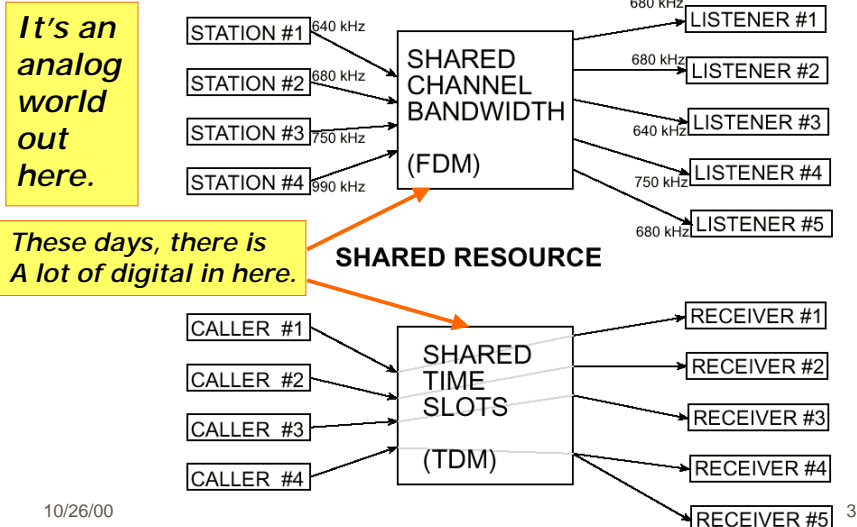


Lecture 16  
 Continuous-Time Signals  
 and Systems  
 27-Oct-00

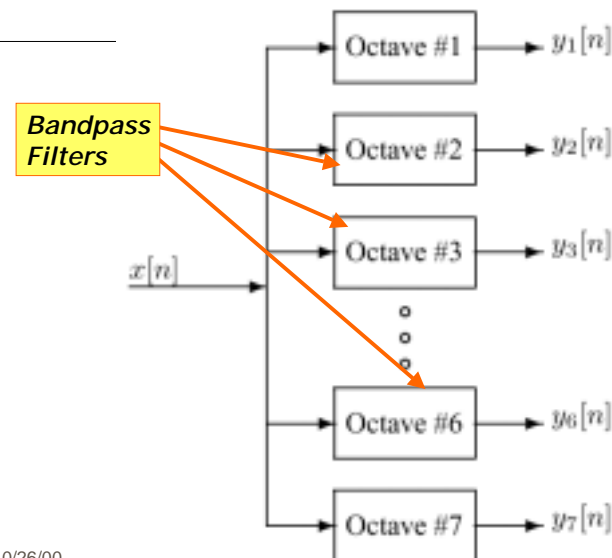
Info: Web-CT, Lab, HW

- Quiz #2: resolve grades by **7-Nov**
  - Average = 75, Median = 78
  - Quiz #3 will be 20-Nov (Monday)
- Get NEW CHAPTERS
  - PDF or Bookstore
- Prob Set #8 is due Next week
- Lab #8 on Bandpass Filtering
  - Due next week

The way communication systems work



Octave Filter Bank



## Lecture 16

### Continuous-Time Signals and Systems

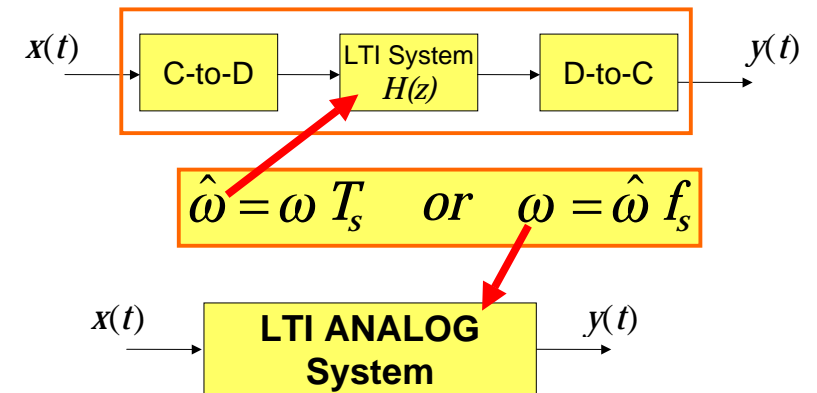
## READING ASSIGNMENTS

- This Lecture:
  - Chapter 10, pp. 1000-1022
- Other Reading:
  - Recitation: Ch. 10, pp. 1022-1029
  - Next Lecture: Chapter 10, all

