EE-2025 Fall-2000

Lecture 12

Digital Filtering of Analog Signals

6-Oct-00

Info: Web-CT, Lab, HW

- Quiz #2 on 20-Oct (Friday)
  - Coverage: HW #3, #4, #5, #6, and #7
- MATLAB Help on Monday & Tuesday
  - 6 PM, VL-456
  - Also, Tuesday at 11am in VL-361
- Lab Quiz #2 next week during Lab #7
  - 3 short problems again

COURSE OBJECTIVE

- Students will be able to:
- Understand **mathematical** descriptions of signal processing **algorithms** and express those algorithms as computer **implementations** (MATLAB)

LECTURE

Perseverance

- **A** lowly virtue whereby mediocrity achieves a glorious success...A. Bierce
- **B**ear in mind, if you are going to amount to anything, that your success does not depend upon the brilliance and the impetuosity with which you take hold, but upon the ever lasting and sanctified bull doggedness with which you hang on after you have taken hold...Dr. A. B. Meldrum

LECTURE

EE-2025 Georgia Tech

Lecture 12

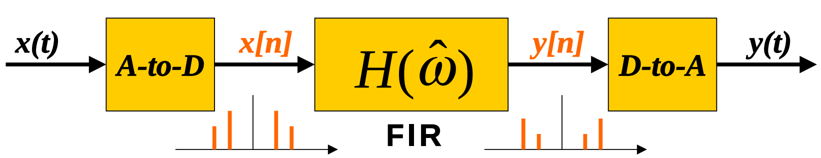
Digital Filtering of Analog Signals

READING ASSIGNMENTS

- This Lecture:
  - Chapter 6, pp. 188-194
- Other Reading:
  - Recitation: Ch. 6, pp. 176-188
    - FREQUENCY RESPONSE EXAMPLES
  - Next Lecture: Chapter 7, start

LECTURE OBJECTIVES

- Two Domains: Time & Frequency
- Track the spectrum of x[n] thru an FIR Filter.
- **UNIFICATION**: How does Frequency Response affect x(t) to produce y(t) ?



PREVIOUS LECTURE REVIEW

- SINUSOIDAL INPUT SIGNAL
  - OUTPUT has SAME FREQUENCY
  - DIFFERENT Amplitude and Phase
- FREQUENCY RESPONSE of FIR
  - MAGNITUDE vs. Frequency
  - PHASE vs. Freq
  - PLOTTING:

MAG

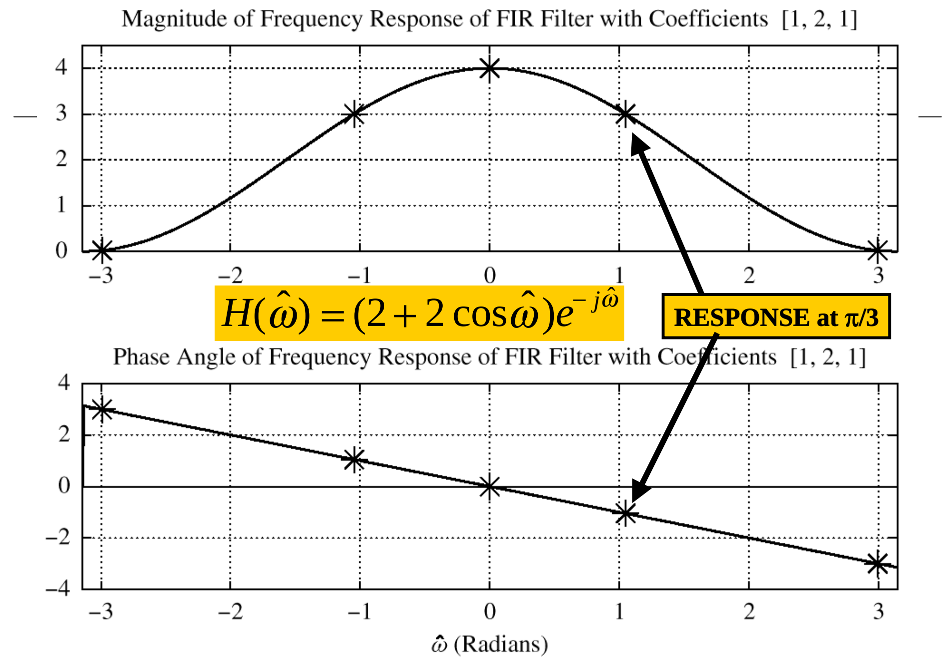
PHASE

$$H(\hat{\omega}) = |H(\hat{\omega})| e^{j\varphi(\hat{\omega})}$$

FREQ. RESPONSE PLOTS

- DENSE GRID **(ww)** from $-\pi$ to $+\pi$
  - `ww = -pi:(pi/100):pi;`
- **`yy = freqz(bb,1,ww)`**
  - VECTOR **bb** contains Filter Coefficients
  - DSP-First: `yy = freekz(bb,1,ww)`

$$\mathcal{H}(\hat{\omega}) = \sum_{k=0}^{M} b_k e^{-j\hat{\omega}k}$$



TIME & FREQ DOMAINS

- LTI: Linear & Time-Invariant
  - COMPLETELY CHARACTERIZED by:
    - IMPULSE RESPONSE h[n] (time domain)
    - FREQUENCY RESPONSE

x[n] → $H(\hat{\omega}) = |H(\hat{\omega})| e^{j\varphi(\hat{\omega})}$ → y[n]

- Two DOMAINS: time & frequency
  - Go back and forth QUICKLY

TIME & FREQUENCY

$$y[n] = \sum_{k=0}^{M} b_k x[n-k] = \sum_{k=0}^{M} h[k]x[n-k]$$

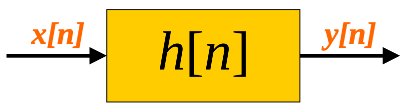
FIR DIFFERENCE EQUATION is the TIME-DOMAIN

$$H(\hat{\omega}) = \sum_{k=0}^{M} h[k] e^{-j\hat{\omega}k}$$

$$H(\hat{\omega}) = h[0]e^{-j0} + h[1]e^{-j\hat{\omega}} + h[2]e^{-j\hat{\omega}2} + h[3]e^{-j\hat{\omega}3} + \cdots$$

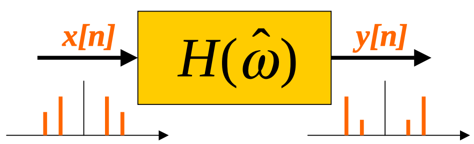
Ex: DELAY by 2 SYSTEM

Find $h[n]$ and $H(\hat{\omega})$ for $y[n] = x[n-2]$



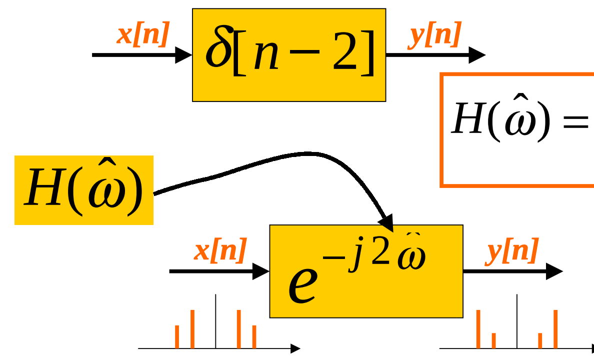
$b_k = \{0,0,1\}$

$h[n] = \delta[n-2]$



DELAY by 2 SYSTEM

Find $h[n]$ and $H(\hat{\omega})$ for $y[n] = x[n-2]$



$$H(\hat{\omega}) = \sum_k \delta[k-2] e^{-j\hat{\omega}k}$$

k=2 ONLY

GENERAL DELAY PROPERTY

Find $h[n]$ and $H(\hat{\omega})$ for $y[n] = x[n-n_d]$

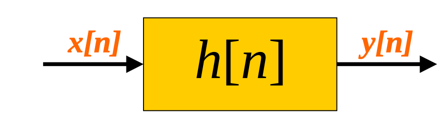
$h[n] = \delta[n-n_d]$

$$H(\hat{\omega}) = \sum_k \delta[k-n_d] e^{-j\hat{\omega}k} = e^{-j\hat{\omega}n_d}$$

