# EE-2025 Fall-00

**LECTURE #3**
**Phasor Addition Theorem**
**28-Aug-00**

# Web-CT Info

- Check the Bulletin Board for msgs
  - **MAKE YOUR OWN POSTINGS**
- Lectures are being posted
  - PDF format (4 per page)
- Get PDF file of Lab#1 from WebCT
  - Lab FAQs are also being posted
- On-Line HW #0 closes tonight @ midnite
- HW #1 due next week (**in Recitation**)



# Homework Formatting

- Include a Cover page with
  - Name
  - Lab section, ie, L05, L21, etc.
  - Recitation Prof's name
  - **See example on Web-CT**
- Write on ONE side only
  - Use Engineer's paper or plain paper
- STAPLE



# Lab Info

- NT passwd = SSN or old password
- Lab FAQs are also being posted
- Bring **Headphones** to Lab
- Lab #1 Verifications: Turn in at end of Lab
- MATLAB Help: Mon,T,Wed 6PM VL-456

## Lab Info: next week

- Lab #1 Report
  - Due week of 4-Sept
    - Monday Labs, turn in on Mon 11-Sept @ Lab
  - Turn in at <u>**BEGINNING**</u> of your lab time
  - Write-up sections 2 and 3
- Learn your Lab TA's format requirements
- Week of Sept 4th (Labor Day)
  - Monday Labs will be shifted to 11-Sept



## PRINTING QUOTA

- ECE Labs have printers, but...
- Limit your printing to essentials
  - Your account has a quota
- 10 pages/week
  - 2000 students
  - 3 courses/student
  - 15 weeks/semester
  - 900,000 pages !

LECTURE

# EE-2025 Georgia Tech

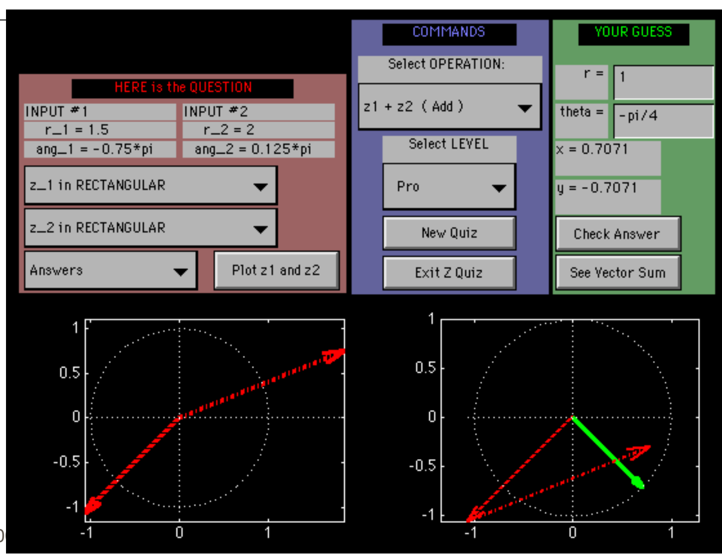
## LECTURE #3
## Phasor Addition Theorem

# READING ASSIGNMENTS

- This Lecture:
  - Chapter 2, pp. 31-43
- Other Reading:
  - Appendix A: Complex Numbers
  - Appendix B: MATLAB
  - Next Lecture: start Chapter 3



# Z DRILL (Complex Arith)






# LECTURE OBJECTIVES

- Phasors = Complex Amplitude
  - Complex Numbers **represent** Sinusoids

$$z(t) = Xe^{j\omega t} = (Ae^{j\varphi})e^{j\omega t}$$

- Develop the ABSTRACTION:
  - Adding Sinusoids = Complex Addition
  - **PHASOR ADDITION THEOREM**

# AVOID Trigonometry

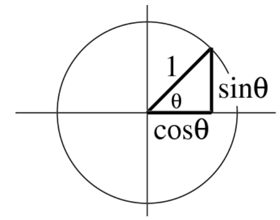
- Algebra, even complex, is EASIER !!!
- Can you recall $\cos(\theta_1+\theta_2)$ ?
- Use the real part of $e^{j(\theta_1+\theta_2)}$

$$e^{j(\theta_1+\theta_2)} = e^{j\theta_1} e^{j\theta_2}$$
$$= (\cos\theta_1 + j\sin\theta_1)(\cos\theta_2 + j\sin\theta_2)$$
$$= \boxed{\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2} + j(\ldots)$$