## LECTURE OBJECTIVES

- Bye bye to D-T Systems for a while
- The UNIT IMPULSE signal
  - Definition
  - Properties
- Continuous-time signals and systems
  - Example systems
  - Review: **L**inearity and **T**ime-**I**nvariance
  - Convolution integral: **impulse** response

## D-T Filtering of C-T Signals



# ANALOG SIGNALS $x(t)$

## INFINITE LENGTH

- SINUSOIDS:  $(t = \text{time in secs})$
- PERIODIC SIGNALS

## ONE-SIDED, e.g., for $t > 0$

- UNIT STEP:  $u(t)$

## FINITE LENGTH

- SQUARE PULSE

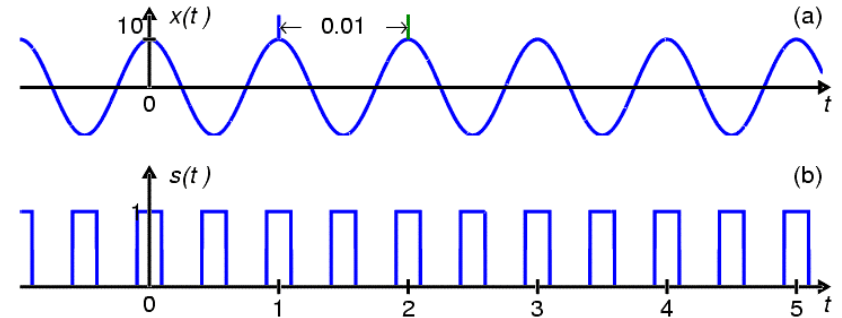
## IMPULSE SIGNAL: $\delta(t)$

## DISCRETE-TIME: $x[n]$ is list of numbers

# CT Signals: PERIODIC

$$x(t) = 10\cos(200\pi t)$$

Sinusoidal signal



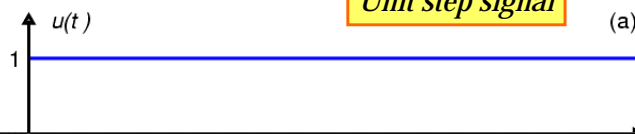
INFINITE DURATION

Square Wave

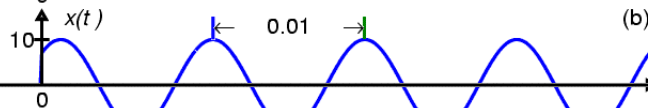
# CT Signals: ONE-SIDED

$$u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$$

Unit step signal

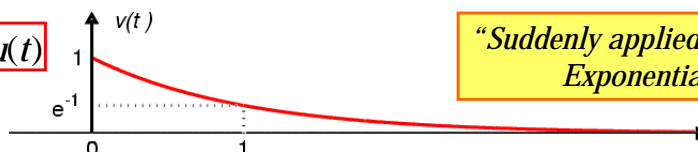


One-Sided Sinusoid



$$v(t) = e^{-t}u(t)$$

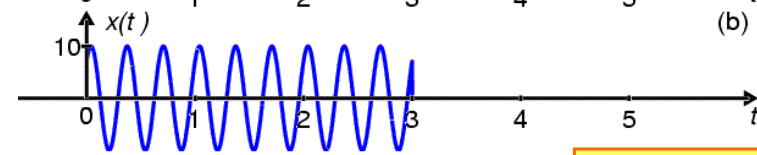
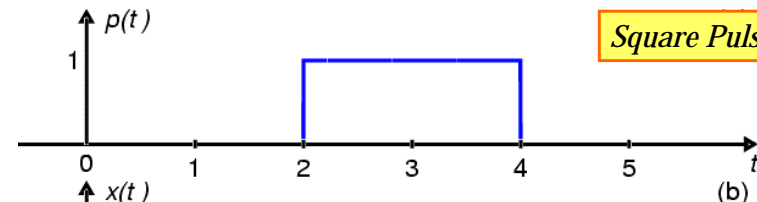
"Suddenly applied" Exponential



# CT Signals: FINITE LENGTH

$$p(t) = u(t-2) - u(t-4)$$

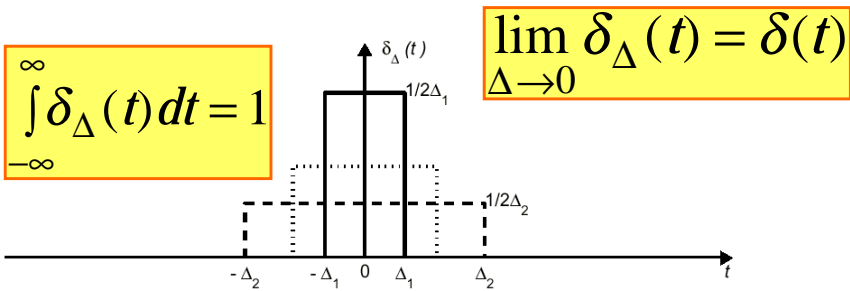
Square Pulse signal



Sinusoid multiplied by a square pulse

# What is an Impulse?

- A signal that is concentrated at one point.



