EE-2025 Fall-2000

Lecture 7
Sampling & Aliasing
15-Sept-00

Quiz Information

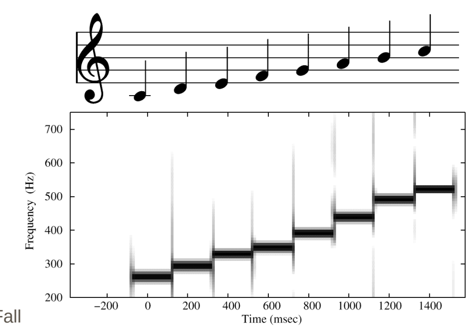
- Monday: bring your ID card
- Coverage:
  - Problem Sets #1 and #2
  - Lectures #1-5
- Rules
  - One 8.5x11" page of **HAND-WRITTEN** notes
  - Must take exam in your assigned lecture
  - 11am section cannot leave early

Information

- Quiz Review
  - Sunday nite at 7 pm in ECE Auditorium
  - Bring questions (it's not a lecture)
- Problem Set #3 due next week
- Lab #4 is Music Synthesis
  - Worth 150 Points
  - Formal lab Report
  - Listening Tests the following week.

CD-ROM DEMOS

- USE THE DEMOS
- Chapter 3: Spectrum LECTURE
  - DEMOS of SPECTROGRAM
  - BEAT NOTES/AM
  - SPEECH
  - MUSIC
  - FM & Chirps



EE-2025 Georgia Tech

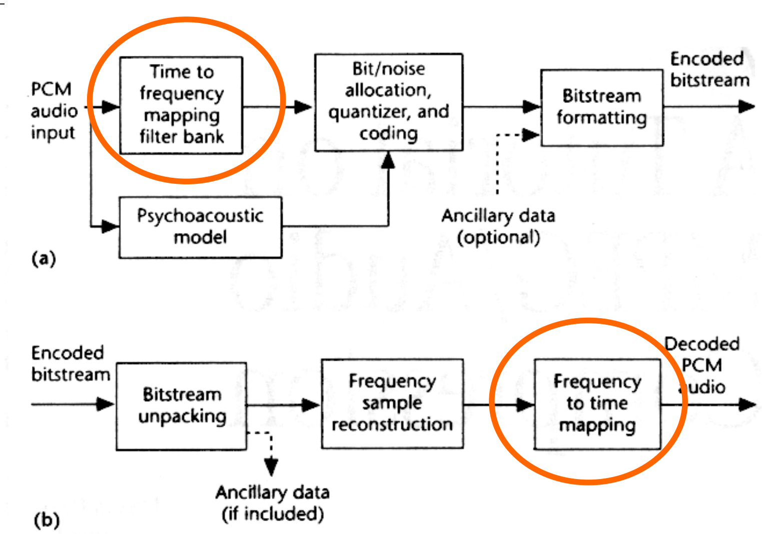
Lecture 7

Sampling & Aliasing

READING ASSIGNMENTS

- This Lecture:
  - Chapter 4, pp. 83-94
- Other Reading:
  - Recitation: Chapter 4, pp. 90-100
    - Strobe Demo
  - Next Lecture: Chap. 4, pp. 100-111

MP-3 Block Diagram



LECTURE OBJECTIVES

- SAMPLING can cause ALIASING
  - Sampling Theorem
  - Sampling Rate > 2(Highest Frequency)
- Spectrum for digital signals, x[n]
  - Normalized Frequency

$$\hat{\omega} = \omega T_s = \frac{2\pi f}{f_s} + 2\pi\ell$$

ALIASING

SYSTEMS Process Signals

- PROCESSING GOALS:
  - Change x(t) into y(t)
    - For example, more BASS
  - Improve x(t), e.g., image deblurring
  - Extract Information from x(t)

System IMPLEMENTATION

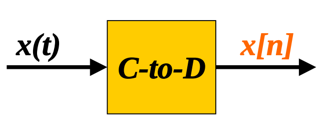
ANALOG/ELECTRONIC:

Circuits: resistors, capacitors, op-amps

- DIGITAL/MICROPROCESSOR
  - Convert x(t) to numbers stored in memory

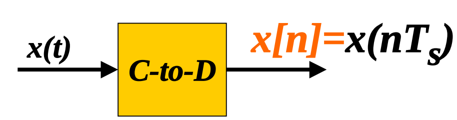
SAMPLING x(t)

