# EE-2025 Fall-2000

Lecture 9

FIR Filtering Intro

25-Sept-00

## Information

- Music Listening this week
  - Have your song ready BEFORE lab
- MATLAB help:
  - Mon & Tues @ 6pm, VL-456
  - Tues @ 11-12 in VL-361
- Problem Set #4 due THIS WEEK



## Quiz Info

- All grading change on Quiz #1 must be completed by 3-Oct
  - After that, the scores are permanent
- Lab Quiz NEXT week: On-Line format
- Quiz #2 on 20-Oct (Friday)

LECTURE



## Questions & Learning

- **I**t is not the answer that enlightens, but the question......Decouvertes
- **N**o man really becomes a fool until he stops asking questions...C. Steinmetz

LECTURE

# EE-2025 Georgia Tech

Lecture 9
FIR Filtering Intro

# READING ASSIGNMENTS

- This Lecture:
  - Chapter 5, pp. 119-131
- Other Reading:
  - Recitation: Ch. 5, pp. 127-133, 142-146
    - CONVOLUTION
  - Next Lecture: Chapter 5, pp. 133-152



# LECTURE OBJECTIVES

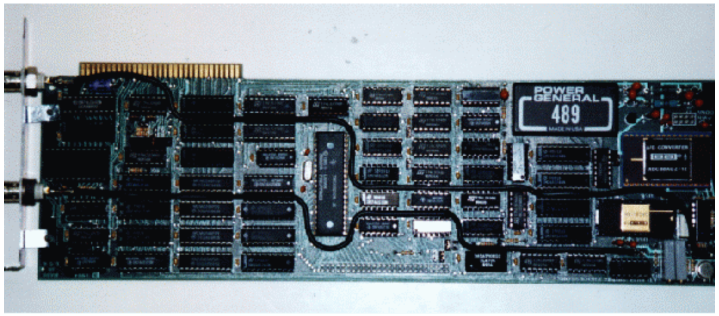
- INTRODUCE FILTERING IDEA
  - Weighted Average
  - Running Average
- FINITE IMPULSE RESPONSE FILTERS
  - **FIR** Filters
  - Show how to compute the output y[n] from the input signal, x[n]



# DIGITAL FILTERING


- CONCENTRATE on the COMPUTER
  - PROCESSING ALGORITHMS
  - SOFTWARE (MATLAB)
  - HARDWARE: DSP chips, VLSI
- DSP: DIGITAL SIGNAL PROCESSING

# The TMS32010, 1983




First PC plug-in board from Atlanta Signal Processors Inc.

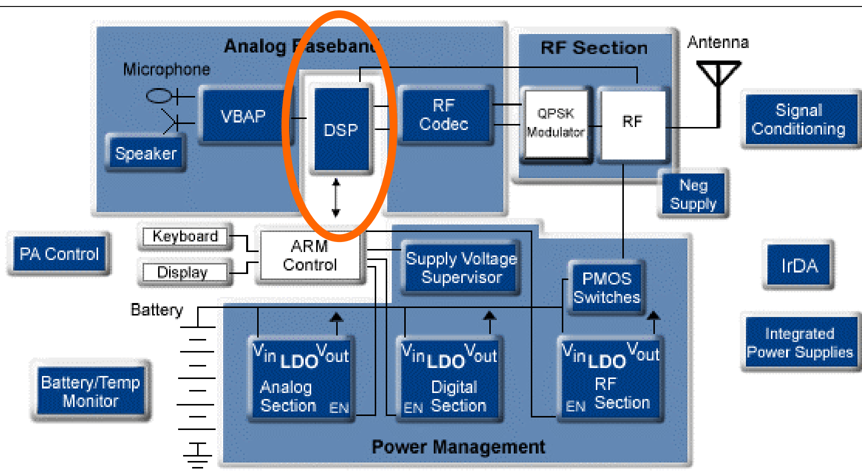


# Rockland Digital Filter, 1971


For the price of a small house, you could have one of these.



