ECE2025 HW#9 Spring 2001

9.1 (a) $\delta(t-10) * \left[\delta(t+10) + 2e^{-t}u(t) + \cos(100\pi t)\right]$

$= \delta(t-10) * \delta(t+10) + \delta(t-10) * 2e^{-t}u(t) + \delta(t-10) * \cos(100\pi t)$

$= \delta(t) + 2e^{-(t-10)}u(t-10) + \cos\left[100\pi(t-10)\right]$

9.1 (b) $\cos(100\pi t)\left[\delta(t) + \delta(t-.002)\right]$

$= \cos(100\pi\cdot 0)\,\delta(t) + \cos(100\pi(.002))\,\delta(t-.002)$

$= 1\,\delta(t) + \cos(0.2\pi)\,\delta(t-.002) = \delta(t) + 0.809\,\delta(t-.002)$

9.1 (c) $\frac{d}{dt}\left[e^{-2(t-2)}u(t-2)\right]$ Use formula for derivative of a product.

$= \frac{d}{dt}\left[e^{-2t}e^{4}u(t-2)\right] = e^{4}(-2)e^{-2t}u(t-2) + e^{-2t}e^{4}\delta(t-2)$

$= -2e^{4}e^{-2t}u(t-2) + e^{-2(2)}e^{4}\delta(t-2)$

$= -2e^{4}e^{-2t}u(t-2) + \delta(t-2)$

9.1 (d) $\int_{-\infty}^{t}\cos(100\pi\tau)\left[\delta(\tau) + \delta(\tau-.002)\right]d\tau$

$= \int_{-\infty}^{t}\cos(100\pi\cdot 0)\,\delta(\tau)\,d\tau + \int_{-\infty}^{t}\cos(100\pi(.002))\,\delta(\tau-.002)\,d\tau$

