

EE-2025

Fall-2000

**Lecture 4
Spectrum Representation
1-Sept-00**

Web-CT Info

- Bulletin Board has all **OFFICIAL** msgs
- Consult the FAQs
- Lectures are being posted
 - PDF format (4 per page)
- Upcoming Events:
 - Quiz #1 on 18-Sept (Monday)

8/30/00

ECE-2025 Fall-00 mhh/jMc

2

James THURBER

- It is better to know some of the questions than all of the answers

Lab Info

- Lab #1 Report
 - Turn in at beginning of your lab time
 - Write-up sections 2 and 3
- Lab #2 has been posted
 - For week of 5-Sept
- Finish Instructor Verification in Lab
- MATLAB Help:
 - Next week: T,W & Thur 6PM VL-456

8/30/00

ECE-2025 Fall-00 mhh/jMc

3

8/30/00

ECE-2025 Fall-00 mhh/jMc

4

Homework Info

- Written HW #1 due NEXT WEEK
 - In Recitation, at the beginning
 - Monday Rec?...turn in HW at lab on Wed.
- HW #1 Solutions posted
 - Thursday nite/Friday morning
- MATLAB Help:
 - Also HW help: 6PM VL-456
 - Will continue throughout the semester

8/30/00

ECE-2025 Fall-00 mhh/jMc

5

Homework Formatting

- Cover page with
 - Name
 - Lab section, ie, L05, L21, etc.
 - Recitation Prof's name
- Write on ONE side only
 - Prefer Engineering paper, or plain
 - NO LINES
- STAPLE

8/30/00

ECE-2025 Fall-00 mhh/jMc

6

HISTORY

- Which company's first successful product was a sine-wave generator?
 - Variable frequency
 - Lab Instrument



8/30/00

ECE-2025 Fall-00 mhh/jMc

Lecture

7

Lecture 4
Spectrum Representation

LECTURE OBJECTIVES

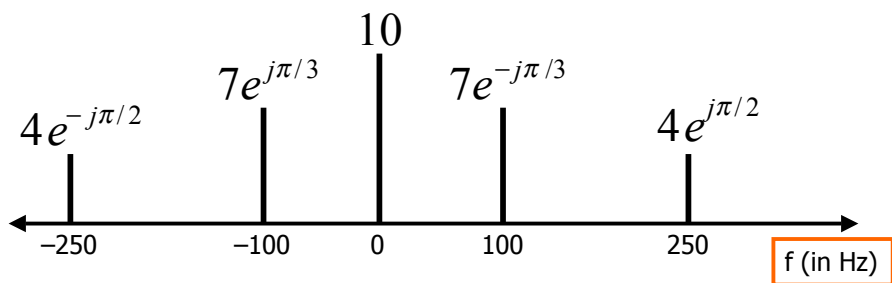
- Sinusoids with **DIFFERENT** Frequencies
 - SYNTHESIZE by Adding Sinusoids