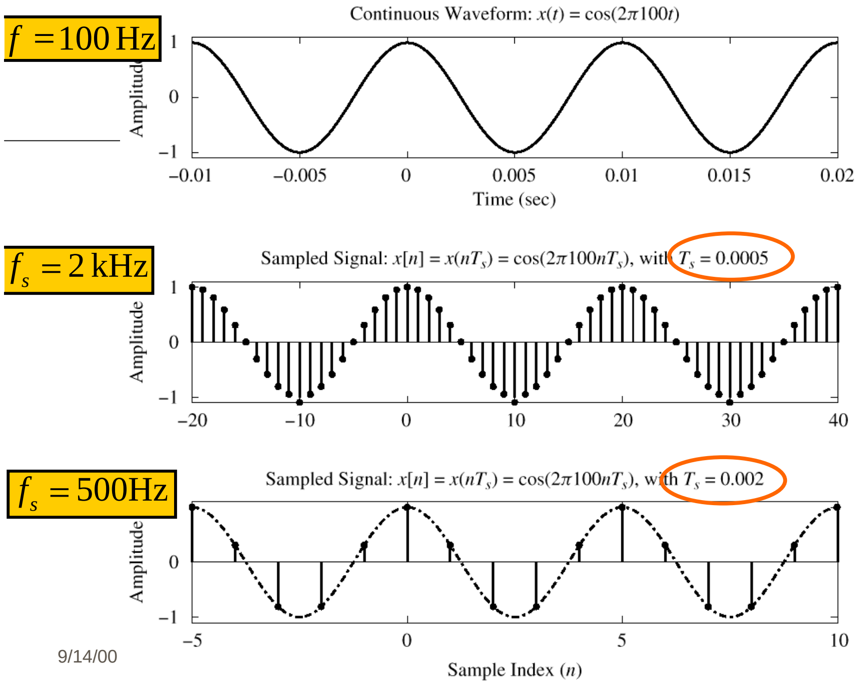
- SAMPLING PROCESS
  - Convert x(t) to numbers x[n]
  - "n" is an integer; x[n] is a sequence
  - "n" is the storage address in memory
- UNIFORM SAMPLING at $t = nT_s$
  - IDEAL: $x[n] = x(nT_s)$



SAMPLING RATE, f_s

- SAMPLING RATE (f_s)
  - $1/T_s$ = NUMBER of SAMPLES PER SECOND
  - T_s = 125 microsec --> f_s = 8000 samples/sec
    - UNITS ARE HERTZ: 8000 Hz
- UNIFORM SAMPLING at $t = nT_s = n/f_s$
  - IDEAL: $x[n] = x(nT_s)=x(n/f_s)$





SAMPLING THEOREM

- HOW OFTEN ?
  - DEPENDS on FREQUENCY of SINUSOID
  - ANSWERED by SHANNON/NYQUIST
  - ALSO DEPENDS on "**RECONSTRUCTION**"

> *Shannon Sampling Theorem*
> A continuous-time signal $x(t)$ with frequencies no higher than $f_{\max}$ can be reconstructed exactly from its samples $x[n] = x(nT_s)$, if the samples are taken at a rate $f_s = 1/T_s$ that is greater than $2f_{\max}$.

STORING DIGITAL SOUND

- ***x*[*n*]** is a SAMPLED SINUSOID
  - A list of numbers stored in memory
- CD rate is 44,100 samples per second
  - 16-bit samples
  - Stereo uses 2 channels
- Number of bytes for 1 minute is
  - 2 X (16/8) X 60 X 44100 = 10.584 Mbytes

DISCRETE-TIME SINUSOID

- Change x(t) into x[n] **DERIVATION**

$$x(t) = A\cos(\omega t + \varphi)$$

$$x[n] = x(nT_s) = A\cos(\omega n T_s + \varphi)$$

$$x[n] = A\cos((\omega T_s)n + \varphi)$$

$$x[n] = A\cos(\hat{\omega} n + \varphi)$$

$$\hat{\omega} = \omega T_s$$

DEFINE DIGITAL FREQUENCY

DIGITAL FREQUENCY $\hat{\omega}$

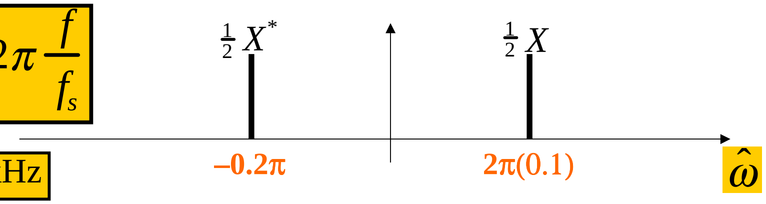
- $\hat{\omega}$ VARIES from **0** to **2π**, as f varies from 0 to the sampling frequency
- DIGITAL FREQUENCY is NORMALIZED
- UNITS are radians, **not** rad/sec