# Euler's FORMULA

- Complex Exponential
  - Real part is cosine
  - Imaginary part is sine
  - Magnitude is one

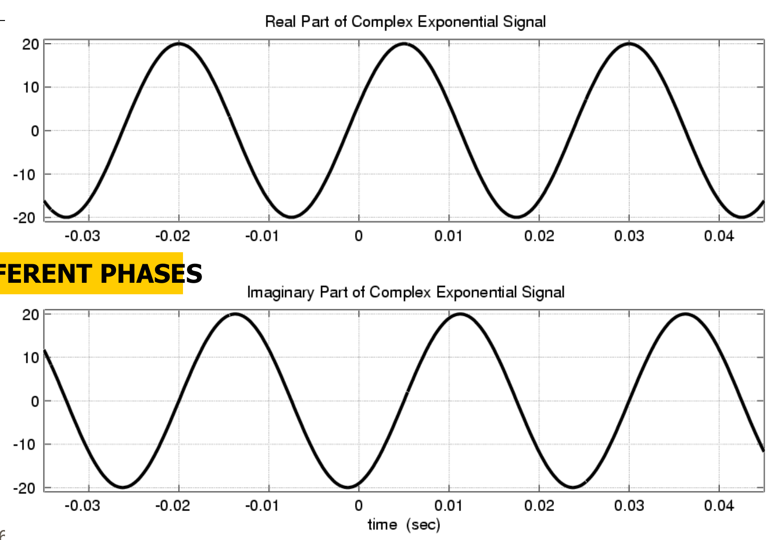



$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$

$$e^{j\omega t} = \cos(\omega t) + j\sin(\omega t)$$



# Real & Imaginary Part Plots



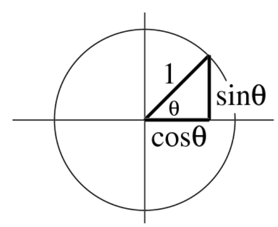

**DIFFERENT PHASES**



# COMPLEX EXPONENTIAL

$$e^{j\omega t} = \cos(\omega t) + j\sin(\omega t)$$

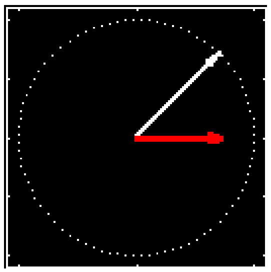
- **Rotating Vector**
  - Angle changes vs. time
  - $\theta = \omega t$
  - ex: $\omega = 10\pi$
  - Rotates $0.1\pi$ in 0.01 secs

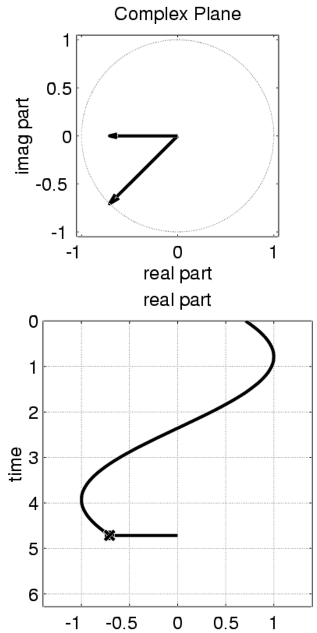



$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$

# Rotating Phasor

See Demo on CD-ROM
Chapter 2








# Cos = REAL PART

- Real Part of Euler's

$$\cos(\omega t) = \Re e\{e^{j\omega t}\}$$

- General Sinusoid

$$x(t) = A\cos(\omega t + \varphi)$$

- So,

$$A\cos(\omega t + \varphi) = \Re e\{A e^{j(\omega t + \varphi)}\}$$
$$= \Re e\{A e^{j\varphi} e^{j\omega t}\}$$



