GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Spring 2001
Problem Set #7

Assigned: 16-Feb-01
Due Date: Week of 26-Feb-01

There will be a lab quiz at the beginning of Lab #6 (19-22-Feb).

Quiz #2 on 2-March (Friday). Covers Problem Sets #4-#7.

Reading: In *DSP First*, Chapter 5 on *FIR Filters* and Chapter 6 on *Frequency Response.*

$\Longrightarrow$ **Please check the "Bulletin Board" often. All official course announcements are posted there.**

ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web. Some problems have solutions similar to those found on the CD-ROM.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.

PROBLEM 7.1:

For each of the following systems, determine if they are (1) linear; (2) time-invariant; (3) causal.

(a) $y[n] = 3x[n-1] + x[n] + 3x[n+1]$

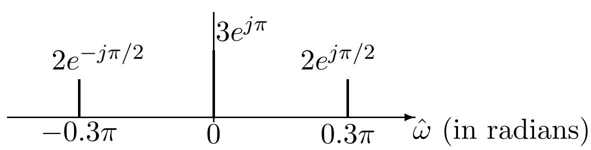
(b) $y[n] = x[n]\cos(.3\pi n)$

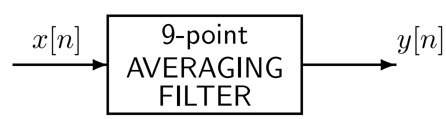
(c) $y[n] = |x[-n]|$

Check the solution to Problem 6.1 on Problem Set #6 for Fall 2000.

PROBLEM 7.2:

A discrete-time signal $x[n]$ has the two-sided spectrum representation shown below.





(a) Write an equation for $x[n]$. Make sure to express $x[n]$ as a real-valued signal.

(b) Determine the formula for the output signal $y[n]$.

See Problem 6.1 of Spring 1999 for solution to this problem.

PROBLEM 7.3*:

A discrete-time system is defined by the input/output relation

$$y[n] = 2x[n+2] + 6x[n] + 2x[n-2]. \tag{1}$$

(a) Determine whether or not the system defined by Equation (1) is (i) linear; (ii) time-invariant; (iii) causal. Explain your answers.

(b) Obtain an expression for the frequency response of this system.

(c) Make a sketch of the frequency response (magnitude and phase) as a function of frequency. *Hint: Use symmetry to simplify your expression before determining the magnitude and phase.*

(d) For the system of Equation (1), determine the output $y_1[n]$ when the input is

$$x_1[n] = 10 - 10\cos(0.5\pi(n-1))$$

Hint: Use the frequency response and superposition to solve this problem.

PROBLEM 7.4:

A second discrete-time system is defined by the input/output relation

$$y[n] = (x[n+1])^3. \tag{2}$$

(a) Determine whether or not the system defined by (2) is (i) linear; (ii) time-invariant; (iii) causal.

(b) For the system of Equation (2), determine the output $y_1[n]$ when the input is

$$x_1[n] = 2\cos(0.6\pi n) = e^{j0.6\pi n} + e^{-j0.6\pi n}.$$

Express your answer in terms of cosine functions. Do not leave any powers of cosine functions in your answers. Note that this system produces output contain frequencies that are not present in the input signal. Explain how this system might cause "aliasing" of sinusoidal components of the input.

See Problem 6.4 of Problem Set #6, Fall 2000 for a problem like this.

PROBLEM 7.5*:

The diagram in Fig. 1 depicts a *cascade connection* of two linear time-invariant systems; i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

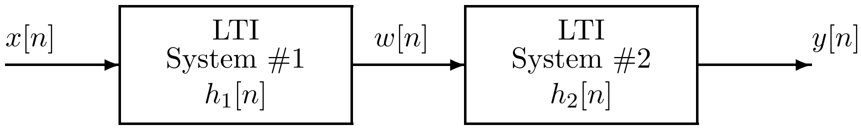


Figure 1: Cascade connection of two LTI systems.

(a) Suppose that LTI System #1 is described by the difference equation

$$w[n] = x[n] - \alpha x[n-1].$$

Determine the impulse response $h_1[n]$ of the first system.

