HOMEWORK # 12
SOLUTIONS

ECE 2025
SPRING '01

12.1)

$$\cos\alpha \cos\beta = \frac{\cos(\alpha+\beta) + \cos(\alpha-\beta)}{2}$$

$$\cos(\omega_c t)\cos(\omega_c t+\phi) = \frac{1}{2}\cos(2\omega_c t+\phi) + \frac{1}{2}\cos\phi$$

$$w(t) = x(t)\cos(\omega_c t+\phi)\cos(\omega_c t) =$$

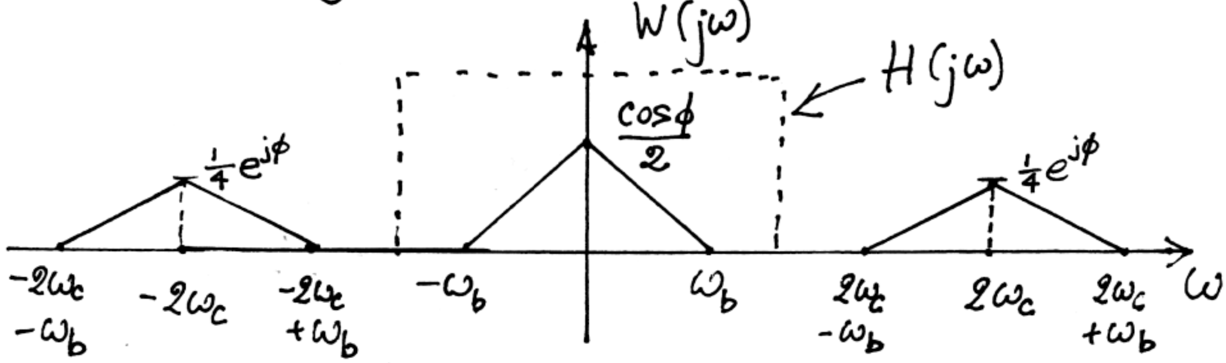
$$= \frac{1}{2}x(t)\cos(2\omega_c t+\phi) + \frac{1}{2}x(t)\cos\phi =$$

$$= \frac{1}{4}x(t)e^{j\phi}e^{j2\omega_c t} + \frac{1}{4}x(t)e^{-j\phi}e^{-j2\omega_c t} +$$

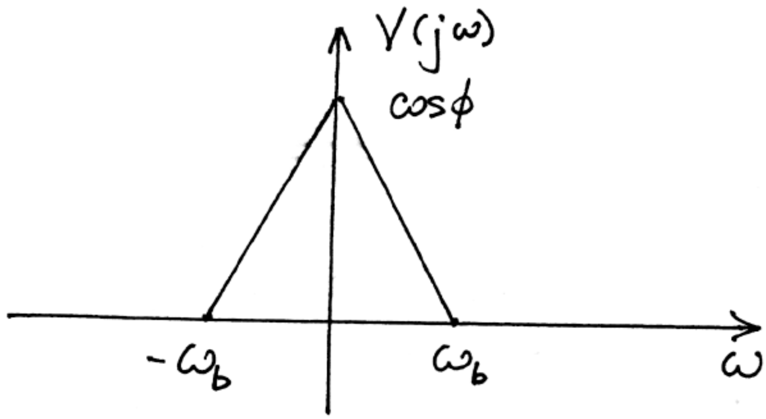
$$+ \frac{1}{2}x(t)\cos\phi$$

$$W(j\omega) = \frac{1}{4}e^{j\phi}X[j(\omega-2\omega_c)] + \frac{1}{4}e^{-j\phi}X[j(\omega+2\omega_c)] +$$

