinusoids is given by the equation

$$x(t) = 10\cos(800\pi t + \pi/4) + 7\cos(1200\pi t - \pi/3) - 3\cos(1600\pi t)$$

Determine the lowest sampling frequency $f_s = 1/T_s$ such that the signal $x(t)$ can theoretically be reconstructed exactly from its samples $x[n] = x(nT_s)$.

PROBLEM 6.2*:

Consider a continuous-time signal

$$x(t) = A\cos(\omega_0 t + \phi)$$

We know that this signal is periodic with period $T_0 = 2\pi/\omega_0$; i.e. $x(t + T_0) = x(t)$ for all t. Now suppose that $x(t)$ is sampled to obtain the sequence

$$x[n] = x(nT_s) = A\cos(\omega_0 nT_s + \phi) = A\cos(\hat{\omega}_0 n + \phi)$$

where $\hat{\omega_0} = \omega_0 T_s$.

Now a discrete-time signal is periodic with period N if $x[n + N] = x[n]$ for all n, where N is necessarily an integer.

(a) Will $x[n]$ be periodic for all possible sampling rates? If not, what condition on T_s will ensure that $x[n]$ is periodic with period N?

(b) If $\omega_0 = 2000\pi$, what value of T_s will result in a periodic sequence with period $N = 100$?

PROBLEM 6.3*:

This problem is concerned with finding the output of an FIR filter for a given input signal. A linear time-invariant system is described by the difference equation

$$y[n] = \sum_{k=0}^{4} (5-k)x[n-k]$$

(a) Determine the filter coefficients $\{b_k\}$ of this FIR filter.

(b) Find the impulse response, $h[n]$, for this FIR filter. The impulse response is a discrete-time signal, so make a (stem) plot of $h[n]$ versus n.

(c) Use the above difference equation to compute the output $y[n]$ when the input is

$$x[n] = \begin{cases} 0 & n < 0 \\ 10 & 0 \leq n \leq 5 \\ 1 & 6 \leq n \leq 10 \\ 0 & n \geq 11. \end{cases}$$

Make a plot of both $x[n]$ and $y[n]$ vs. n. (Hint: you might find it useful to check your results with MATLAB's `conv()` function.)

PROBLEM 6.4*:

The *unit step* sequence, denoted by $u[n]$, is defined as

$$u[n] = \begin{cases} 0 & n < 0 \\ 1 & n \geq 0 \end{cases}$$

(a) Make a plot of $u[n]$ for $-5 \leq n \leq 10$. Describe the plot of $u[n]$ outside this range.

(b) We can use the unit step sequence as a convenient representation for sequences that are given by formulas over a range of values. For example, make a plot of the sequence

$$x[n] = (.5)^n(u[n] - u[n-5])$$

for $-5 \leq n \leq 10$. *Hint: First determine the values of the sequence* $(u[n] - u[n-5])$.

(c) Suppose that $x[n]$ in part (b) is the input to a 4-point running average system. Compute and plot $y[n]$, the output of the system for $-5 \leq n \leq 10$.

PROBLEM 6.5:

Consider a system defined by $\quad y[n] = \sum_{k=5}^{10} b_k x[n-k]$

Notice that the filter coefficients b_0, b_1, b_2, ..., b_4 are all zero.

Suppose that the input $x[n]$ is non-zero only for $5 \leq n \leq 20$. Show that $y[n]$ is non-zero at most over a finite interval of the form $N_3 \leq n \leq N_4$. Determine N_3 and N_4.

Hint: consult Figs. 5.5 and 5.6 in the book for the sliding window interpretation of the FIR filter.

PROBLEM 6.6*:

The diagram in Fig. 1 depicts a *cascade connection* of two linear time-invariant systems; i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

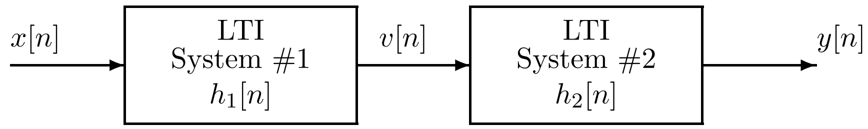

Figure 1: Cascade connection of two LTI systems.

Suppose that System #1 has impulse response,

$$h_1[n] = \begin{cases} 0 & n < 0 \\ 0.1 & n = 0 \leq n \leq 9 \\ 0 & n > 9 \end{cases}$$

and System #2 is described by the difference equation

$$y[n] = v[n] - v[n-1]$$

(a) Determine the difference equation of System #1; i.e., the equation that relates $v[n]$ to $x[n]$.

(b) When the input signal $x[n]$ is an impulse, $\delta[n]$, determine the signal $v[n]$ and make a plot.

(c) Determine $h_2[n]$, the impulse response of System #2.

(d) Determine the impulse response of the overall cascade system, i.e., find $y[n]$ when $x[n] = \delta[n]$.

(e) From the impulse response of the overall cascade system as obtained in part (d), obtain a single difference equation that relates $y[n]$ directly to $x[n]$ in Fig. 1.

PROBLEM 6.7*:

For each of the following systems, determine if they are (1) linear; (2) time-invariant; (3) causal.

(a) $y[n] = x[n-2] + 2x[n] + x[n+2]$

(b) $y[n] = nx[n]$

(c) $y[n] = (x[-n])^2$