EE-2025 Fall-2000

Lecture 15

Recap FIR Filters: H(z) and the Frequency Response

16-Oct-00

Info: Quiz #2

- Quiz #2 is 20-Oct (Friday)
  - Coverage: HW #3 -- #7
  - One page of notes (8.5 by 11, two sided)
  - Calculator
- Review Session 7:30pm on Thursday

- Prob Set #7 is due this week
  - Solution will be posted Thurs nite @ 7:30pm

Info: Web-CT, Lab, HW

- Get NEW CHAPTERS
  - Continuous-Time Signals & Systems
  - PDF (Web-CT) or Bookstore

- Lab #8 on Bandpass Filtering
  - Spans two weeks over the break

LECTURE

EE-2025 Georgia Tech

Lecture 15

Recap FIR Filters: H(z) and the Frequency Response

READING ASSIGNMENTS

- This Lecture:
  - Chapter 7, pp. 220-230
- Other Reading:
  - Recitation & Lab: Ch. 7, pp. 220-239
    - ZEROS (and POLES)
  - Next Lecture: Notes on Continuous-Time

LECTURE OBJECTIVES

- ZEROS and POLES
- Relate H(z) to FREQUENCY RESPONSE

$$H(\hat{\omega}) = H(z)\Big|_{z=e^{j\hat{\omega}}}$$

- THREE DOMAINS:
  - Show Relationship for FIR:

$$h[n] \leftrightarrow H(z) \leftrightarrow H(e^{j\hat{\omega}})$$

DESIGN PROBLEM

- Example:
  - Design a Lowpass FIR filter (Find b_k)
  - Reject completely 0.7π, 0.8π, and 0.9π
  - Estimate the filter length needed to accomplish this task. How many b_k ?
- Z POLYNOMIALS provide the TOOLS

THREE DOMAINS

Z-TRANSFORM-DOMAIN

POLYNOMIALS: H(z)

$$\{b_k\}$$

TIME-DOMAIN

FREQ-DOMAIN

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

$$H(\hat{\omega}) = \sum_{k=0}^{M} b_k e^{-j\hat{\omega}k}$$

FREQUENCY RESPONSE ?

- Same Form:

$\hat{\omega}$ − Domain

$$H(\hat{\omega}) = \sum_{k=0}^{M} b_k e^{-j\hat{\omega}k}$$

$$H(\hat{\omega}) = \sum_{k=0}^{M} b_k \left(e^{j\hat{\omega}}\right)^{-k}$$

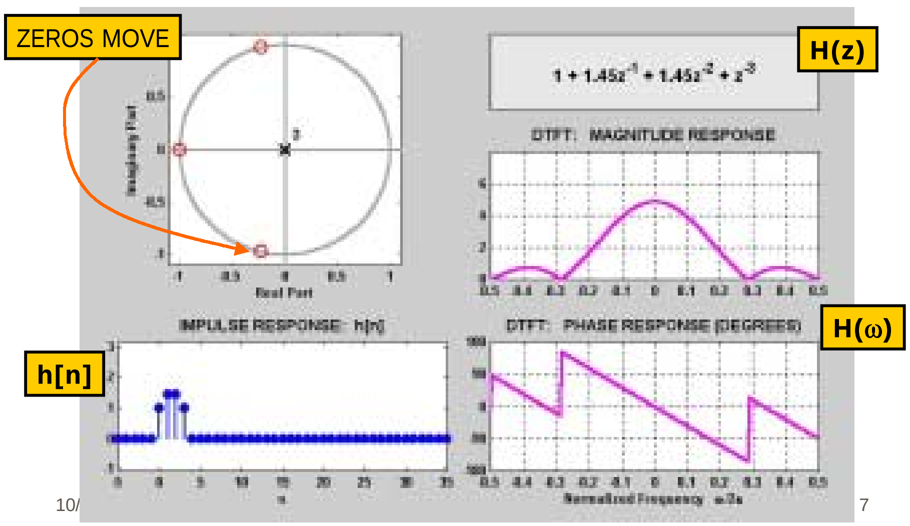
$$z = e^{j\hat{\omega}}$$

z − Domain

$$H(z) = \sum_{k=0}^{M} b_k z^{-k}$$

SAME COEFFICIENTS

3 DOMAINS MOVIE: FIR



NULLING PROPERTY of H(z)