$$+ \frac{\cos\phi}{2}X(j\omega)$$



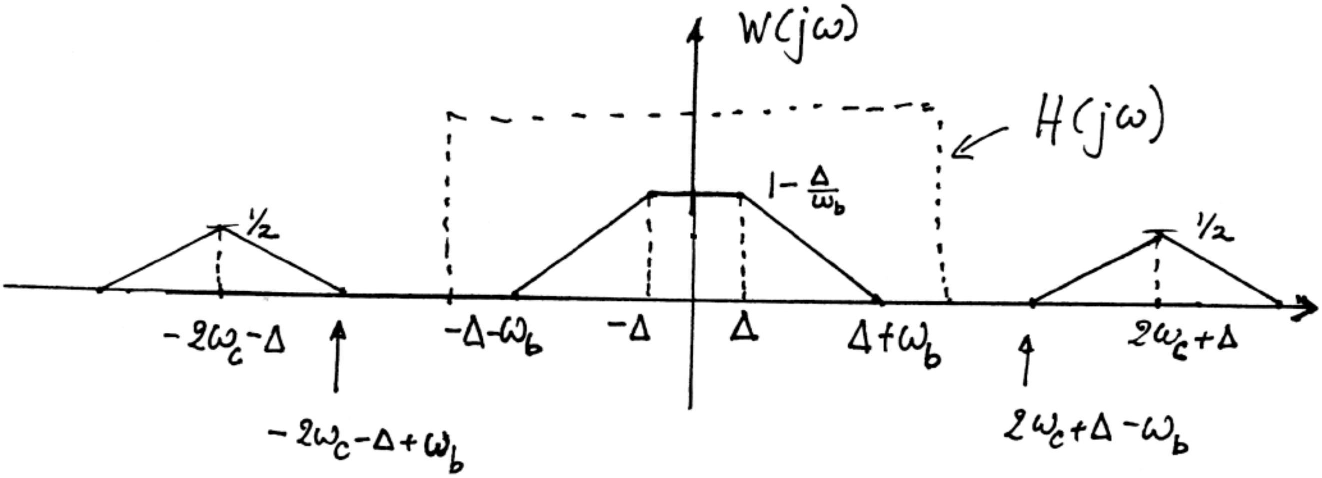
②



$$v(t) = x(t)\cos\phi$$

b) $$w(t) = x(t)\cos(\omega_c t)\cos[(\omega_c+\Delta)t] =$$

$$= \frac{x(t)}{2}\cos(\Delta\cdot t) + \frac{x(t)}{2}\cos[(2\omega_c+\Delta)t]$$



$$v(t) = x(t)\cos(\Delta\cdot t)$$

12.2)

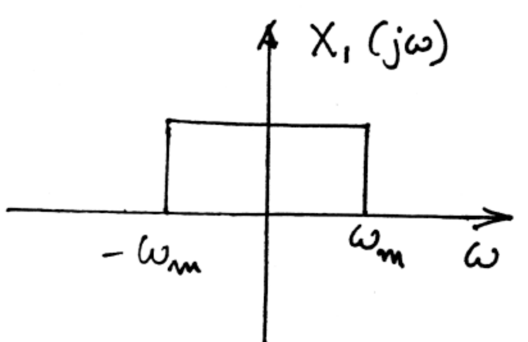
a) $x_1(t)\cos(\omega_c t) \longleftrightarrow \frac{1}{2}X_1[j(\omega-\omega_c)] + \frac{1}{2}X_1[j(\omega+\omega_c)]$

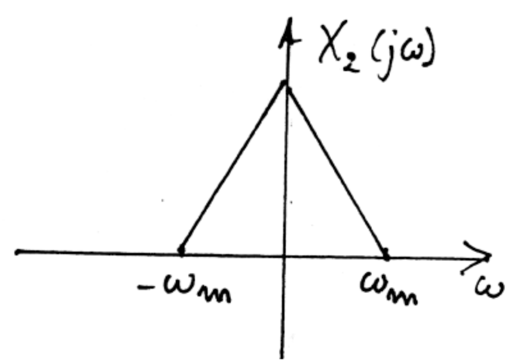
$$x_2(t)\sin(\omega_c t) \longleftrightarrow \frac{1}{2j}X_2[j(\omega-\omega_c)] - \frac{1}{2j}X_2[j(\omega+\omega_c)]$$