$= 1\,u(t) + \cos(0.2\pi)\,u(t-.002) = u(t) + 0.809\,u(t-.002)$

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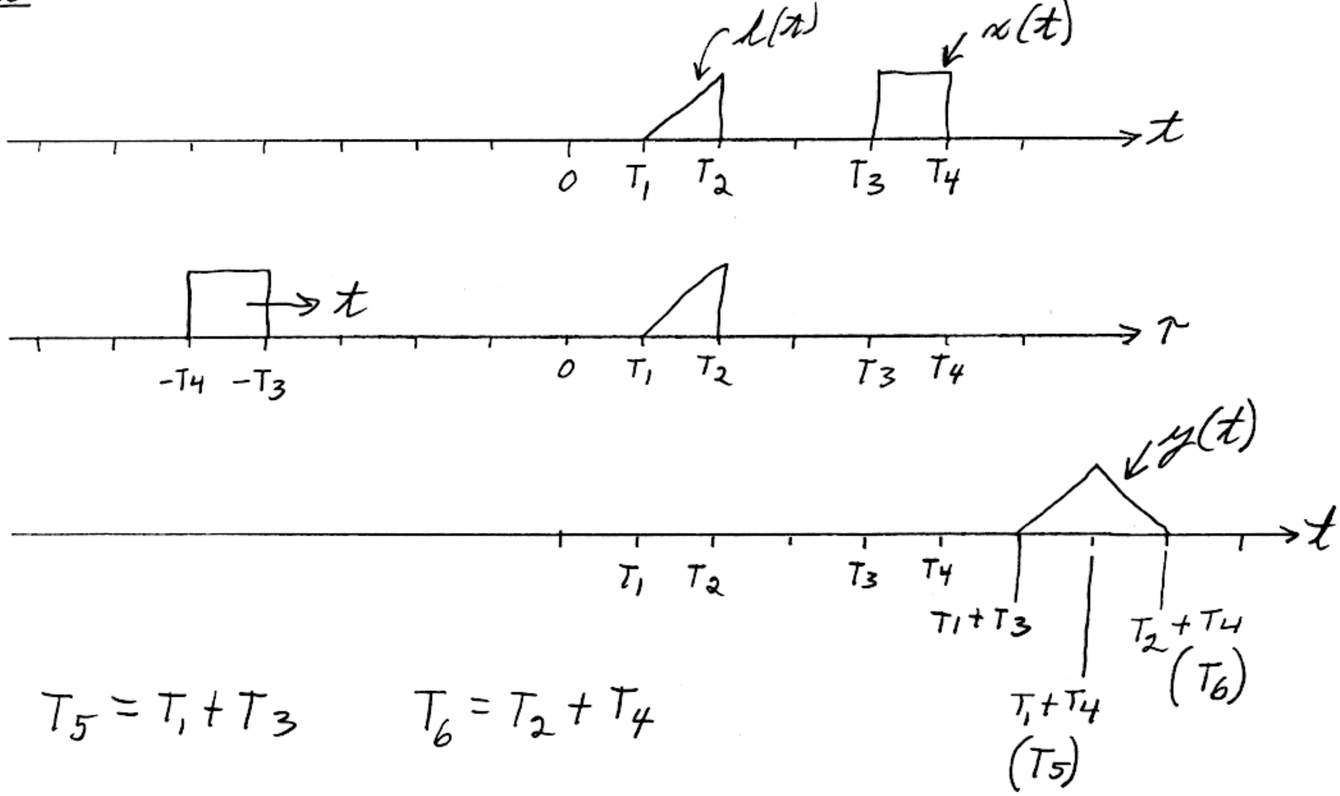
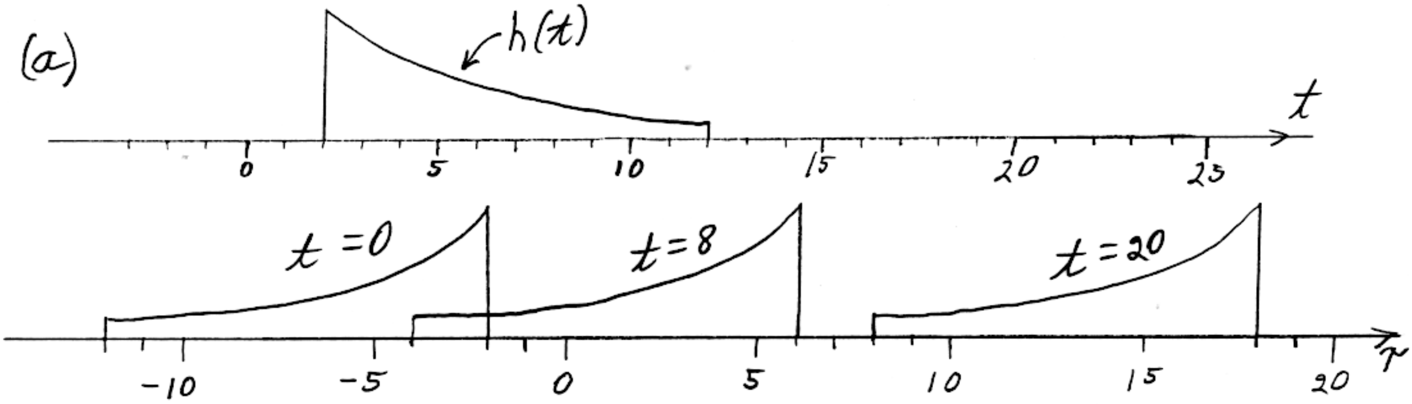
9.2