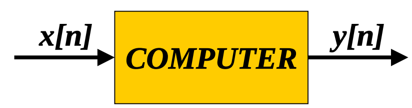
# Digital Cell Phone




Free (?) with 2 year contract



# DISCRETE-TIME SYSTEM




- OPERATE on x[n] to get y[n]
- WANT a GENERAL CLASS of SYSTEMS
  - ANALYZE the SYSTEM
    - TOOLS: TIME-DOMAIN & FREQUENCY-DOMAIN
  - SYNTHESIZE the SYSTEM

# D-T SYSTEM EXAMPLES

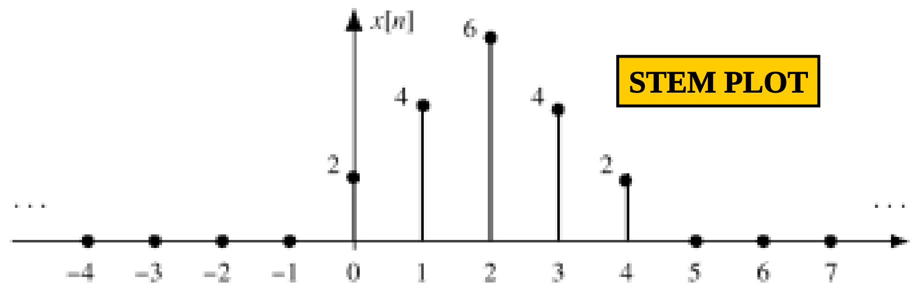

- EXAMPLES:
  - POINTWISE OPERATORS
    - SQUARING: $y[n] = (x[n])^2$
  - RUNNING AVERAGE
    - RULE: “the output at time n is the average of three consecutive input values”



# DISCRETE-TIME SIGNAL

- x[n] is a LIST of NUMBERS
  - INDEXED by “n”

**STEM PLOT**






# 3-PT AVERAGE SYSTEM

- ADD 3 CONSECUTIVE NUMBERS
  - Do this for each “n”

the following input–output equation

**Make a TABLE**

$$y[n] = \tfrac{1}{3}(x[n] + x[n+1] + x[n+2])$$

| $n$ | $n < -2$ | $-2$ | $-1$ | 0 | 1 | 2 | 3 | 4 | 5 | $n > 5$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $x[n]$ | 0 | 0 | 0 | 2 | 4 | **6** | **4** | **2** | 0 | 0 |
| $y[n]$ | 0 | $\frac{2}{3}$ | 2 | 4 | $\frac{14}{3}$ | **4** | 2 | $\frac{2}{3}$ | 0 | 0 |