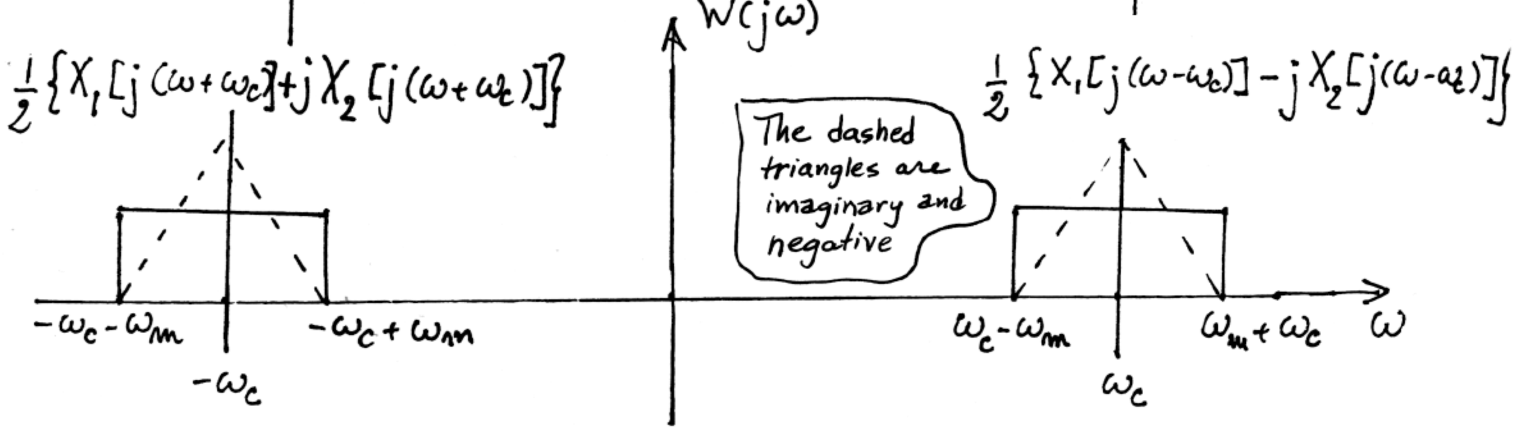
$$w(t) = x_1(t)\cos(\omega_c t) + x_2(t)\sin(\omega_c t) \longleftrightarrow$$

$$W(j\omega) = \frac{1}{2}\left\{X_1[j(\omega-\omega_c)] - jX_2[j(\omega-\omega_c)]\right\} +$$

$$+\frac{1}{2}\left\{X_1[j(\omega+\omega_c)] + jX_2[j(\omega+\omega_c)]\right\}$$







b) $\omega_a = \omega_c - \omega_m$

$\omega_b = \omega_c + \omega_m$

c) $$v(t) = w(t)\sin(\omega_c t) = x_1(t)\sin(\omega_c t)\cos(\omega_c t) +$$

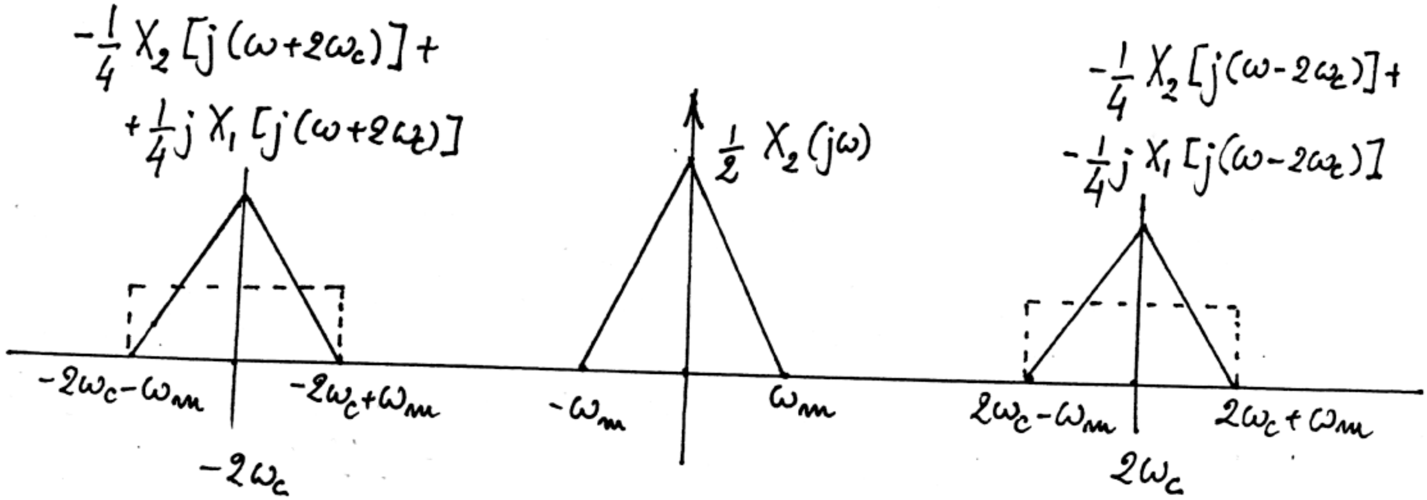
$$+ x_2(t)\sin^2(\omega_c t) = \frac{1}{2}x_1(t)\sin(2\omega_c t) +$$