- When H(z)=0 on the unit circle.
  - Find inputs x[n] that give zero output

$$H(z) = 1 - 2z^{-1} + 2z^{-2} - z^{-3}$$

$$H(e^{j\hat{\omega}}) = 1 - 2e^{-j\hat{\omega}} + 2e^{-j2\hat{\omega}} - e^{-j3\hat{\omega}}$$

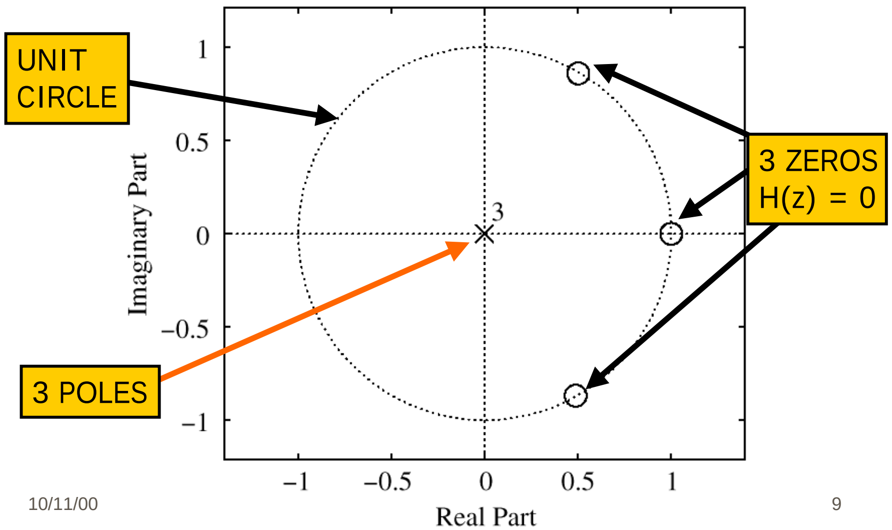
x[n] → $H(z)$ → y[n]

$$H(e^{j\pi/3}) = ?$$

$$x[n] = e^{j(\pi/3)n}$$

$$y[n] = H(e^{j(\pi/3)}) \cdot e^{j(\pi/3)n}$$

PLOT ZEROS in z-DOMAIN



NULLING PROPERTY of H(z)

- Evaluate H(z) at the input "frequency"

$$H(e^{j\hat{\omega}}) = 1 - 2e^{-j\hat{\omega}} + 2e^{-j2\hat{\omega}} - e^{-j3\hat{\omega}}$$

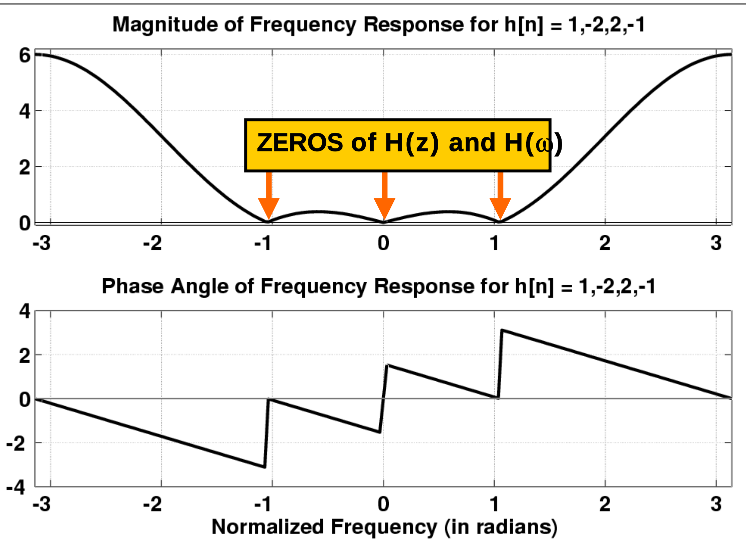
$$y[n] = H(e^{j\pi/3}) \cdot e^{j(\pi/3)n}$$

$$y[n] = (1 - 2e^{-j\pi/3} + 2e^{-j2\pi/3} - e^{-j3\pi/3}) \cdot e^{j(\pi/3)n}$$

$$(1 - 2(\tfrac{1}{2} - j\tfrac{\sqrt{3}}{2}) + 2(-\tfrac{1}{2} - j\tfrac{\sqrt{3}}{2}) + 1)$$

$$y[n] = (1 - 1 + j\sqrt{3} - 1 - j\sqrt{3} + 1) \cdot e^{j(\pi/3)n} = 0$$

FIR Frequency Response



NULLING FILTER

- PLACE ZEROS to make y[n] = 0

$$H(z) = 1 - 2z^{-1} + 2z^{-2} - z^{-3}$$ ← 3 ZEROS H(z) = 0

the output resulting from each of the following three signals will be zero:

$H(z_1) = 0$ $\quad x_1[n] = (z_1)^n = 1$ → $y_1[n] = 0$

$H(z_2) = 0$ $\quad x_2[n] = (z_2)^n = e^{j\pi n/3}$ $\quad y_2[n] = 0$

$H(z_3) = 0$ $\quad x_3[n] = (z_3)^n = e^{-j\pi n/3}$ $\quad y_3[n] = 0$

L-pt RUNNING SUM H(z)

$$H(z)=\sum_{k=0}^{L-1}\tfrac{1}{L}z^{-k}=\frac{1-z^{-L}}{L(1-z^{-1})}=\frac{z^{L}-1}{Lz^{L-1}(z-1)}$$

$$z^{L}-1=0 \quad\Rightarrow\quad z^{L}=1=e^{j2\pi k}$$

$$z=e^{j(2\pi/L)k} \quad \text{for } k=1,2,\ldots L\text{-}1$$

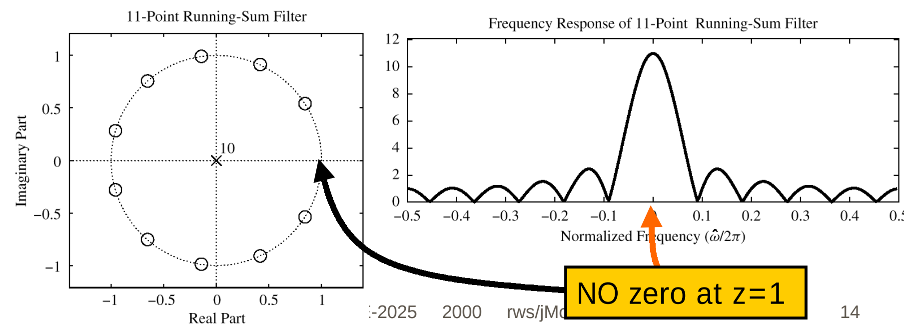
ZEROS on UNIT CIRCLE

(z-1) in denominator cancels k=0 term

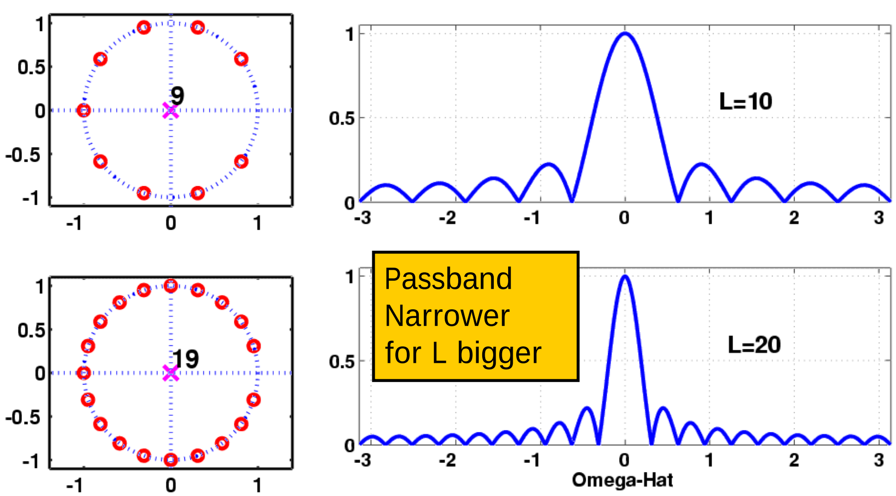
11-pt RUNNING SUM H(z)

