

EE-2025

Fall-2000

Lecture 18

Frequency Response of Continuous-Time Systems

3-Nov-00

Info: Web-CT, Lab, HW

Calendar:

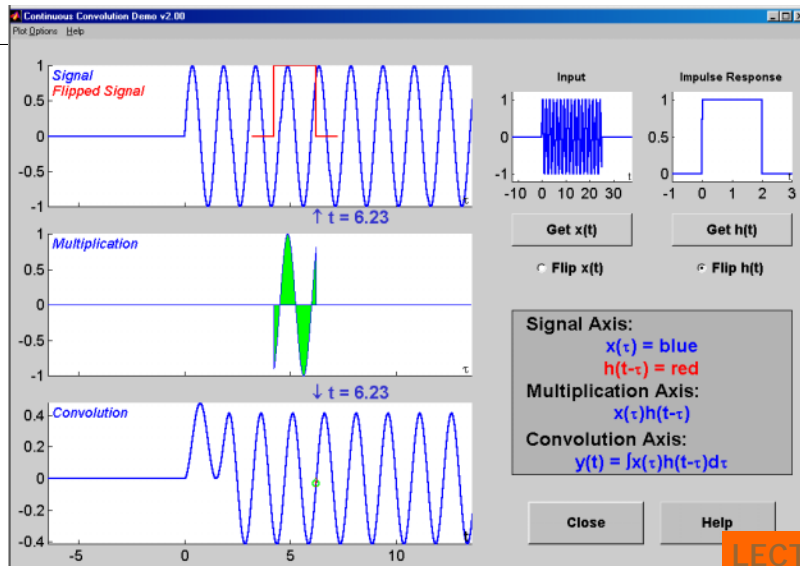
- Quiz #3 is 20-Nov
- CHECK YOUR GRADES !!!
 - Web-CT is the OFFICIAL gradebook
- Prob Set #9 is due next week
- Lab Quiz next week (7-Nov thru 13-Nov)
 - Lab #9 is due next week

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Convolution GUI: Sinusoid



Lecture 18

Frequency Response of
Continuous-Time Systems

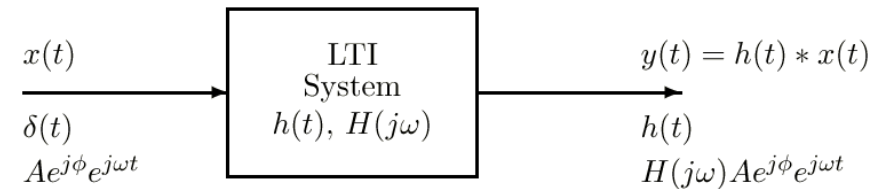
READING ASSIGNMENTS

- This Lecture:
 - Chapter 11, pp. 1100-1118 (all)
 - In NEW Chapters
- Other Reading:
 - Next Lecture & Recitation:
 - Chapter 12, 1200-1230

LECTURE OBJECTIVES

- Review of convolution
 - **THE** operation for **LTI** Systems
- Complex exponential input signals
 - Frequency Response
 - Cosine signals
 - Real part of complex exponential
- Fourier Series thru $H(j\omega)$
 - These are Analog Filters

LTI Systems

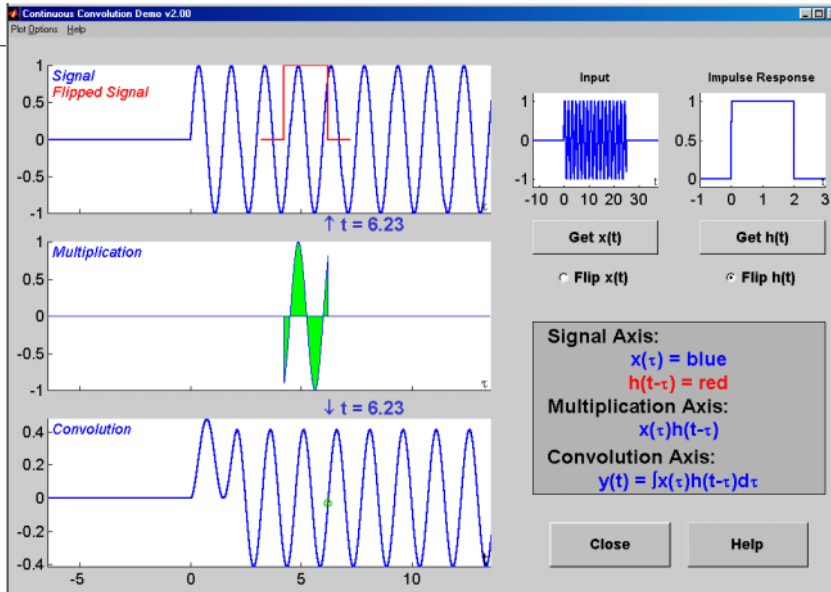


- Convolution defines LTI system

$$y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau$$

- Response to a complex exponential gives frequency response $H(j\omega)$

Convolution GUI: Sinusoid



Thought Process #1

- SUPERPOSITION (Linearity)
 - ┆ Make $x(t)$ a weighted sum of signals
 - ┆ Then $y(t)$ is also a sum—different weights
 - DIFFERENT OUTPUT SIGNALS usually
- Use SINUSOIDS
 - ┆ Make $x(t)$ a weighted sum of sinusoids
 - ┆ Then $y(t)$ is also a sum of sinusoids
 - ┆ Different Magnitudes and Phase
- LTI SYSTEMS: Sinusoidal Response