ONLY ONE non-ZERO TERM for k at k = n_d

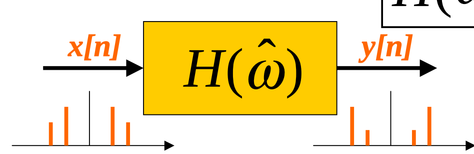
FREQ DOMAIN --> TIME ??

- START with $H(\hat{\omega})$ and find $h[n]$ or b_k



$h[n] = ?$

$$H(\hat{\omega}) = 7e^{-j2\hat{\omega}} \cos(\hat{\omega})$$



FREQ DOMAIN --> TIME

$$H(\hat{\omega}) = 7e^{-j2\hat{\omega}}\cos(\hat{\omega})$$

EULER's Formula

$$= 7e^{-j2\hat{\omega}}(0.5e^{j\hat{\omega}} + 0.5e^{-j\hat{\omega}})$$

$$= (3.5e^{-j\hat{\omega}} + 3.5e^{-j3\hat{\omega}})$$

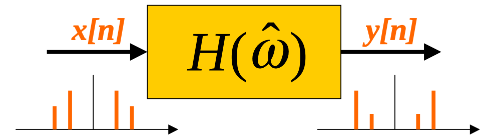
$$h[n] = 3.5\delta[n-1] + 3.5\delta[n-3]$$

$$b_k = \{0,\ 3.5,\ 0, 3.5\}$$

EXAMPLE 6.2

Find $y[n]$ when $H(\hat{\omega})$ is known

& $x[n] = 2e^{j\pi/4}e^{j(\pi/3)n}$



$$H(\hat{\omega}) = (2 + 2\cos\hat{\omega})e^{-j\hat{\omega}}$$

EXAMPLE 6.2 (answer)

Find $y[n]$ when $x[n] = 2e^{j\pi/4}e^{j(\pi/3)n}$

Answer: Eval $H(\hat{\omega})$ at $\hat{\omega} = \pi/3$.

$$H(\hat{\omega}) = (2 + 2\cos\hat{\omega})e^{-j\hat{\omega}}$$

$$H(\hat{\omega}) = 3e^{-j\pi/3} \quad @\ \hat{\omega} = \pi/3$$

$$y[n] = \left(3e^{-j\pi/3}\right) \times 2e^{j\pi/4}e^{j(\pi/3)n} = 6e^{-j\pi/12}e^{j(\pi/3)n}$$

SINUSOID thru FIR

$$x[n] = X_0 + \sum_{k=1}^{N}\left(\frac{X_k}{2}e^{j\hat{\omega}_k n} + \frac{X_k^*}{2}e^{-j\hat{\omega}_k n}\right)$$