$$x(t) = \sum_{k=1}^N A_k \cos(2\pi f_k t + \varphi_k)$$

- **SPECTRUM** Representation
 - Graphical Form shows **DIFFERENT** Freqs

FREQUENCY DIAGRAM

- Plot Complex Amplitude vs. Freq



Another FREQ. Diagram

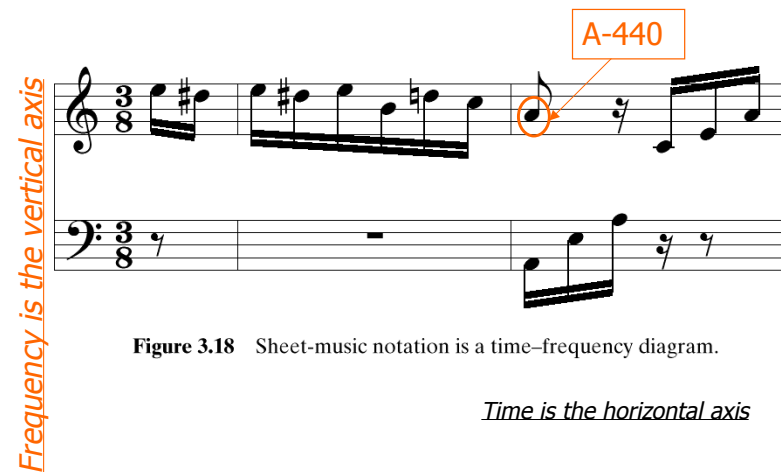


Figure 3.18 Sheet-music notation is a time–frequency diagram.


MOTIVATION

■ Synthesize **Complicated** Signals

■ Musical Notes

- Piano uses 3 strings for many notes
- Chords: play several notes simultaneously

■ Human Speech

- Vowels have dominant frequencies 
- Application: computer generated speech

■ Can **all** signals be generated this way?

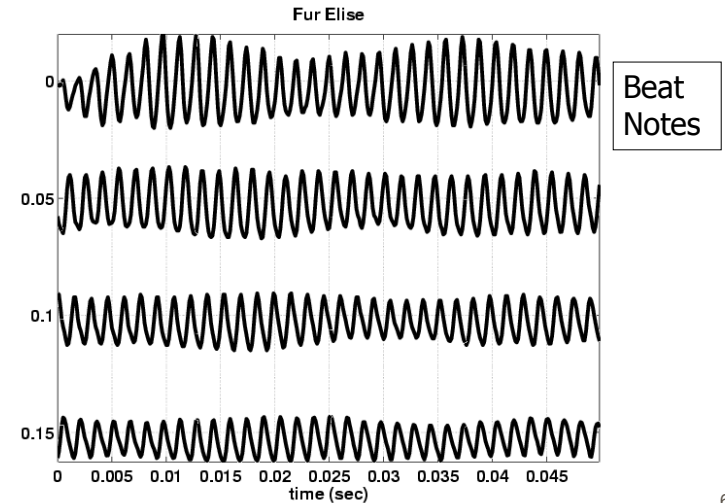
- Sum of sinusoids?

8/30/00

ECE-2025 rws/jMc

5

Fur Elise WAVEFORM



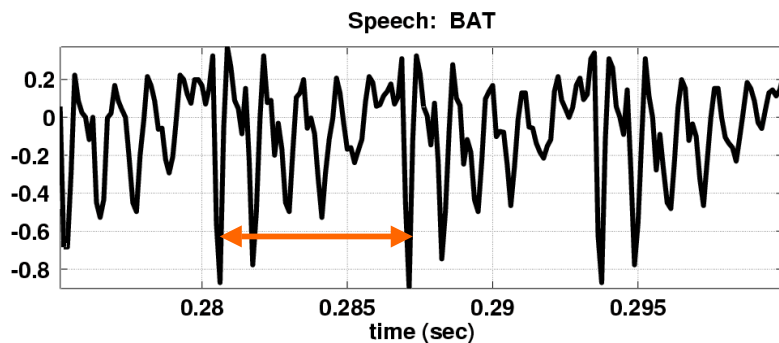
8/30/00

6

Speech Signal: BAT

■ Nearly **Periodic** in Vowel Region

- Period is (Approximately) $T = 0.0065$ sec



8/30/00

ECE-2025 rws/jMc

7

Euler's Formula Reversed

■ Solve for **cosine** (or sine)

$$e^{j\omega t} = \cos(\omega t) + j \sin(\omega t)$$

$$e^{-j\omega t} = \cos(-\omega t) + j \sin(-\omega t)$$

$$e^{-j\omega t} = \cos(\omega t) - j \sin(\omega t)$$

$$e^{j\omega t} + e^{-j\omega t} = 2 \cos(\omega t)$$

$$\cos(\omega t) = \frac{1}{2} (e^{j\omega t} + e^{-j\omega t})$$

8/30/00

ECE-2025 rws/jMc

8

INVERSE Euler's Formula

- Solve for **cosine** (or sine)