$$+ \frac{1}{2}x_2(t)\left[1 - \cos(2\omega_c t)\right] = \frac{1}{2}x_2(t) +$$

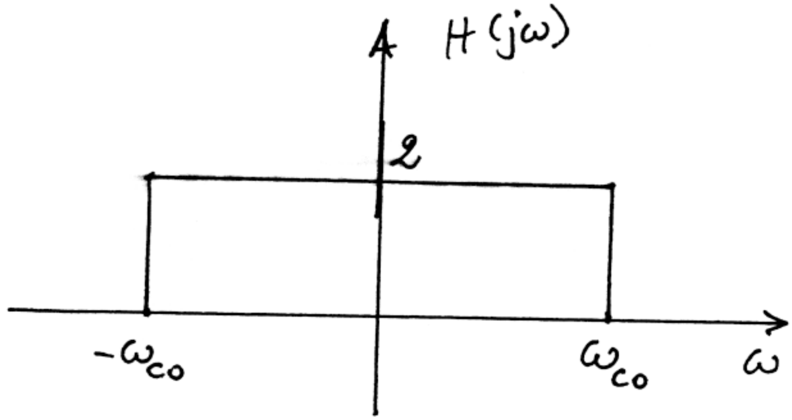
$$+ \frac{1}{2}x_1(t)\sin(2\omega_c t) - \frac{1}{2}x_2(t)\cos(2\omega_c t)$$

$$V(j\omega) = \frac{1}{2}X_2[j\omega] + \frac{1}{4j}\left\{X_1[j(\omega - 2\omega_c)] +\right.$$

$$\left. - X_1[j(\omega + 2\omega_c)]\right\} - \frac{1}{4}\left\{X_2[j(\omega - 2\omega_c)] + X_2[j(\omega + 2\omega_c)]\right\}$$

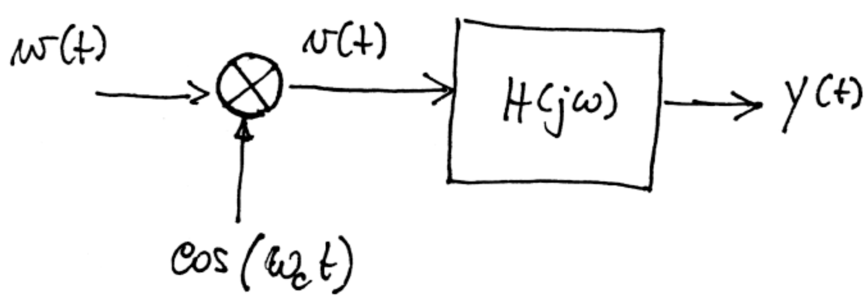


d)



Ideal low-pass filter with gain equal to 2 and $\omega_m \leq \omega_{co} \leq 2\omega_c - \omega_m$ (in fact, a non-ideal low-pass filter will also work).

e)



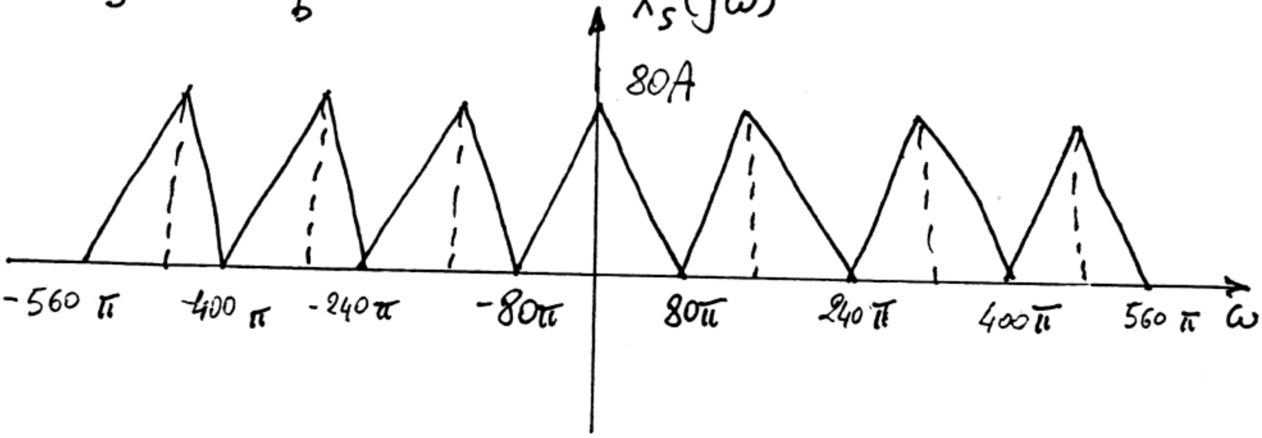
$$v(t) = x_1(t)\cos^2(\omega_c t) + x_2(t)\sin(\omega_c t)\cos(\omega_c t) =$$