$$\hat{\omega} = \omega T_s = \frac{2\pi f}{f_s}$$

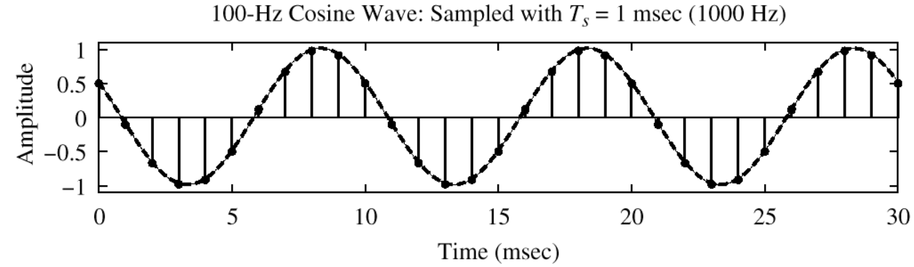
SPECTRUM (DIGITAL)

$$\hat{\omega} = 2\pi \frac{f}{f_s}$$

$f_s = 1\text{kHz}$



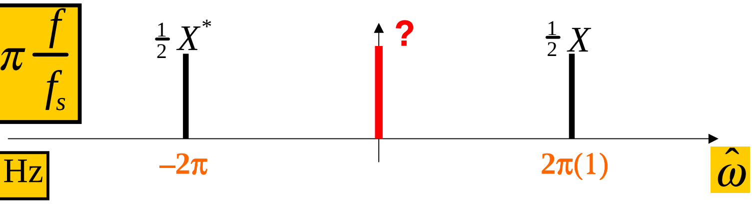
$$x[n] = \cos(2\pi(100)(n/1000) + \varphi)$$



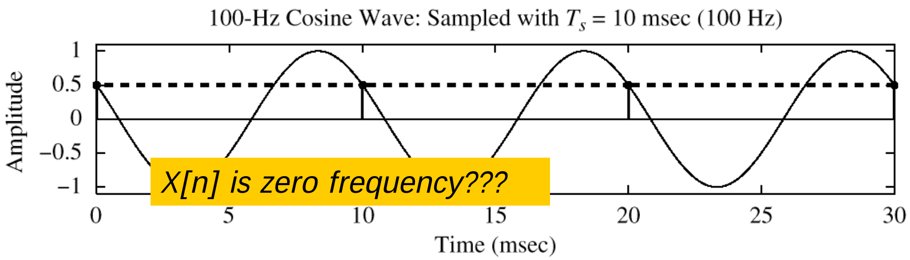
SPECTRUM (DIGITAL) ???

$$\hat{\omega} = 2\pi \frac{f}{f_s}$$

$f_s = 100\text{ Hz}$



$$x[n] = \cos(2\pi(100)(n/100) + \varphi)$$



The REST of the STORY

- Spectrum of x[n] has more than one line for each complex exponential
  - Called **ALIASING**
  - **MANY SPECTRAL LINES**
- SPECTRUM is PERIODIC with period = **2π**
  - Because

$$A\cos(\hat{\omega}n + \varphi) = A\cos((\hat{\omega} + 2\pi)n + \varphi)$$

ALIASING DERIVATION

- Other Frequencies give the same $\hat{\omega}$

$$\text{If } x(t) = A\cos(2\pi(f + \ell f_s)t + \varphi)$$

$$\text{and we substitute: } t \leftarrow \tfrac{n}{f_s}$$

$$\text{then: } x[n] = A\cos(2\pi(f + \ell f_s)\tfrac{n}{f_s} + \varphi)$$

$$\text{or, } x[n] = A\cos(2\pi\tfrac{f}{f_s}n + 2\pi\ell n + \varphi)$$

ALIASING DERIVATION–2

- Other Frequencies give the same $\hat{\omega}$

$$\text{If } x(t) = A\cos(2\pi(f + \ell f_s)t + \varphi) \qquad t \leftarrow \frac{n}{f_s}$$

$$\text{and we want: } x[n] = A\cos(\hat{\omega}n + \varphi)$$

$$\text{then: } \hat{\omega} = \frac{2\pi(f + \ell f_s)}{f_s} = \frac{2\pi f}{f_s} + \frac{2\pi \ell f_s}{f_s}$$

$$\hat{\omega} = \omega T_s = \frac{2\pi f}{f_s} + 2\pi\ell$$

ALIASING CONCLUSIONS

- ADDING f_s or $2f_s$ or $-f_s$ TO THE FREQ of x(t) gives exactly the same x[n]
- The samples, x[n] = x(n/ f_s) are EXACTLY THE SAME VALUES
- GIVEN x[n], WE CAN'T DISTINGUISH f_o FROM ($f_o + f_s$) or ($f_o + 2f_s$)