$$= X_0 + \sum_{k=1}^{N}|X_k|\cos(\hat{\omega}_k n + \angle X_k)$$

MULTIPLY MAGS

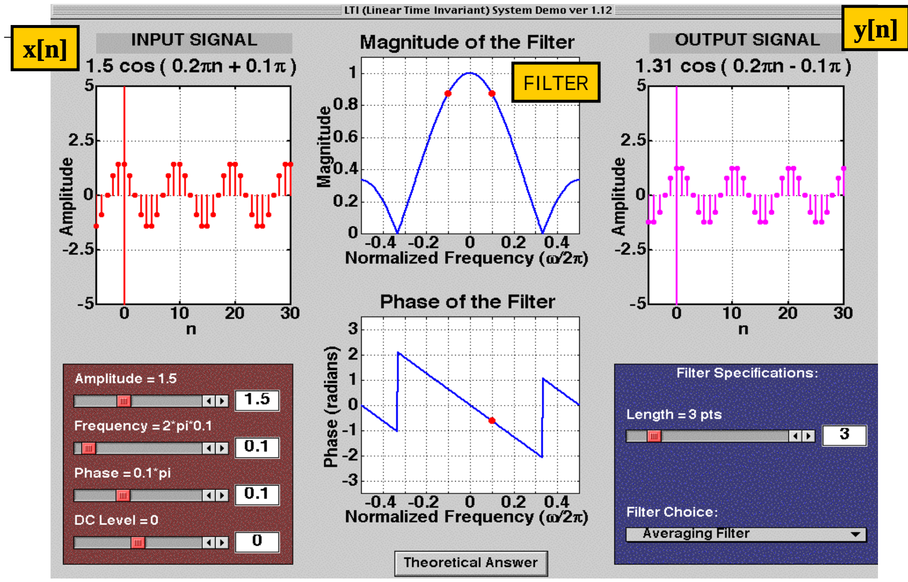
ADD PHASES

if $\mathcal{H}(-\hat{\omega}) = \mathcal{H}^*(\hat{\omega})$, the corresponding output is

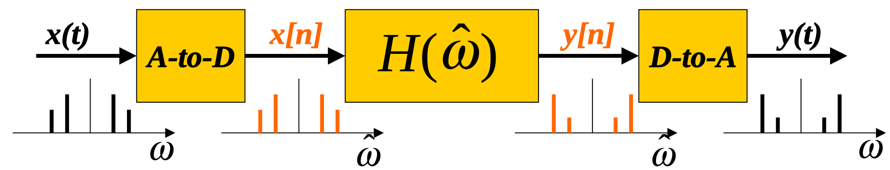
$$y[n] = \mathcal{H}(0)X_0 + \sum_{k=1}^{N}\left(\mathcal{H}(\hat{\omega}_k)\frac{X_k}{2}e^{j\hat{\omega}_k n} + \mathcal{H}(-\hat{\omega}_k)\frac{X_k^*}{2}e^{-j\hat{\omega}_k n}\right)$$

$$= \mathcal{H}(0)X_0 + \sum_{k=1}^{N}|\mathcal{H}(\hat{\omega}_k)||X_k|\cos\left(\hat{\omega}_k n + \angle X_k + \angle\mathcal{H}(\hat{\omega}_k)\right)$$

LTI Demo with Sinusoids

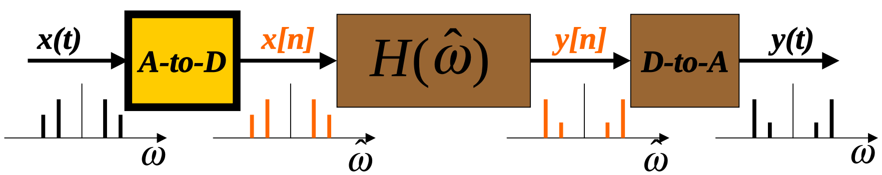


DIGITAL "FILTERING"



- ω SPECTRUM of x(t) (SUM of SINUSOIDS)
- $\hat{\omega}$ SPECTRUM of x[n]
  - Is ALIASING a PROBLEM ?
- SPECTRUM y[n] (FIR Gain or Nulls)
- ω Then, OUTPUT y(t) = SUM of SINUSOIDS