$T_5 = T_1 + T_3 \qquad T_6 = T_2 + T_4$

9.3 $h(t) = e^{-0.1(t-2)}\left[u(t-2) - u(t-12)\right]$

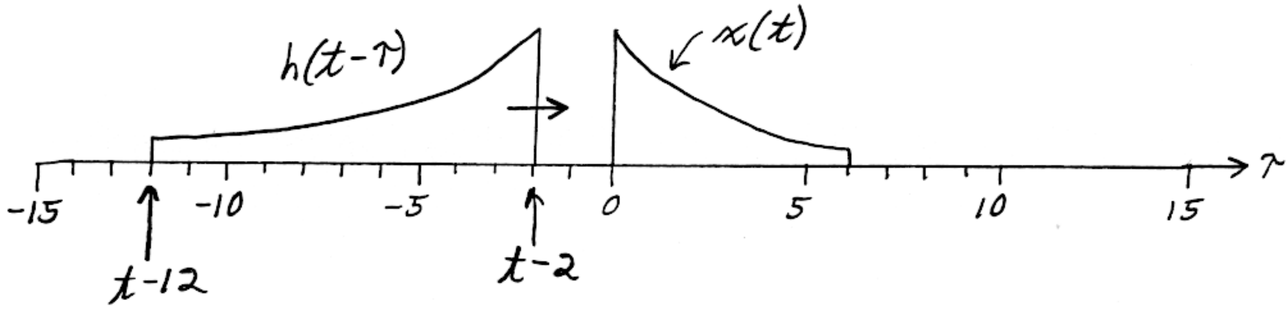
(a)



(b) $x(t) = \delta(t-2)$ This simply delays $h(t)$, $y(t) = h(t-2)$

$y(t) = e^{-0.1(t-4)}\left[u(t-4) - u(t-14)\right]$

9.3(c) $x(t) = e^{-0.25t}[u(t) - u(t-6)]$



Region 1 $t-2<0, \therefore t<2$ $\quad y(t)=0$

Region 2 $0<t-2<6, \therefore 2<t<8$ $\quad \int_0^{t-2}$

Region 3 $t-2>6 \quad t-12<0$
$\therefore 8<t<12$ $\quad \int_0^{6}$

Region 4 $0<t-12<6$
$\therefore 12<t<18$ $\quad \int_{t-12}^{6}$

Region 5 $t-12>6$ $\quad y(t)=0$
$\therefore t>18$

In each case, the integrand is

$$x(t)h(t-\tau) = e^{-0.1(t-2)} e^{-0.15\tau}$$

Region 1 $\boxed{y(t)=0 \text{ for } t<2}$

<u>Region 2</u> $y(t) = e^{-0.1(t-2)} \int_0^{t-2} e^{-0.15\tau} d\tau =$

$$= \frac{e^{-0.1(t-2)}}{-0.15} \left[e^{-0.15\tau} \right|_0^{t-2} = \boxed{6.667\left[e^{-0.1(t-2)} - e^{-0.25(t-2)}\right] \text{ for } 2 < t < 8}$$

<u>Region 3</u> $y(t) = e^{-0.1(t-2)} \int_0^{6} e^{-0.15\tau} d\tau$

$$= \frac{e^{-0.1(t-2)}}{-0.15} \left[e^{-0.15\tau} \right|_0^{6} = \frac{e^{-0.1(t-2)}}{0.15} \left[e^{0} - e^{-0.15(6)} \right]$$

$$= \boxed{3.956 e^{-0.1(t-2)} \text{ for } 8 < t < 12}$$

<u>Region 4</u>

$$y(t) = e^{-0.1(t-2)} \int_{t-12}^{6} e^{-0.15\tau} d\tau = \frac{e^{-0.1(t-2)}}{-0.15} \left[e^{-0.15\tau} \right|_{t-12}^{6}$$

$$= 6.667 e^{-0.1(t-2)} \left[e^{-0.15(t-12)} - e^{-0.15(6)} \right]$$

$$= \boxed{49.2628 e^{-0.25t} - 2.7108 e^{-0.1(t-2)} \text{ for } 12 < t < 18}$$

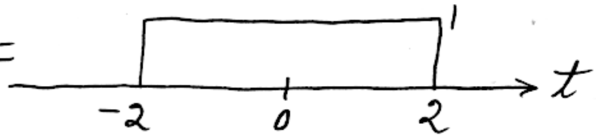
<u>Region 5</u>

$$\boxed{y(t) = 0 \text{ for } t > 18}$$

9.4 $\displaystyle y(t) = \int_{t-2}^{t+2} x(\tau)\, d\tau$

(a) let $x(\tau) = \delta(\tau)$ $\displaystyle h(t) = \int_{t-2}^{t+2} \delta(\tau)\, d\tau = u(\tau)\Big|_{t-2}^{t+2}$