**n=0** $y[0] = \tfrac{1}{3}(x[0] + x[1] + x[2])$

**n=1** $y[1] = \tfrac{1}{3}(x[1] + x[2] + x[3])$

**INPUT SIGNAL**

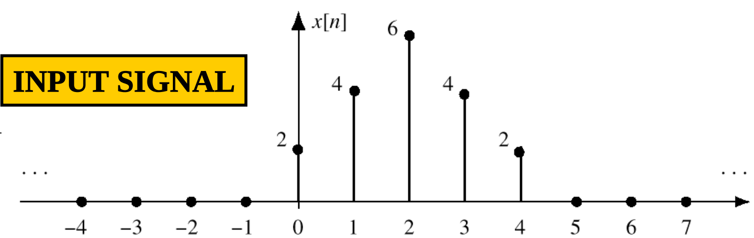



**Figure 5.2** Finite-length input signal, $x[n]$.

$$y[n] = \tfrac{1}{3}(x[n] + x[n+1] + x[n+2])$$

**OUTPUT SIGNAL**

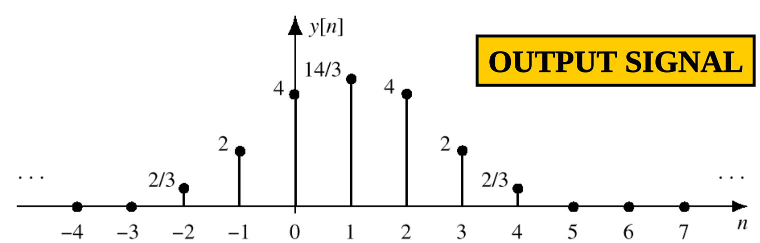



**Figure 5.3** Output of running average, $y[n]$.

# PAST, PRESENT, FUTURE

Sec. 5.2 The Running Average Filter **123**

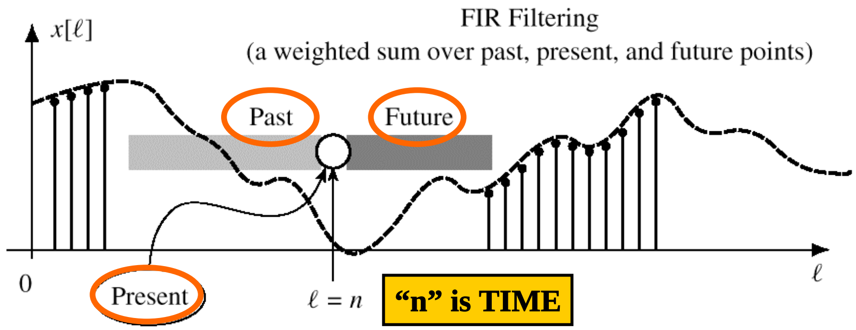



**Figure 5.4** The running-average filter calculation at time index $n$ uses values within a sliding window (shaded). Dark shading indicates the future ($\ell > n$); light shading, the past ($\ell < n$).

# ANOTHER 3-pt AVERAGER

- Uses "PAST" VALUES of x[n]
  - IMPORTANT IF "n" represents REAL TIME
    - WHEN x[n] & y[n] ARE STREAMS

$$y[n] = \tfrac{1}{3}(x[n] + x[n-1] + x[n-2])$$

| $n$ | $n < -2$ | $-2$ | $-1$ | $0$ | $1$ | $2$ | $3$ | $4$ | $5$ | $6$ | $7$ | $n > 7$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $x[n]$ | 0 | 0 | 0 | 2 | 4 | **6** | **4** | **2** | 0 | 0 | 0 | 0 |
| $y[n]$ | 0 | 0 | 0 | $\frac{2}{3}$ | 2 | 4 | $\frac{14}{3}$ | **4** | 2 | $\frac{2}{3}$ | 0 | 0 |



# GENERAL FIR FILTER

- FILTER COEFFICIENTS $\{b_k\}$
  - DEFINE THE FILTER

$$y[n] = \sum_{k=0}^{M} b_k\, x[n-k]$$

  - For example, $\{b_k\} = \{3, -1, 2, 1\}$

$$y[n] = \sum_{k=0}^{3} b_k x[n-k]$$

$$= 3x[n] - x[n-1] + 2x[n-2] + x[n-3]$$



# GENERAL FIR FILTER

- FILTER COEFFICIENTS $\{b_k\}$

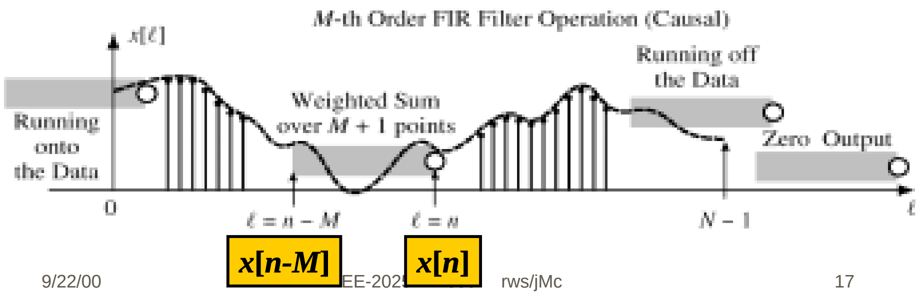
$$y[n] = \sum_{k=0}^{M} b_k\, x[n-k]$$

- FILTER ORDER is M
- FILTER LENGTH is L = M+1
  - NUMBER of FILTER COEFFS is L

# GENERAL FIR FILTER

- SLIDE a WINDOW across x[n]

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$





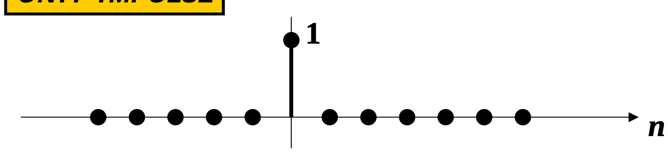

# FILTERED STOCK SIGNAL


# SPECIAL INPUT SIGNALS

- x[n] = SINUSOID **FREQUENCY RESPONSE**
- x[n] has only one NON-ZERO VALUE

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

**UNIT-IMPULSE**






# UNIT IMPULSE SIGNAL δ[n]

| $n$ | ... | −2 | −1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | ... |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\delta[n]$ | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\delta[n-3]$ | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |

**n=3**

**δ[n] is NON-ZERO When its argument is equal to ZERO**

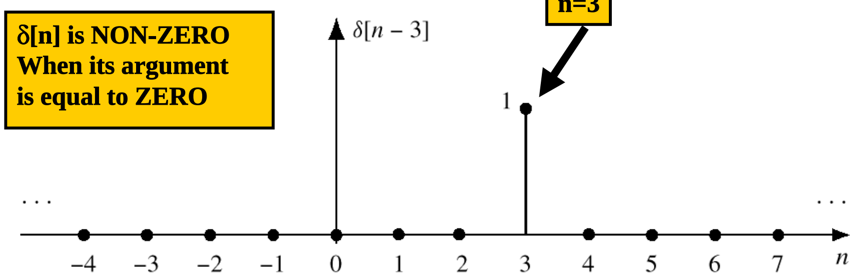
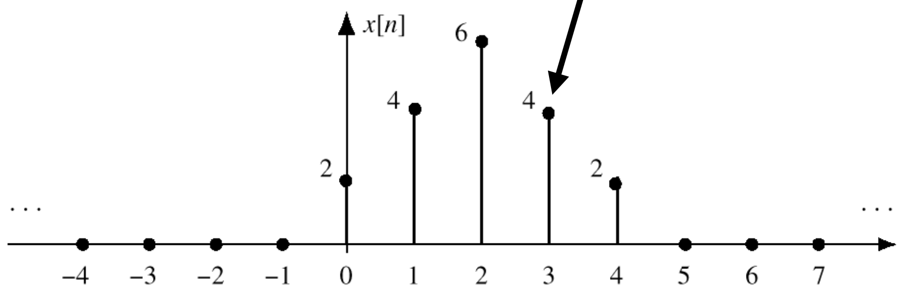



**Figure 5.7** Shifted impulse sequence, $\delta[n-3]$.

# MATH FORMULA for x[n]

- Use SHIFTED IMPULSES to write x[n]

$$x[n] = 2\delta[n] + 4\delta[n-1] + 6\delta[n-2] + 4\delta[n-3] + 2\delta[n-4]$$






# SUM of SHIFTED IMPULSES

| $n$ | ... | −2 | −1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | ... |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $2\delta[n]$ | 0 | 0 | 0 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $4\delta[n-1]$ | 0 | 0 | 0 | 0 | 4 | 0 | 0 | 0 | 0 | 0 | 0 |
| $6\delta[n-2]$ | 0 | 0 | 0 | 0 | 0 | 6 | 0 | 0 | 0 | 0 | 0 |
| $4\delta[n-3]$ | 0 | 0 | 0 | 0 | 0 | 0 | 4 | 0 | 0 | 0 | 0 |
| $2\delta[n-4]$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 0 | 0 | 0 |
| $x[n]$ | 0 | 0 | 0 | 2 | 4 | 6 | 4 | 2 | 0 | 0 | 0 |

$$x[n] = \sum_k x[k]\delta[n-k]$$ ← **This formula ALWAYS works**

$$= \ldots + x[-1]\delta[n+1] + x[0]\delta[n] + x[1]\delta[n-1] + \ldots \quad (5.3.6)$$

# 4-pt AVERAGER

- CAUSAL SYSTEM: USE PAST VALUES
  - y[n] = (x[n]+x[n-1]+x[n-2]+x[n-3])/4
- INPUT = UNIT IMPULSE SIGNAL = **δ[n]**
  - x[n] = **δ[n]**
  - **y[n] = 0.25 δ[n] + 0.25δ[n-1] + 0.25δ[n-2] + 0.25δ[n-3]**
- OUTPUT is called "IMPULSE RESPONSE"
  - h[n] = {...,0,0,0.25,0.25,0.25,0.25,0,0,...}



# FIR IMPULSE RESPONSE

- Convolution = Filter Definition
  - Filter Coeffs = Impulse Response

| $n$ | $n<0$ | 0 | 1 | 2 | 3 | ... | $M$ | $M+1$ | $n>M+1$ |
|---|---|---|---|---|---|---|---|---|---|
| $x[n]=\delta[n]$ | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $y[n]=h[n]$ | 0 | $b_0$ | $b_1$ | $b_2$ | $b_3$ | ... | $b_M$ | 0 | 0 |

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

$$y[n] = \sum_{k=0}^{M} h[k]x[n-k]$$

**CONVOLUTION**