GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

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Vectorizing a Cosine Series

This document shows how to vectorize a cosine series expansion.

Problem:

We want to evaluate the following function for several values of t with MATLAB.

$$x(t) = \sum_{k=1}^{N} A_k \cos(2\pi f_k t + \phi_k) = \mathrm{Re}\left\{ \sum_{k=1}^{N} X_k e^{j2\pi f_k t} \right\}$$

where $X_k = A_k e^{j\phi_k}$ and the variables A_k, f_k, and ϕ_k are given. Knowing that MATLAB is inefficient when doing heavy calculations within loops, we want to do this calculation using matrix-vector techniques.

Solution:

Lets assume that we want to calculate $x(t)$ for t_1, t_2, and t_3. Expanding the summation, we see that we need to find

$$\begin{aligned}
x(t_1) &= A_1 \cos(2\pi f_1 t_1 + \phi_1) + A_2 \cos(2\pi f_2 t_1 + \phi_2) + \cdots + A_N \cos(2\pi f_N t_1 + \phi_N) \\
x(t_2) &= A_1 \cos(2\pi f_1 t_2 + \phi_1) + A_2 \cos(2\pi f_2 t_2 + \phi_2) + \cdots + A_N \cos(2\pi f_N t_2 + \phi_N) \\
x(t_3) &= A_1 \cos(2\pi f_1 t_3 + \phi_1) + A_2 \cos(2\pi f_2 t_3 + \phi_2) + \cdots + A_N \cos(2\pi f_N t_3 + \phi_N)
\end{aligned}$$

Using matrix-vector notation we can rewrite this as

$$\underbrace{\begin{bmatrix} x(t_1) \\ x(t_2) \\ x(t_3) \end{bmatrix}}_{\boldsymbol{x}} = \underbrace{\begin{bmatrix} \cos(2\pi f_1 t_1 + \phi_1) & \cos(2\pi f_2 t_1 + \phi_2) & \cdots & \cos(2\pi f_N t_1 + \phi_N) \\ \cos(2\pi f_1 t_2 + \phi_1) & \cos(2\pi f_2 t_2 + \phi_2) & \cdots & \cos(2\pi f_N t_2 + \phi_N) \\ \cos(2\pi f_1 t_3 + \phi_1) & \cos(2\pi f_2 t_3 + \phi_2) & \cdots & \cos(2\pi f_N t_3 + \phi_N) \end{bmatrix}}_{\boldsymbol{C}} \underbrace{\begin{bmatrix} A_1 \\ A_2 \\ \vdots \\ A_N \end{bmatrix}}_{\boldsymbol{a}}$$

or more simply as

$$\boldsymbol{x} = \boldsymbol{C}\boldsymbol{a} .$$

Since the cosine function works element-by-element, we can pull it outside of the matrix **C** and instead calculate

$$\boldsymbol{x} = \cos(\boldsymbol{G})\boldsymbol{a}$$

where

$$\boldsymbol{G} = \begin{bmatrix} 2\pi f_1 t_1 + \phi_1 & 2\pi f_2 t_1 + \phi_2 & \cdots & 2\pi f_N t_1 + \phi_N \\ 2\pi f_1 t_2 + \phi_1 & 2\pi f_2 t_2 + \phi_2 & \cdots & 2\pi f_N t_2 + \phi_N \\ 2\pi f_1 t_3 + \phi_1 & 2\pi f_2 t_3 + \phi_2 & \cdots & 2\pi f_N t_3 + \phi_N \end{bmatrix}$$

$$= 2\pi \begin{bmatrix} f_1 t_1 & f_2 t_1 & \cdots & f_N t_1 \\ f_1 t_2 & f_2 t_2 & \cdots & f_N t_2 \\ f_1 t_3 & f_2 t_3 & \cdots & f_N t_3 \end{bmatrix} + \begin{bmatrix} \phi_1 & \phi_2 & \cdots & \phi_N \\ \phi_1 & \phi_2 & \cdots & \phi_N \\ \phi_1 & \phi_2 & \cdots & \phi_N \end{bmatrix}$$

$$= 2\pi \boldsymbol{t}\boldsymbol{f}' + \boldsymbol{1}\boldsymbol{\phi}'$$

and

$$\boldsymbol{t} = \begin{bmatrix} t_1 \\ t_2 \\ t_3 \end{bmatrix}, \qquad \boldsymbol{f} = \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_N \end{bmatrix}, \qquad \boldsymbol{1} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \qquad \boldsymbol{\phi} = \begin{bmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_N \end{bmatrix}.$$

The prime mark in the above equation denotes the matrix (vector) transpose operation. Thus,

$$\boldsymbol{f}' = \begin{bmatrix} f_1 & f_2 & \cdots & f_N \end{bmatrix}$$

and

$$\boldsymbol{\phi}' = \begin{bmatrix} \phi_1 & \phi_2 & \cdots & \phi_N \end{bmatrix}.$$