# COMPLEX AMPLITUDE

- General Sinusoid

$$x(t) = A\cos(\omega t + \varphi) = \Re e\{A e^{j\varphi} e^{j\omega t}\}$$

- Sinusoid = REAL PART of $(Ae^{j\phi})e^{j\omega t}$

$$x(t) = \Re e\{X e^{j\omega t}\} = \Re e\{z(t)\}$$

- **Complex AMPLITUDE = X**

$$z(t) = X e^{j\omega t} \qquad X = A e^{j\varphi}$$



# POP QUIZ: Complex Amp

- Find the COMPLEX AMPLITUDE for:

$$x(t) = \sqrt{3}\cos(77\pi t + 0.5\pi)$$

- Use EULER's FORMULA:

$$x(t) = \Re e\{\sqrt{3} e^{j(77\pi t + 0.5\pi)}\}$$
$$= \Re e\{\sqrt{3} e^{j0.5\pi} e^{j77\pi t}\}$$

$$X = \sqrt{3} e^{j0.5\pi}$$

# WANT to ADD SINUSOIDS

- ALL SINUSOIDS HAVE SAME FREQUENCY
- HOW to GET {Amp,Phase} of RESULT ?

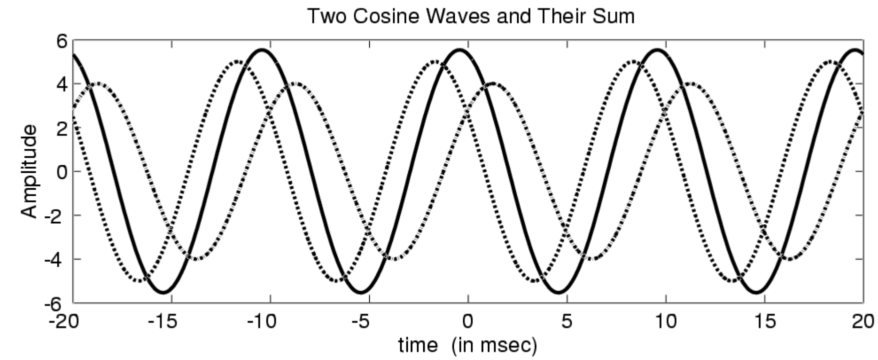
$$x_1(t) = 1.7\cos(2\pi(10)t + 70\pi/180)$$
$$x_2(t) = 1.9\cos(2\pi(10)t + 200\pi/180)$$

$$x_3(t) = x_1(t) + x_2(t)$$
$$= 1.532\cos(2\pi(10)t + 141.79\pi/180)$$



# ADD SINUSOIDS

- Sum Sinusoid has **SAME** Frequency






# PHASOR ADDITION RULE

$$x(t) = \sum_{k=1}^{N} A_k \cos(\omega_0 t + \phi_k)$$
$$= A\cos(\omega_0 t + \phi)$$

Get the new complex amplitude by complex addition

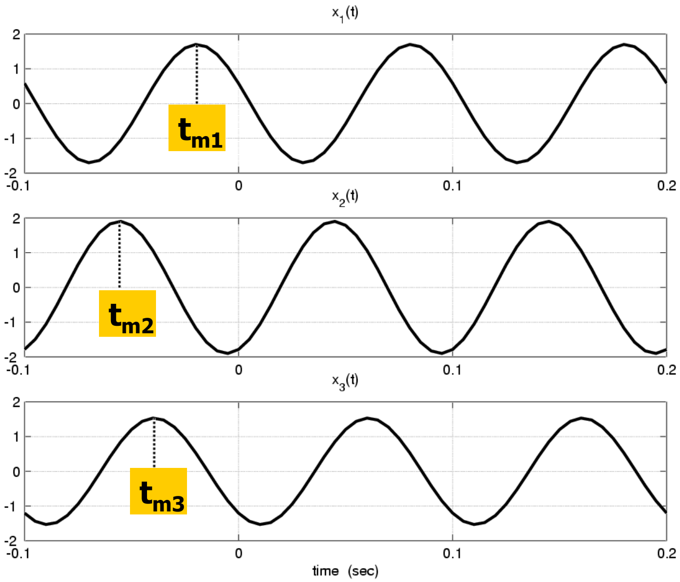
$$\sum_{k=1}^{N} A_k e^{j\phi_k} = Ae^{j\phi}$$