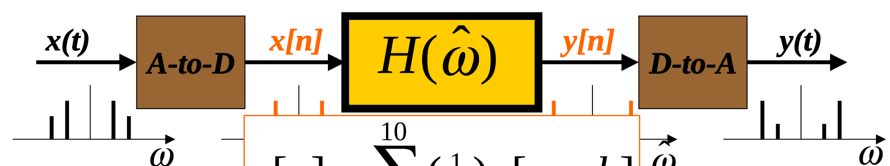
FREQUENCY SCALING



- TIME SAMPLING: $t = nT_s$
  - IF <u>NO</u> ALIASING:
  - FREQUENCY SCALING

$$\hat{\omega} = \omega T_s = \frac{\omega}{f_s}$$

11-pt AVERAGER Example



$$y[n] = \sum_{k=0}^{10} (\tfrac{1}{11}) x[n-k]$$

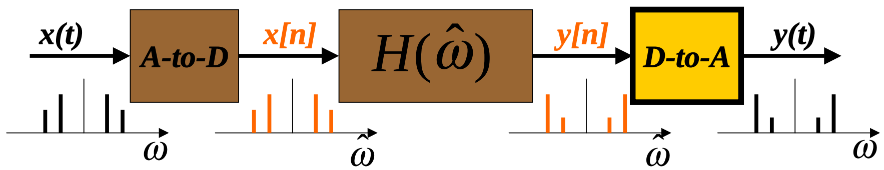
- 250 Hz
- 25 Hz

$$H(\hat{\omega}) = \frac{\sin(\frac{11}{2}\hat{\omega})}{11\sin(\frac{1}{2}\hat{\omega})} e^{-j5\hat{\omega}}$$

?

$$x(t) = \cos(2\pi(25)t) + \cos(2\pi(250)t - \tfrac{1}{2}\pi)$$

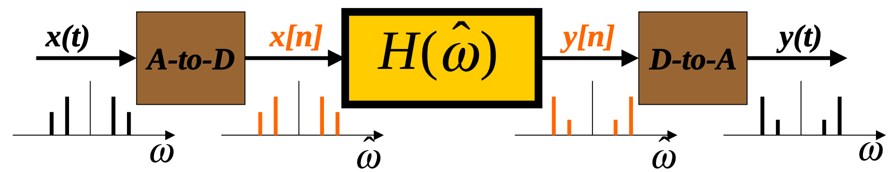
D-A FREQUENCY SCALING



- TIME SAMPLING: $t = nT_s \Rightarrow n \leftarrow t f_s$
- RECONSTRUCT up to $0.5f_s$
  - FREQUENCY SCALING

$$\omega = \hat{\omega} f_s$$

TRACK the FREQUENCIES

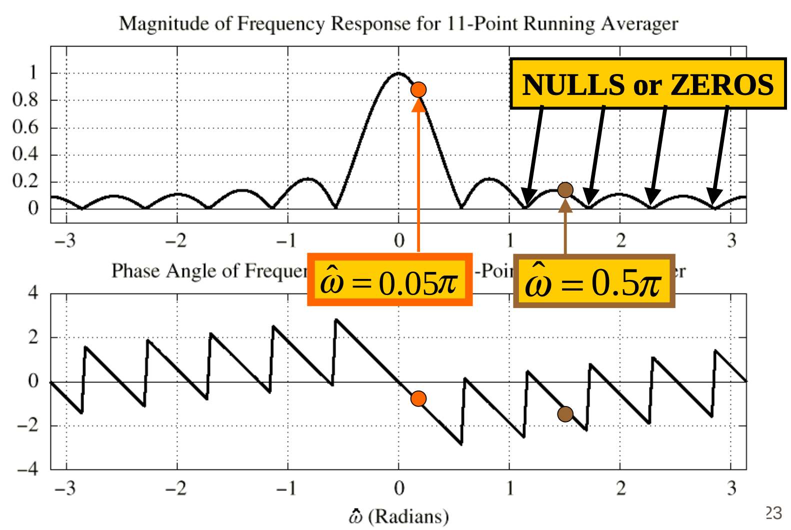


250 Hz	0.5π	$H(0.5\pi)$	0.5π	250 Hz
25 Hz	$.05\pi$	$H(0.05\pi)$	$.05\pi$	25 Hz

Fs = 1000 Hz

NO new freqs

11-pt AVERAGER



EVALUATE Freq. Response

$$H(\hat{\omega}) = \frac{\sin(\hat{\omega}11/2)}{11\sin(\hat{\omega}/2)} e^{-j\hat{\omega}5}$$

At $\hat{\omega} = 0.5\pi$

$$H(\hat{\omega}) = \frac{\sin((0.5\pi)11/2)}{11\sin(0.5\pi/2)} e^{-j(0.5\pi)5}$$

$$= \frac{\sin(2.75\pi)}{11\sin(0.25\pi)} e^{-j2.5\pi}$$

$$= 0.0909 e^{-j0.5\pi}$$

EVALUATE Freq. Response

$$x(t) = \cos(2\pi(25)t) + \sin(2\pi(250)t)$$

evaluating at 25 and 250 Hz.