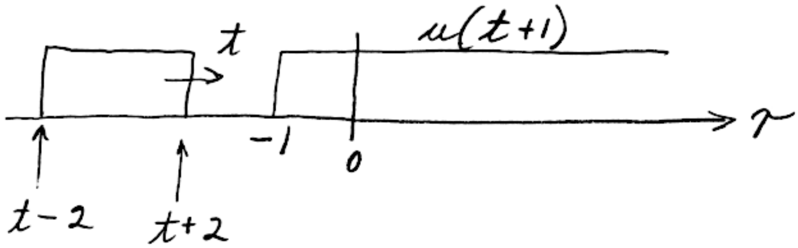
$= u(t+2) - u(t-2) =$



(b) Yes, it is stable because $\displaystyle\int_{-\infty}^{\infty} |h(t)|\, dt < \infty$

(c) No, it is not causal because $h(t) = 1$ for $-2 < t < 0$

(d) $\displaystyle y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau = \int_{-\infty}^{\infty} u(t+1)\left[u(t-\tau+2) - u(t-\tau-2)\right] d\tau$



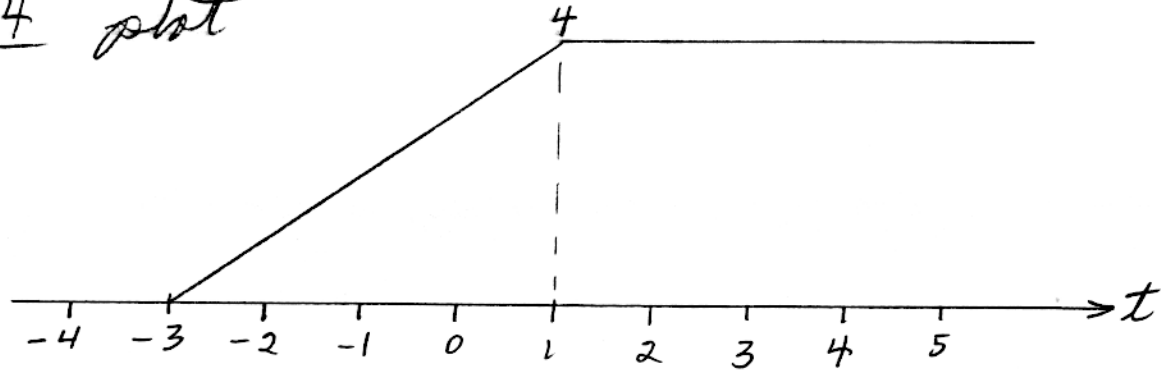
Region 1 $\quad t+2 < -1 \qquad t < -3 \quad y(t) = 0$

Region 2 $\quad t+2 \geq -1$ and $t-2 < -1 \qquad 1 > t \geq -3$

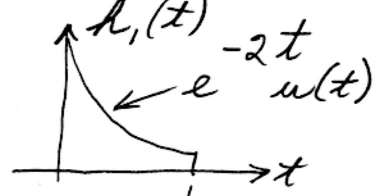
$\displaystyle y(t) = \int_{-1}^{t+2} 1 \cdot d\tau = \tau\Big|_{-1}^{t-2} = t+2+1 = t+3$

Region 3 $\quad t-2 > -1 \qquad t > 1 \quad \displaystyle y(t) = \int_{t-2}^{t+2} 1 \cdot d\tau = 4$