# Phasor Addition Proof

$$\sum_{k=1}^{N} A_k \cos(\omega_0 t + \phi_k) = \sum_{k=1}^{N} \Re e\left\{A_k e^{j(\omega_0 t + \phi_k)}\right\}$$
$$= \Re e\left\{\sum_{k=1}^{N} A_k e^{j\phi_k} e^{j\omega_0 t}\right\}$$
$$= \Re e\left\{\left(\sum_{k=1}^{N} A_k e^{j\phi_k}\right) e^{j\omega_0 t}\right\}$$
$$= \Re e\left\{\left(Ae^{j\phi}\right) e^{j\omega_0 t}\right\} = A\cos(\omega_0 t + \phi)$$

# ADD SINUSOIDS EXAMPLE





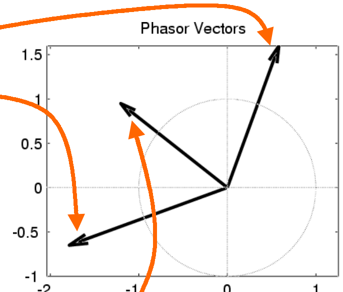

# Convert Time-Shift to Phase

- Measure peak times:
  - $t_{m1}$=-0.0194, $t_{m2}$=-0.0556, $t_{m3}$=-0.0394
- Convert to phase (T=0.1)
  - $\phi_1 = -\omega t_{m1} = -2\pi(t_{m1}/T) = 70\pi/180$,
  - $\phi_2 = 200\pi/180$
- Amplitudes
  - $A_1$=1.7, $A_2$=1.9, $A_3$=1.532



# Phasor Add: Numerical

- Convert Polar to Cartesian
  - $X_1$ = 0.5814 + j1.597
  - $X_2$ = -1.785 - j0.6498
  - $X_3$ = -1.204 + j0.9476



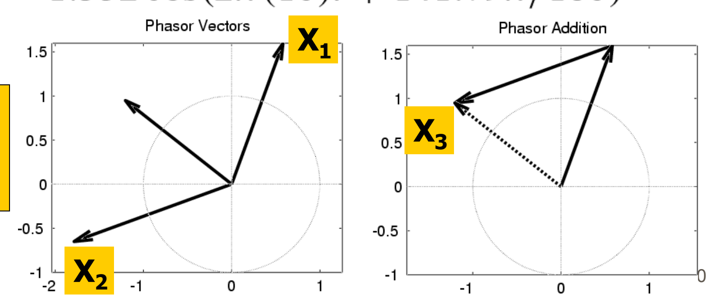

- Convert back to Polar
  - $X_3$ = 1.532 at angle 141.79π/180
  - This is the sum



# ADD SINUSOIDS

$$x_1(t) = 1.7\cos(2\pi(10)t + 70\pi/180)$$
$$x_2(t) = 1.9\cos(2\pi(10)t + 200\pi/180)$$
$$x_3(t) = x_1(t) + x_2(t)$$
$$= 1.532\cos(2\pi(10)t + 141.79\pi/180)$$

*VECTOR (PHASOR) ADD*



# POP QUIZ: Add Sinusoids

- ADD THESE 2 SINUSOIDS:

$$x_1(t) = \cos(77\pi t)$$

$$x_2(t) = \sqrt{3}\cos(77\pi t + 0.5\pi)$$

- COMPLEX ADDITION:

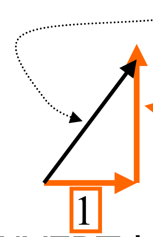
$$1e^{j0} + \sqrt{3}e^{j0.5\pi}$$



# POP QUIZ (answer)

- COMPLEX ADDITION:

$$1 + j\sqrt{3} = 2e^{j\pi/3}$$




$$j\sqrt{3} = \sqrt{3}e^{j0.5\pi}$$

- CONVERT back to cosine form:

$$x_3(t) = 2\cos(77\pi t + \tfrac{\pi}{3})$$