$$\mathcal{H}(2\pi(25)/1000) = \frac{\sin(\pi(25)(11)/1000)}{11\sin(\pi(25)/1000)} e^{-j2\pi(25)(5)/1000}$$

$$= 0.8811 e^{-j\pi/4}$$

$$\mathcal{H}(2\pi(250)/1000) = \frac{\sin(\pi(250)(11)/1000)}{11\sin(\pi(250)/1000)} e^{-j2\pi(250)(5)/1000}$$

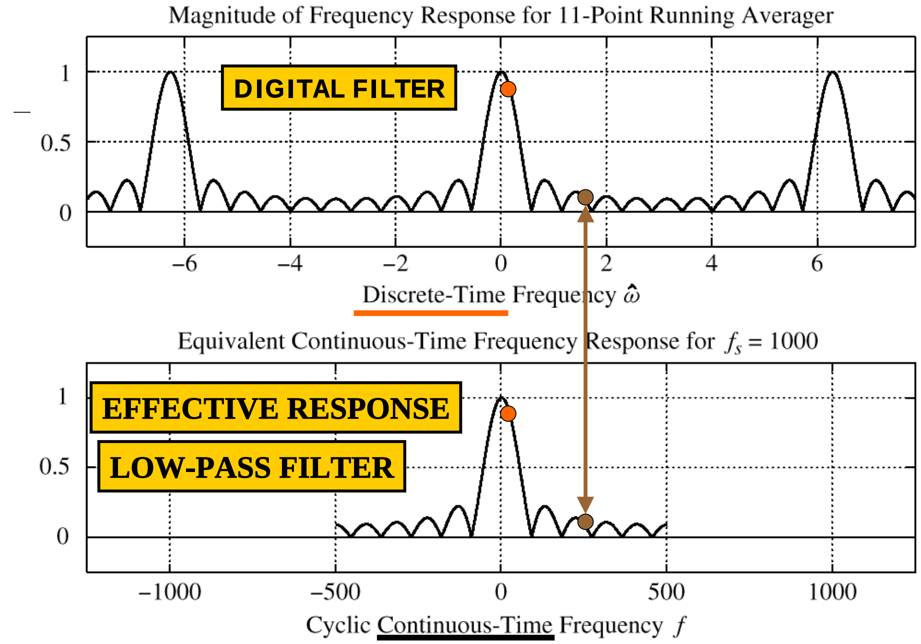
$$= 0.0909 e^{-j\pi/2}$$

$$y(t) = 0.8811\cos(2\pi(25)t - \pi/4) + 0.0909\sin(2\pi(250)t - \pi/2)$$

MAG SCALE

PHASE CHANGE

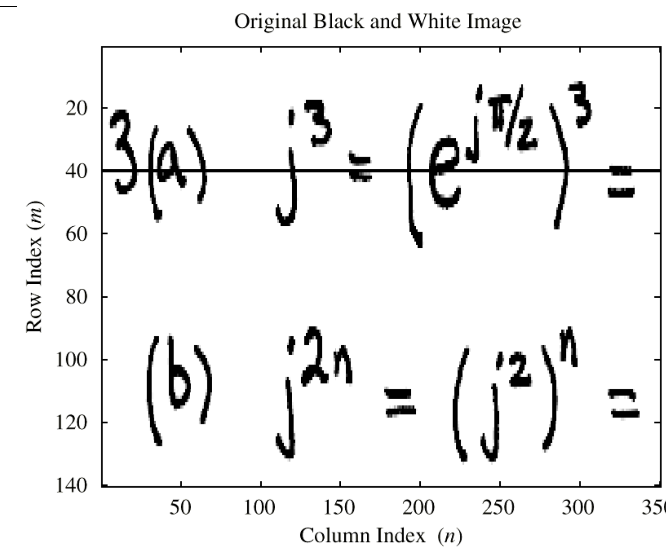
$f_s = 1000$



FILTER TYPES