9.4 plot



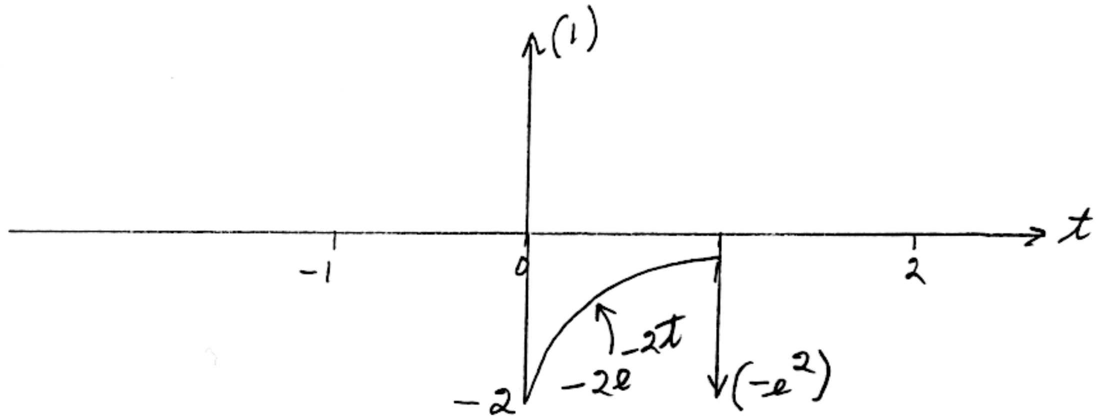
9.5 $y(t) = x(t) * h_1(t) * h_2(t)$



$$h_1(t) = e^{-2t}\big(u(t) - u(t-1)\big)$$

$$y(t) = \frac{d\,h_1(t)}{dt} = -2e^{-2t}\big(u(t)-u(t-1)\big) + e^{-2t}\big(\delta(t)-\delta(t-1)\big)$$

$$= -2e^{-2t}\left[u(t)-u(t-1)\right] + \quad \delta(t) - e^{2}\delta(t-1)$$

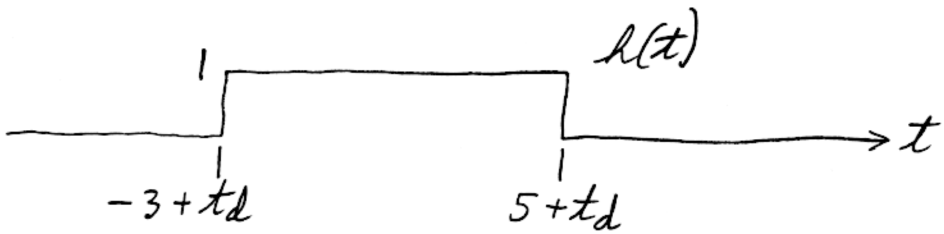


9.6 $v(t) = u(t+3) - u(t-5)$

(a)

$$h(t) = y(t)\Big|_{x(t)=\delta(t)} = v(t) * \delta(t - t_d) = \delta(t-t_d) * v(t)$$

$$\therefore\ h(t) = u(t+3-t_d) - u(t-5-t_d)$$



(b) $t_d \geq 3$ because $h(t) = 0$ for $t < 0$

(c) #1 and #2 are not stable because

$$\int_{-\infty}^{\infty} |h_1(t)|\,dt \to \infty \quad \text{and} \quad \int_{-\infty}^{\infty} |h_2(t)|\,dt \to \infty$$

#3 is stable

The overall system is stable because

$$\int_{-\infty}^{\infty} |h(t)|\,dt < \infty$$

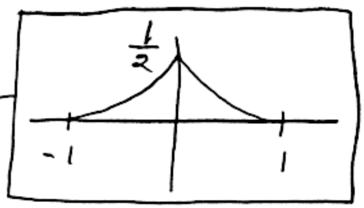
9.7 (a) This system can be represented as