$$\cos(\omega t) = \frac{1}{2} (e^{j\omega t} + e^{-j\omega t})$$

$$\sin(\omega t) = \frac{1}{2j} (e^{j\omega t} - e^{-j\omega t})$$

SPECTRUM Interpretation

- Cosine = sum of 2 complex exponentials:

$$A \cos(7t) = \frac{A}{2} e^{j7t} + \frac{A}{2} e^{-j7t}$$

- One has a positive frequency
- The other has **negative** freq.
- Amplitude of each is half as big

NEGATIVE FREQUENCY

- Is negative frequency real?
- Doppler Radar provides an example
 - Police radar measures speed by using the Doppler shift principle
 - Let's assume 400Hz <---> 60 mph
 - **+400Hz** means towards the radar
 - **-400Hz** means away (opposite **direction**)
 - Think of a train whistle

SPECTRUM of SINE

- Sine = sum of 2 complex exponentials:

$$A \sin(7t) = \frac{A}{2j} e^{j7t} - \frac{A}{2j} e^{-j7t}$$

$$= \frac{1}{2} A e^{-j0.5\pi} e^{j7t} + \frac{1}{2} A e^{j0.5\pi} e^{-j7t}$$

$$\frac{-1}{j} = j = e^{j0.5\pi}$$

- Positive freq. has phase = -0.5π
- Negative freq. has phase = $+0.5\pi$

GRAPHICAL SPECTRUM

EXAMPLE of SINE

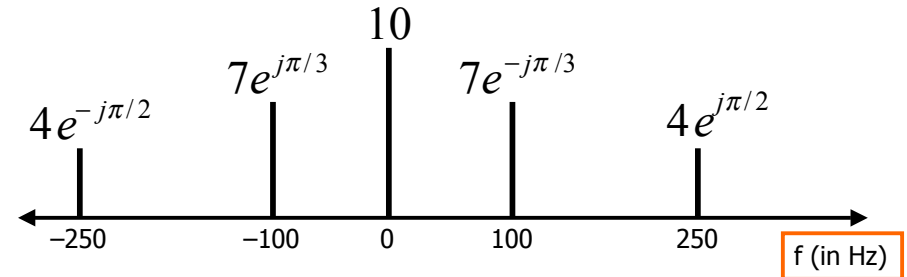
$$A \sin(7t) = \frac{1}{2} A e^{-j0.5\pi} e^{j7t} + \frac{1}{2} A e^{j0.5\pi} e^{-j7t}$$



AMPLITUDE, PHASE & FREQUENCY are shown

SPECTRUM ---> SINUSOID

■ Add the spectrum components:



What is the formula for the signal $x(t)$?

Gather (A, ω, ϕ) information

■ Frequencies:

- -250 Hz
- -100 Hz
- 0 Hz
- 100 Hz
- 250 Hz

■ Amplitude & Phase

- 4 $-\pi/2$
- 7 $+\pi/3$
- 10 **0**
- 7 $-\pi/3$
- 4 $+\pi/2$

Note the **conjugate phase**

DC is another name for zero-freq component

DC component always has zero phase (for real $x(t)$)

Add Spectrum Components-1

■ Frequencies:

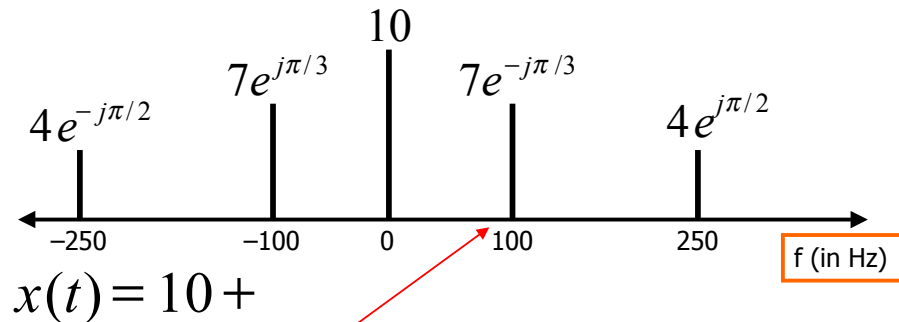
- -250 Hz
- -100 Hz
- 0 Hz
- 100 Hz
- 250 Hz