# Defining the Impulse

- Assume the properties apply to the limit:

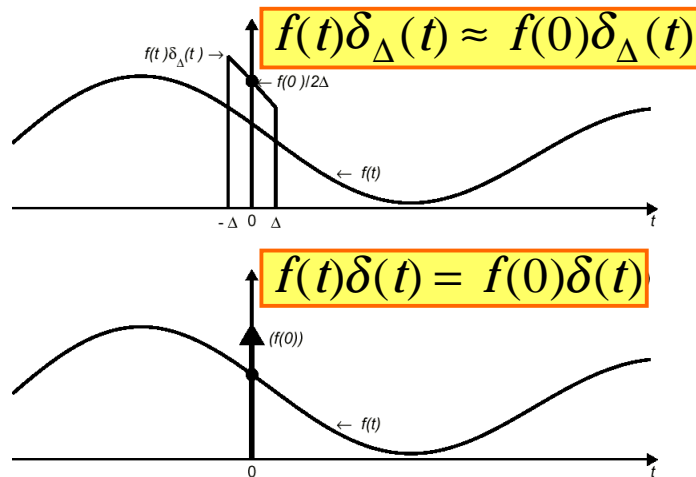
$$\lim_{\Delta \rightarrow 0} \delta_{\Delta}(t) = \delta(t)$$

- One "INTUITIVE" definition is:

$$\delta(t) = 0, \quad t \neq 0 \quad \text{Concentrated at } t=0$$

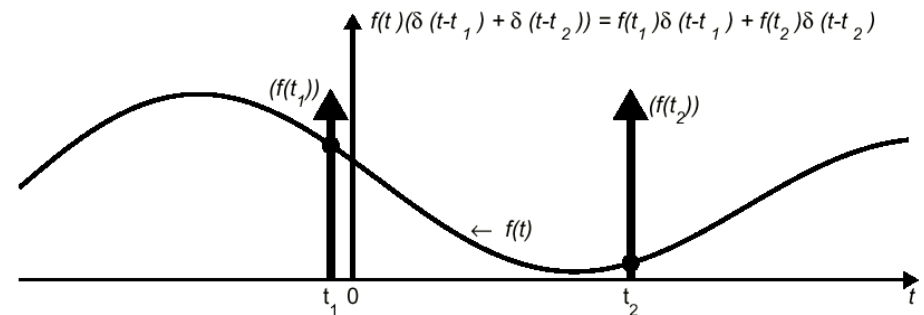
$$\int_{-\infty}^{\infty} \delta(\tau) d\tau = 1 \quad \text{Unit area}$$

# Sampling Property



# General Sampling Property

$$f(t)\delta(t-t_0) = f(t_0)\delta(t-t_0)$$



## Properties of the Impulse

$$\delta(t - t_0) = 0, \quad t \neq t_0 \quad \text{Concentrated at one time}$$

$$\int_{-\infty}^{\infty} \delta(t - t_0) dt = 1 \quad \text{Unit area}$$

$$f(t)\delta(t - t_0) = f(t_0)\delta(t - t_0) \quad \text{Sampling Property}$$

$$\int_{-\infty}^{\infty} f(t)\delta(t - t_0) dt = f(t_0) \quad \text{Extract one value of } f(t)$$

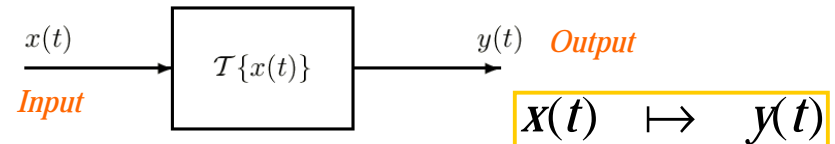
$$\frac{du(t)}{dt} = \delta(t) \quad \text{Derivative of unit step}$$

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## Continuous-Time Systems



### Examples:

■ Delay  $y(t) = x(t - t_d)$

■ Modulator  $y(t) = [A + x(t)] \cos \omega_c t$

■ Integrator  $y(t) = \int_{-\infty}^t x(\tau) d\tau$

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## CT BUILDING BLOCKS

- INTEGRATOR (CIRCUITS)
- DIFFERENTIATOR
- DELAY by  $t_0$
- MODULATOR (e.g., AM Radio)
- MULTIPLIER & ADDER

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## Ideal Delay:

- Mathematical Definition:

$$y(t) = x(t - t_d)$$

- To find the IMPULSE RESPONSE,  $h(t)$ , let  $x(t)$  be an impulse, so

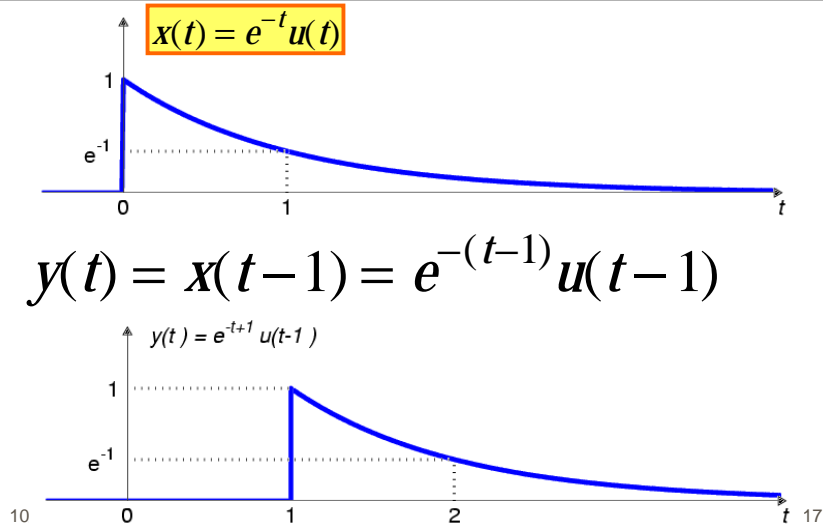
$$h(t) = \delta(t - t_d)$$

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## Output of Ideal Delay of 1 sec



## Integrator:

- Mathematical Definition:

$$y(t) = \int_{-\infty}^t x(\tau) d\tau \quad \text{Running Integral}$$

- To find the IMPULSE RESPONSE,  $h(t)$ , let  $x(t)$  be an impulse, so

$$h(t) = \int_{-\infty}^t \delta(\tau) d\tau = u(t)$$

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## Integrator:

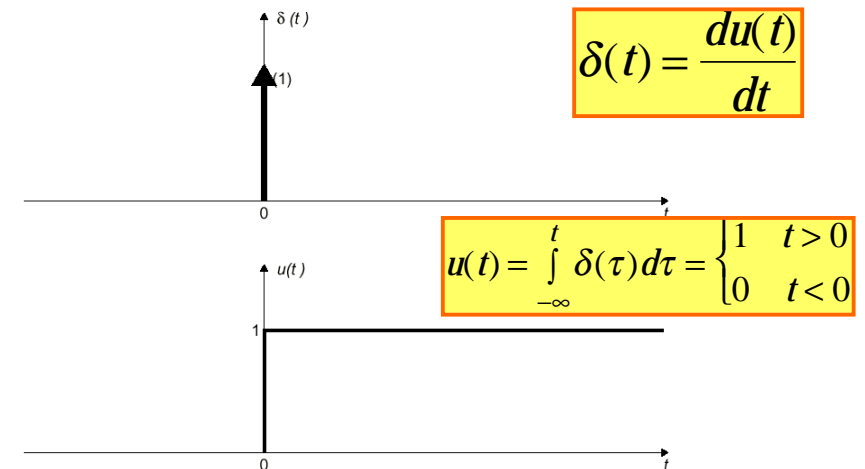
$$y(t) = \int_{-\infty}^t x(\tau) d\tau$$

- Integrate the impulse

$$\int_{-\infty}^t \delta(\tau) d\tau = u(t)$$

- IF  $t < 0$ , we get zero
- IF  $t > 0$ , we get one
  - Thus we have  $h(t) = u(t)$  for the integrator