$$H(z)=\sum_{k=0}^{10} z^{-k}$$

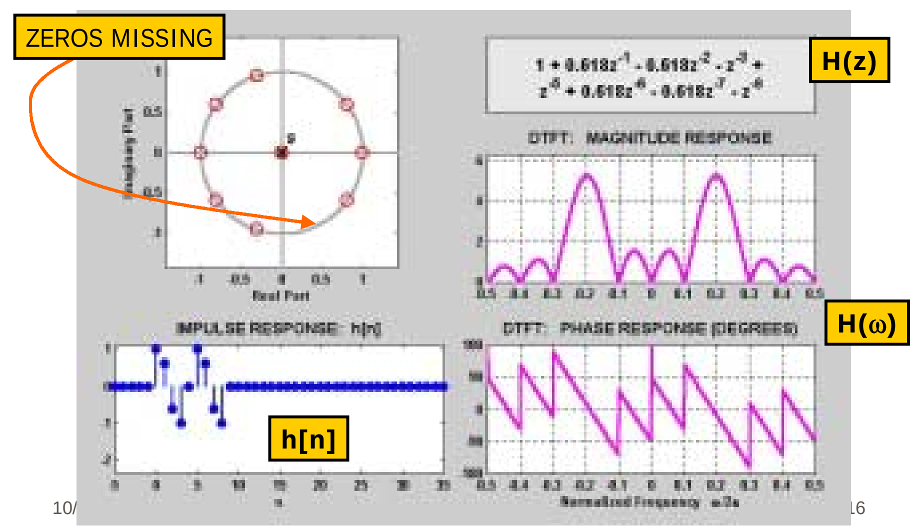
$$H(z)=(1-e^{j2\pi/11}z^{-1})(1-e^{j4\pi/11}z^{-1})\cdots(1-e^{j20\pi/11}z^{-1})$$



FILTER DESIGN: CHANGE L



3 DOMAINS MOVIE: FIR



Pop Quiz: Sinusoidal Response

- Given:

$$H(\hat{\omega}) = e^{-j\hat{\omega}} \cos(\hat{\omega})$$

- Find y[n] when

$$x[n] = \cos(0.25\pi n)$$

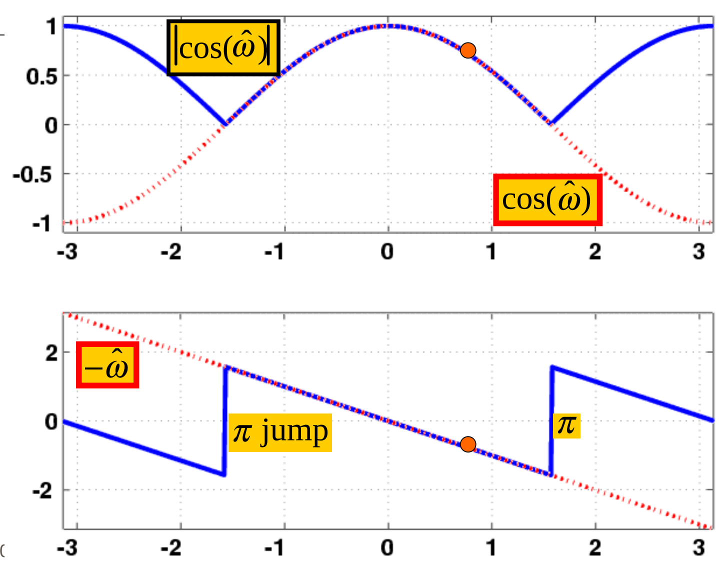
PLOT MAG & PHASE

- Given: $H(\hat{\omega}) = e^{-j\hat{\omega}} \cos(\hat{\omega})$
- Derive Magnitude and Phase

$$|H(\hat{\omega})| = |e^{-j\hat{\omega}}||\cos(\hat{\omega})| = |\cos(\hat{\omega})|$$

$$\angle H(\hat{\omega}) = \begin{cases} -\hat{\omega} & \cos(\hat{\omega}) \geq 0 \\ -\hat{\omega} + \pi & \text{if} \cos(\hat{\omega}) < 0 \end{cases}$$

Ans: FREQ RESPONSE



POP QUIZ: OUTPUT y[n]

- Find y[n] when $x[n] = \cos(0.25\pi n)$

$$y[n] = |H| \cos(0.25\pi n + \angle H)$$

$$= 0.707 \cos(0.25\pi n - \tfrac{\pi}{4})$$

$$H(\hat{\omega}) = e^{-j\hat{\omega}} \cos(\hat{\omega}) \quad \text{at } \hat{\omega} = \tfrac{\pi}{4}$$

$$H(\tfrac{\pi}{4}) = e^{-j\pi/4} \cos(\tfrac{\pi}{4}) = 0.707 e^{-j\pi/4}$$