■ Amplitude & Phase

- 4 $-\pi/2$
- 7 $+\pi/3$
- 10 0
- 7 $-\pi/3$
- 4 $+\pi/2$

$$x(t) = 10 + 7e^{-j\pi/3} e^{j2\pi(100)t} + 7e^{j\pi/3} e^{-j2\pi(100)t} + 4e^{j\pi/2} e^{j2\pi(250)t} + 4e^{-j\pi/2} e^{-j2\pi(250)t}$$

Add Spectrum Components-2



$$x(t) = 10 +$$

$$7e^{-j\pi/3} e^{j2\pi(100)t} + 7e^{j\pi/3} e^{-j2\pi(100)t} \\ + 4e^{j\pi/2} e^{j2\pi(250)t} + 4e^{-j\pi/2} e^{-j2\pi(250)t}$$

Simplify Components

$$x(t) = 10 + \\ 7e^{-j\pi/3} e^{j2\pi(100)t} + 7e^{j\pi/3} e^{-j2\pi(100)t} \\ + 4e^{j\pi/2} e^{j2\pi(250)t} + 4e^{-j\pi/2} e^{-j2\pi(250)t}$$

Use Euler's Formula to get **REAL** sinusoids:

$$A \cos(\omega t + \varphi) = \frac{A}{2} e^{j\varphi} e^{j\omega t} + \frac{A}{2} e^{-j\varphi} e^{-j\omega t}$$

8/30/00

ECE-2025 rws/jMc

18

FINAL ANSWER

$$x(t) = 10 + 14 \cos(2\pi(100)t - \pi/3) \\ + 8 \cos(2\pi(250)t + \pi/2)$$

So, we get the general form:

$$x(t) = \sum_{k=1}^N A_k \cos(2\pi f_k t + \varphi_k)$$

Summary: GENERAL FORM

$$x(t) = A_0 + \sum_{k=1}^N A_k \cos(2\pi f_k t + \varphi_k)$$

$$x(t) = X_0 + \sum_{k=1}^N \Re\{X_k e^{j2\pi f_k t}\}$$

$$X_k = A_k e^{j\varphi_k}$$

Frequency = f_k

$$\Re\{z\} = \frac{1}{2} z + \frac{1}{2} z^*$$

$$x(t) = X_0 + \sum_{k=1}^N \left\{ \frac{1}{2} X_k e^{j2\pi f_k t} + \frac{1}{2} X_k^* e^{-j2\pi f_k t} \right\}$$

8/30/00

20

8/30/00

ECE-2025 rws/jMc

19

Example: Synthetic Vowel

Sum of 5 Frequency Components

f_k (Hz)	X_k	Mag	Phase (rad)
200	$(771 + j12202)$	12,226	1.508
400	$(-8865 + j28048)$	29,416	1.876
500	$(48001 - j8995)$	48,836	-0.185
1600	$(1657 - j13520)$	13,621	-1.449
1700	$4723 + j0$	4723	0

Table 3.1: Complex amplitudes for harmonic signal that approximates the vowel sound “ah”.