$$= \frac{1}{2}x_1(t)[1 + \cos(2\omega_c t)] + \frac{1}{2}x_2(t)\sin(2\omega_c t) =$$

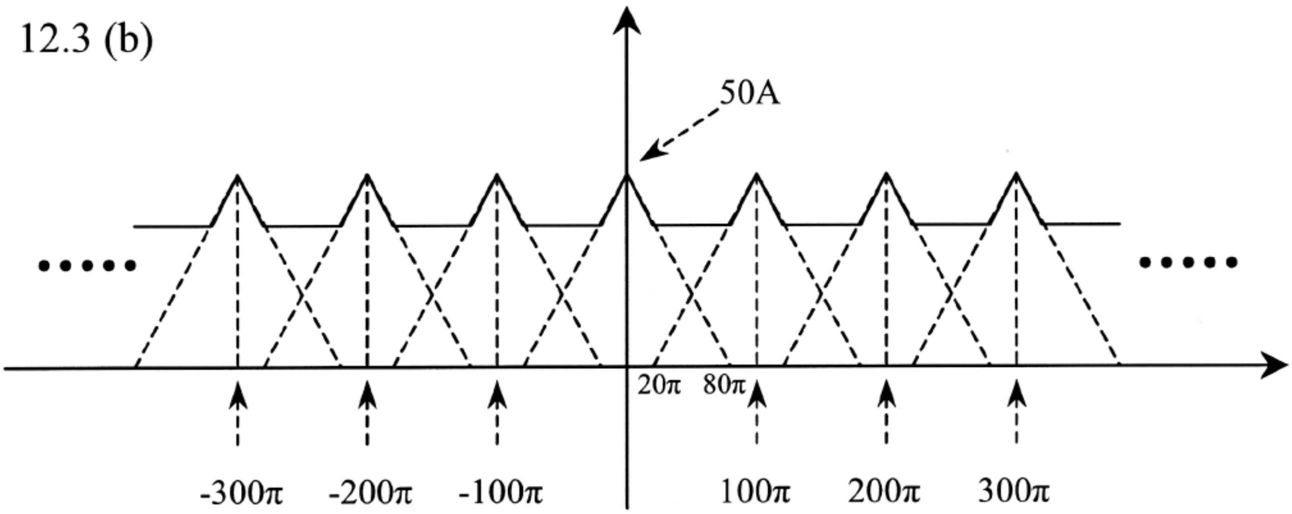
$$= \frac{1}{2}x_1(t) + \frac{1}{2}x_1(t)\cos(2\omega_c t) + \frac{1}{2}x_2(t)\sin(2\omega_c t)$$

12.3)

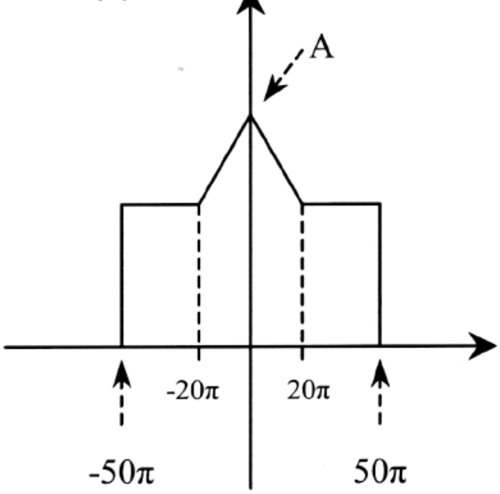
a) $\omega_s \geq 2\omega_b = 160\pi$



12.3 (b)