NORMALIZED FREQUENCY

- DIGITAL FREQUENCY

Normalized Radian Frequency

$$\hat{\omega} = \omega T_s = \frac{2\pi f}{f_s} + 2\pi\ell$$

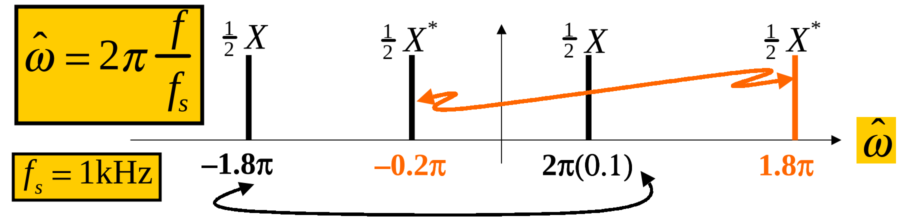
Normalized Cyclic Frequency

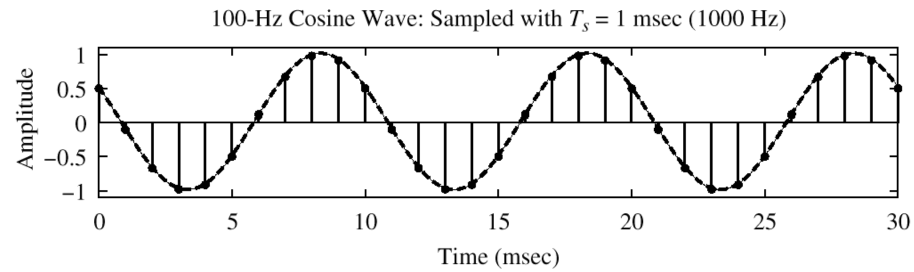
$$\hat{f} = \hat{\omega}/(2\pi) = f T_s = f/f_s$$

SPECTRUM for x[n]

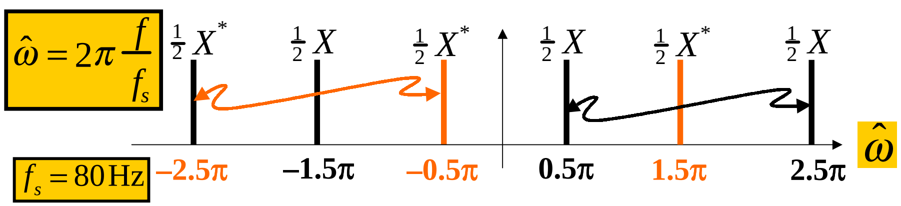
- PLOT versus NORMALIZED FREQU ENCY
- INCLUDE ALL SPECTRUM LINES
  - ALIASES
    - ADD MULTIPLES of 2π
    - SUBTRACT MULTIPLES of 2π
  - FOLDED ALIASES
    - (to be discussed later)
    - ALIASES of NEGATIVE FREQS

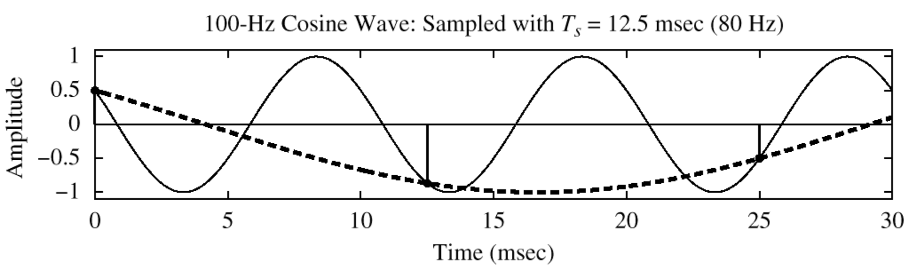
SPECTRUM (MORE LINES)





SPECTRUM (ALIASING CASE)





SPECTRUM (FOLDING CASE)