## Graphical Representation



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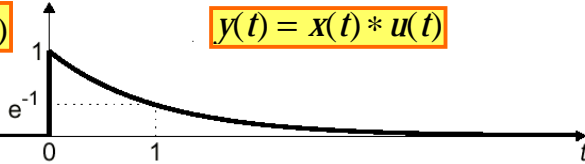
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## Output of Integrator

$$x(t) = e^{-t}u(t)$$



$$y(t) = x(t) * u(t)$$

$$y(t) = \int_{-\infty}^t e^{-\tau} u(\tau) d\tau = \begin{cases} 0 & t < 0 \\ \int_0^t e^{-\tau} d\tau & t \geq 0 \end{cases}$$

$$= (1 - e^{-t})u(t)$$

## Differentiator:

- Mathematical Definition:

$$y(t) = \frac{dx(t)}{dt}$$

- To find  $h(t)$ , let  $x(t)$  be an impulse, so

$$h(t) = \frac{d\delta(t)}{dt} = \delta^{(1)}(t) \quad \text{Doublet}$$

## Differentiator Output: $y(t) = \frac{dx(t)}{dt}$

$$x(t) = e^{-t}u(t)$$



$$y(t) = \frac{d}{dt}(e^{-t}u(t)) = \frac{d}{dt}(e^{-t})u(t) + e^{-t} \frac{d}{dt}(u(t))$$

$$= -e^{-t}u(t) + e^{-t}\delta(t)$$

$$= -e^{-t}u(t) + e^{-0}\delta(t) = -e^{-t}u(t) + \delta(t)$$

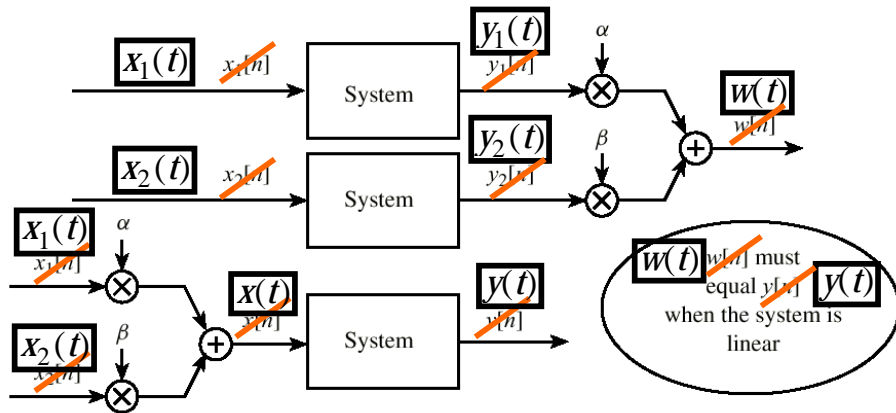
## Linear and Time-Invariant (LTI) Systems

- If a continuous-time system is both linear and time-invariant, then the output  $y(t)$  is related to the input  $x(t)$  by a **convolution integral**

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau = x(t) * h(t)$$

where  $h(t)$  is the **impulse response** of the system.

## Testing for Linearity

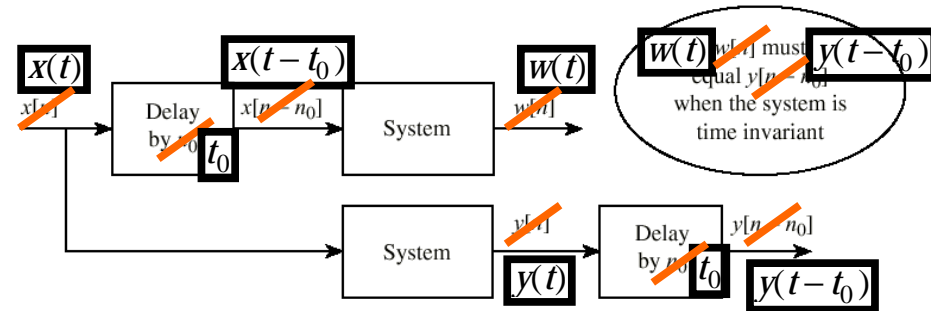


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## Testing Time-Invariance



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Integrator: 
$$y(t) = \int_{-\infty}^t x(\tau) d\tau$$

- Linear

$$\int_{-\infty}^t [ax_1(\tau) + bx_2(\tau)] d\tau = ay_1(t) + by_2(t)$$

- And Time-Invariant

$$w(t) = \int_{-\infty}^t x(\tau - t_0) d\tau \quad \text{let } \sigma = \tau - t_0$$

$$\Rightarrow w(t) = \int_{-\infty}^{t-t_0} x(\sigma) d\sigma = y(t-t_0)$$

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Modulator: 
$$y(t) = [A + x(t)] \cos \omega_c t$$

- Not** linear--obvious because

$$[A + ax_1(t) + bx_2(t)] \neq$$

$$[A + ax_1(t)] + [A + bx_2(t)]$$

- Not** time-invariant

$$w(t) = [A + x(t-t_0)] \cos \omega_c t \neq y(t-t_0)$$

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