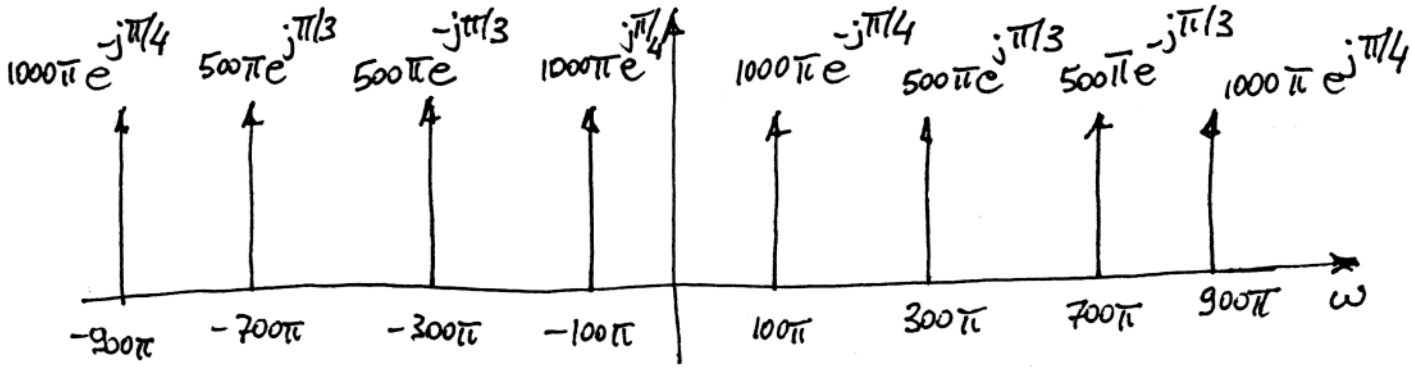
12.3 (c)



12.4)

a) $$X(j\omega) = 2\pi\left[e^{-j\pi/4}\delta(\omega-100\pi) + e^{j\pi/4}\delta(\omega+100\pi)\right] +$$

$$+ \pi\left[e^{j\pi/3}\delta(\omega-300\pi) + e^{-j\pi/3}\delta(\omega+300\pi)\right]$$



$$x_r(t) = x(t)$$

b) $$x_r(t) = 2\cos(100\pi t - \pi/4) + \cos(200\pi t - \pi/3)$$

c) Choose ω_s so that 300π gets aliased to 0,

i.e. $\omega_s = 300\pi = \frac{2\pi}{T_s} \Rightarrow T_s = \frac{1}{150}$ sec.

$$A = \frac{1}{2}e^{j\pi/3} + \frac{1}{2}e^{-j\pi/3} = \cos(\pi/3) = 0.5 .$$

12.5)

a) $\frac{2\pi}{T_s} = \omega_s = 2\,\omega_b = 2000\pi$ rad/sec.