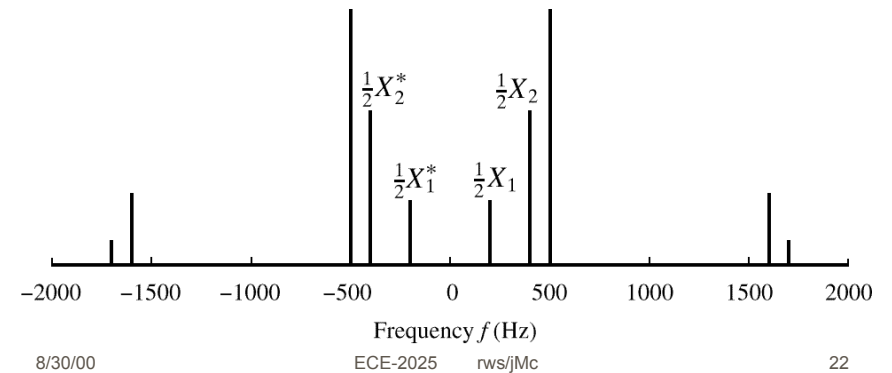
8/30/00

ECE-2025 rws/jMc

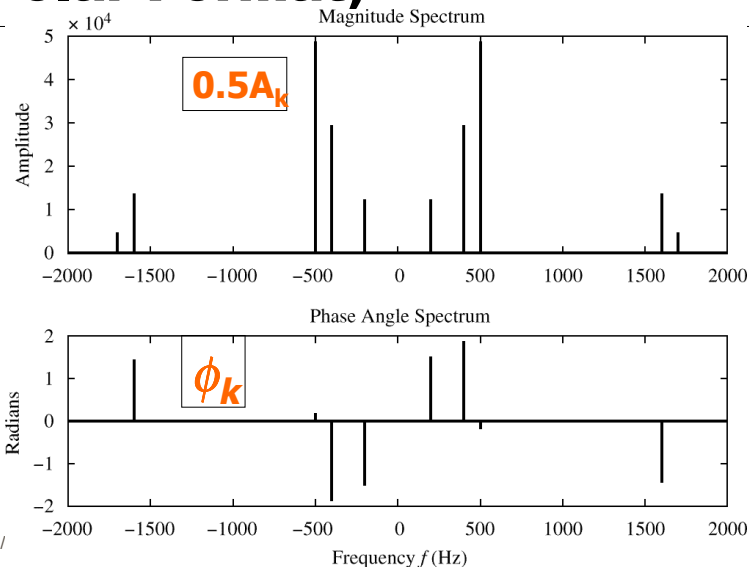
21

SPECTRUM of VOWEL

- Note: Spectrum has $0.5X_k$ (except X_{DC})
- Conjugates in negative frequency



SPECTRUM of VOWEL (Polar Format)



Vowel Waveform (sum of all 5 components)

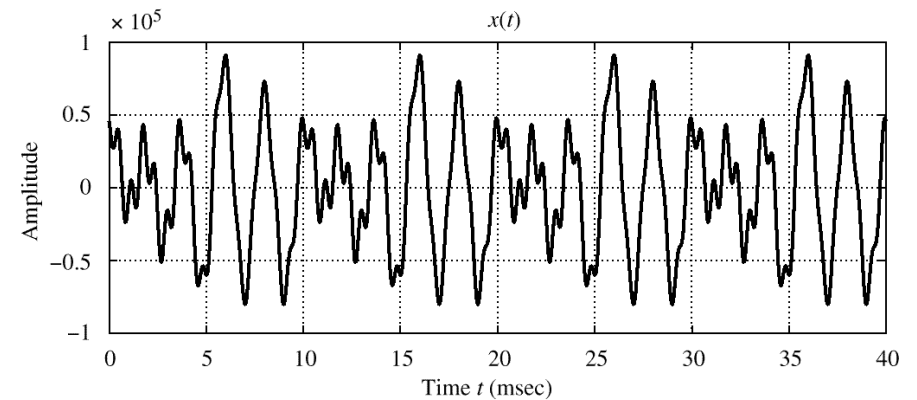


Figure 3.11 Sum of all of the terms in (3.3.4). Note that the period is 10 msec, which equals $1/f_0$.