(b) The LTI System #2 is described by the impulse response

$$h_2[n] = \alpha^n (u[n] - u[n-L]) = \sum_{k=0}^{L-1} \alpha^k \delta[n-k] = \begin{cases} \alpha^n & n = 0, 1, \ldots, L-1 \\ 0 & \text{otherwise.} \end{cases}$$

For the special case of $L = 5$, use convolution to show that the impulse response sequence of the overall cascade system is

$$h[n] = h_1[n] * h_2[n] = \delta[n] - \alpha^5 \delta[n-5].$$

(c) Generalize your result in part (b) for the general case of L any integer value.

(d) Obtain a single difference equation that relates $y[n]$ to $x[n]$ in Fig. 1.

(e) Assuming that $0 < \alpha < 1$, how would you choose L so that $y[n] = x[n]$ in Figure 1; i.e., how would you choose L so that the second system "undoes" the effect of the first system?

You will use the results of this problem in Lab #6.

PROBLEM 7.6*:

Consider again the cascade system in Figure 1 with

$$h_1[n] = \delta[n] - \delta[n-1] \qquad \text{and} \qquad h_2[n] = u[n] - u[n-5].$$

(a) Determine $H_1(\hat{\omega})$, the frequency response of the first system.

(b) Determine $H_2(\hat{\omega})$, the frequency response of the second system.

(c) By convolution, show that $h[n] = h_1[n] * h_2[n] = \delta[n] - \delta[n-5]$ (see part part (c) of Problem 7.5 with $\alpha = 1$). From $h[n]$ determine $H(\hat{\omega})$ the frequency response of the overall system (from $x[n]$ to $y[n]$).

(d) Show that your result in part (c) is the product of the results in parts (a) and (b); i.e., $H_1(\hat{\omega})H_2(\hat{\omega}) = H(\hat{\omega})$.

PROBLEM 7.7*:

A linear time-invariant discrete-time system is described by the difference equation

$$y[n] = x[n] + 2x[n-1] + 3x[n-2] - x[n-4].$$

(a) Draw a block diagram that represents this system in terms of unit-delay elements, coefficient multipliers, and adders as in Figure 5.13 in the text.

(b) Determine the impulse response $h[n]$ for this system. Express your answer as a sum of scaled and shifted unit impulse sequences.

(c) Use convolution to determine the output due to the input

$$x[n] = \delta[n] - \delta[n-1] + \delta[n-2]$$

Plot the output sequence $y[n]$ for $-3 \le n \le 10$.

(d) Now consider another LTI system whose impulse response is

$$h_d[n] = \delta[n] - \delta[n-1] + \delta[n-2].$$

Use convolution again to determine $y_d[n] = x_d[n] * h_d[n]$, the output of this system when the input is

$$x_d[n] = \delta[n] + 2\delta[n-1] + 3\delta[n-2] - \delta[n-4].$$

How does your answer compare to the answer in part (c)? This example illustrates the general commutative property of convolution; i.e., $x[n] * h[n] = h[n] * x[n]$.

PROBLEM 7.8*:

The frequency response of a linear time-invariant filter is given by the formula

$$H(\hat{\omega}) = (1 + 0.8e^{-j\hat{\omega}})(1 - e^{-j\pi/2}e^{-j\hat{\omega}})(1 - e^{j\pi/2}e^{-j\hat{\omega}}). \tag{3}$$

(a) Write the difference equation that gives the relation between the input $x[n]$ and the output $y[n]$. *Hint: Multiply out the factors to obtain a sum of powers of* $e^{-j\hat{\omega}}$.

(b) What is the impulse response of this system?

(c) If the input is of the form $x[n] = Ae^{j\phi}e^{j\hat{\omega}n}$, for what values of $-\pi \le \hat{\omega} \le \pi$ will $y[n] = 0$ for all n?

(d) Use superposition to determine the output of this system when the input is

$$x[n] = 3 + \delta[n-3] + e^{j0.5\pi n} \qquad \text{for } -\infty < n < \infty$$

Hint: Divide the input into three parts and find the outputs separately each by the easiest method and then add the results.