b) $\mathcal{H}(\hat{\omega}) = e^{-j10\hat{\omega}}$

$\hat{\omega} = \omega T_s$

$H_{eff}(j\omega) = e^{-j10\omega T_s} = e^{-j0.01\omega}$

$Y_c(j\omega) = H_{eff}(j\omega)\,X_c(j\omega) = e^{-j0.01\omega}\,X_c(j\omega)$

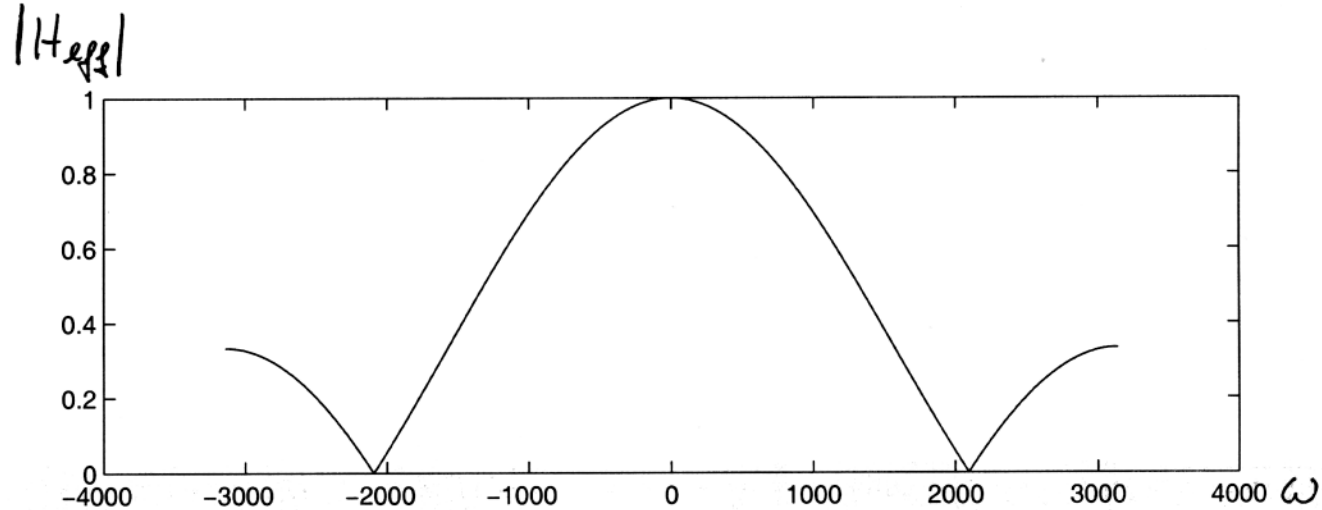
$y_c(t) = x_c(t - 0.01)$

c) $\mathcal{H}(\hat{\omega}) = \frac{1}{3}\left(1 + e^{-j\hat{\omega}} + e^{-j2\hat{\omega}}\right) = \frac{1}{3} e^{-j\hat{\omega}} \dfrac{\sin \frac{3}{2}\hat{\omega}}{\sin \frac{\hat{\omega}}{2}}$

$\hat{\omega} = \omega T_s$

$H_{eff}(j\omega) = \frac{1}{3} e^{-j0.001\omega} \dfrac{\sin(0.0015\omega)}{\sin(0.0005\omega)}$

$(|\omega| \leq 1000\pi)$


|Heff|
1
0.8
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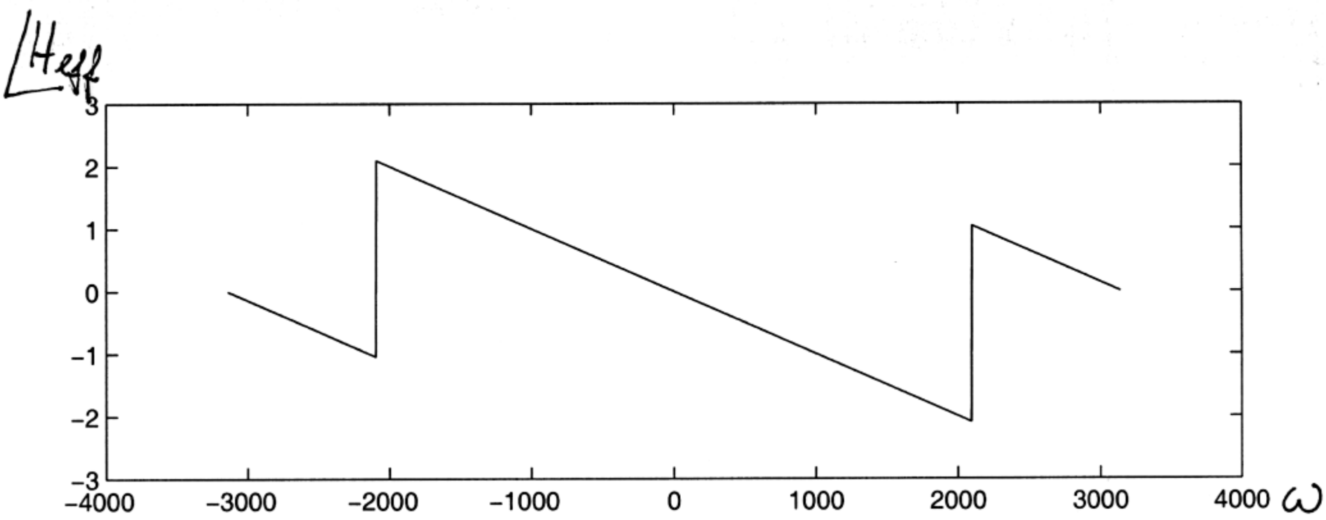
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