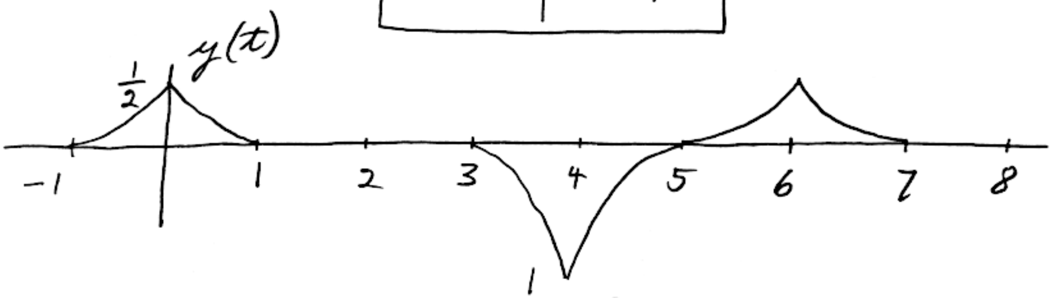
$$\boxed{x'(t)} \to \boxed{u(t)} \xrightarrow{x(t)} \boxed{l(t)} \to y(t)$$

Rearranging:

$$\boxed{x'(t)} \to \underbrace{\boxed{l(t)} \to \boxed{u(t)}} \to y(t)$$

$\delta(t) - 2\delta(t-4) + \delta(t-6) \to$





$y(0) = \frac{1}{2}$ $\quad y = 0$ for $t < -1$

$1 < t < 3$

$t = 5$

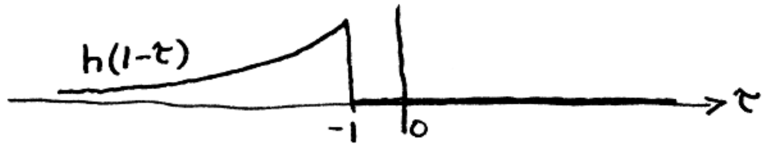
$t > 7$

Prob 9.8

(a) $h(t-\tau)$ for $t=1$ is $h(1-\tau) = e^{-(1-\tau-2)} u(1-\tau-2)$

FLIP & SHIFT by 1

$h(1-\tau) = e^{-(-1-\tau)} u(-1-\tau)$ ← STARTS @ $\tau=-1$



(b) Yes, the system is causal because $h(t)=0$ for $t<0$.
In fact, $h(t)=0$ for $t<2$.

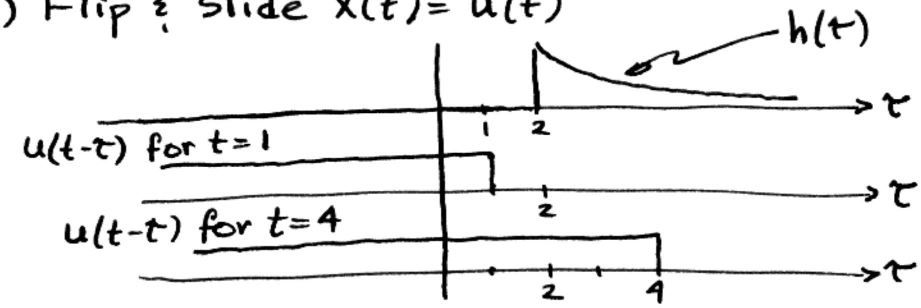
(c) To test for stability we do the integral $\int_{-\infty}^{\infty} |h(t)|\,dt$

$$\int_{-\infty}^{\infty} |e^{-(t-2)} u(t-2)|\,dt = \int_{2}^{\infty} e^{-(t-2)}\,dt = \left.\frac{e^{-(t-2)}}{-1}\right|_{2}^{\infty} = 0 - \frac{e^{0}}{-1} = 1 < \infty$$

Thus the system is stable.

(d) See the result from the convolution below: $t_1 = 2$

(e) Flip & Slide $x(t) = u(t)$



From the drawings, there is NO overlap when $t<2$.
$\Rightarrow y(t) = 0$ for $t<2$.

For $t \geq 2$, we have overlap from $\tau=2$ up to $\tau=t$.

$$y(t) = \int_{2}^{t} 1 \cdot e^{-(\tau-2)}\,d\tau = \left.\frac{e^{-(\tau-2)}}{-1}\right|_{2}^{t}$$

$$y(t) = \frac{e^{-(t-2)}}{-1} - \frac{e^{0}}{-1} = 1 - e^{-(t-2)}$$

$\therefore y(t) = (1 - e^{-(t-2)})\,u(t-2)$