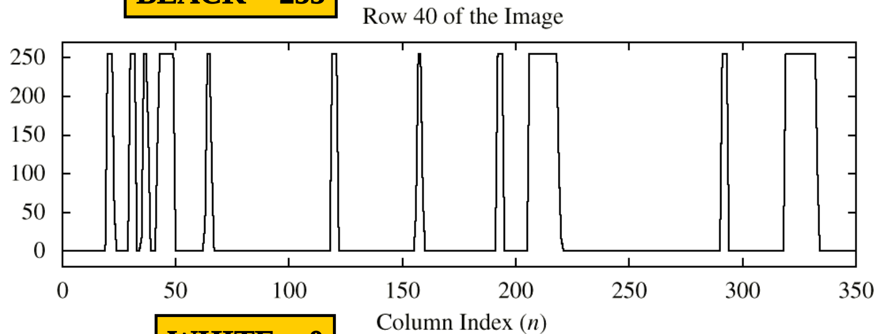
- LOW-PASS FILTER (LPF)
  - BLURRING
  - ATTENUATES HIGH FREQUENCIES
- HIGH-PASS FILTER (HPF)
  - SHARPENING for IMAGES
  - BOOSTS THE HIGHS
  - REMOVES DC
- BAND-PASS FILTER (BPF)

B & W IMAGE



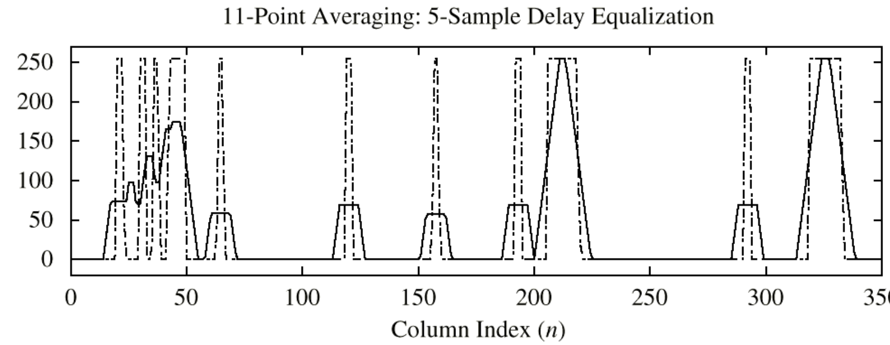
ROW of B&W IMAGE

BLACK = 255



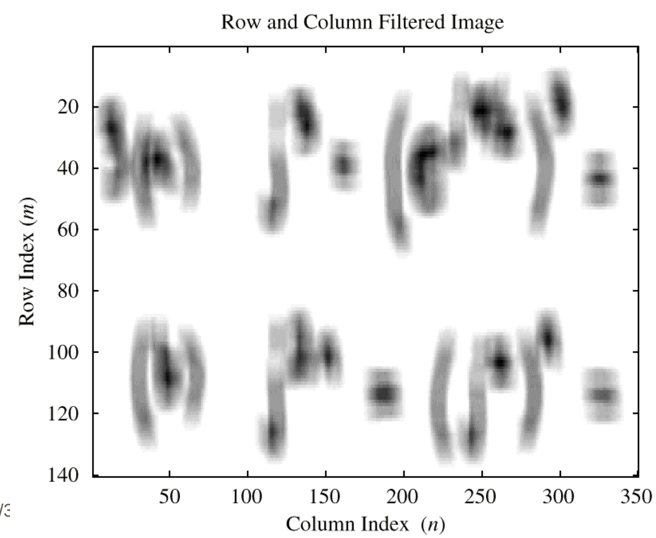
WHITE = 0

FILTERED ROW of IMAGE



ADJUSTED DELAY by 5 samples

FILTERED B&W IMAGE



LPF: BLUR

B&W IMAGE with COSINE

FILTERED: 11-pt AVG