Thought Process #2

- SUPERPOSITION (Linearity)
 - ┆ Make $x(t)$ a weighted sum of signals
- Use SINUSOIDS
 - ┆ Any $x(t)$ = weighted sum of sinusoids
 - ┆ HOW? Use FOURIER ANALYSIS INTEGRAL
 - ┆ To find the weights from $x(t)$
- LTI SYSTEMS:
 - ┆ Frequency Response changes each sinusoidal component

Complex Exponential Input

$$x(t) = Ae^{j\phi} e^{j\omega t} \mapsto y(t) = \boxed{H(j\omega)} Ae^{j\phi} e^{j\omega t}$$

$$y(t) = \int_{-\infty}^{\infty} h(\tau) Ae^{j\phi} e^{j\omega(t-\tau)} d\tau$$

$$y(t) = \left(\int_{-\infty}^{\infty} h(\tau) e^{-j\omega\tau} d\tau \right) Ae^{j\phi} e^{j\omega t}$$

$$H(j\omega) = \int_{-\infty}^{\infty} h(\tau) e^{-j\omega\tau} d\tau$$

↑

Frequency Response

When does $H(j\omega)$ Exist?

- When is $|H(j\omega)| < \infty$?

$$|H(j\omega)| = \left| \int_{-\infty}^{\infty} h(\tau) e^{-j\omega\tau} d\tau \right| \leq \int_{-\infty}^{\infty} |h(\tau)| |e^{-j\omega\tau}| d\tau$$

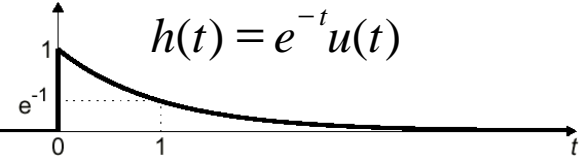
$$|H(j\omega)| \leq \int_{-\infty}^{\infty} |h(\tau)| d\tau < \infty$$

- Thus the frequency system is a **stable** system.

$$h(t) = e^{-at} u(t) \Leftrightarrow H(j\omega) = \frac{1}{a + j\omega}$$

- Suppose that $h(t)$ is:

$$a = 1$$

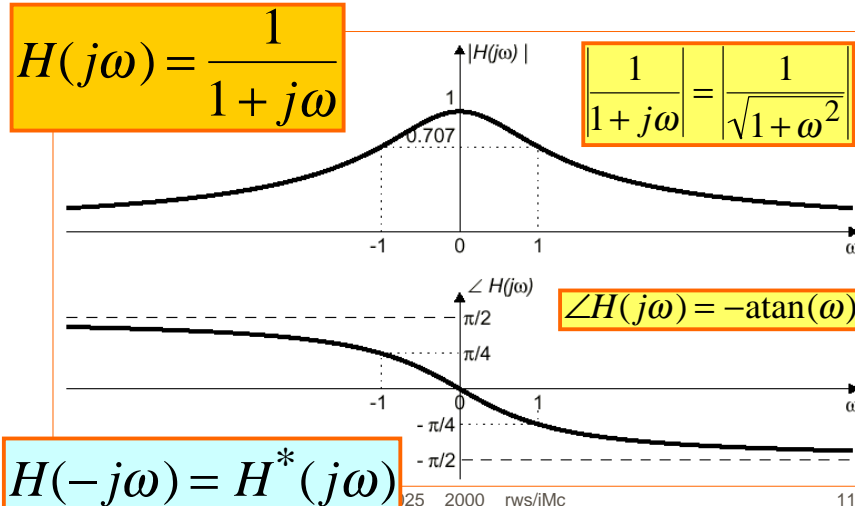


$$H(j\omega) = \int_{-\infty}^{\infty} e^{-a\tau} u(\tau) e^{-j\omega\tau} d\tau = \int_0^{\infty} e^{-(a+j\omega)\tau} d\tau$$

$$a > 0$$

$$H(j\omega) = \frac{e^{-(a+j\omega)\tau}}{-(a+j\omega)} \Big|_0^{\infty} = \frac{e^{-a\tau} e^{-j\omega\tau}}{-(a+j\omega)} \Big|_0^{\infty} = \frac{1}{a+j\omega}$$

Magnitude and Phase Plots



Freq Response of Integrator?

- Impulse Response
 - $h(t) = u(t)$
- NOT a Stable System
 - Frequency response $H(j\omega)$ does NOT exist
- Leaky Integrator (a is small)
 - Cannot build a perfect Integral $a \rightarrow 0$

$$h(t) = e^{-at} u(t) \Leftrightarrow H(j\omega) = \frac{1}{a + j\omega} \rightarrow \frac{1}{j\omega} ?$$

Ideal Delay: $y(t) = x(t - t_d)$

$$H(j\omega) = \int_{-\infty}^{\infty} \delta(\tau - t_d) e^{-j\omega\tau} d\tau = e^{-j\omega t_d}$$

$$H(j\omega) = e^{-j\omega t_d}$$

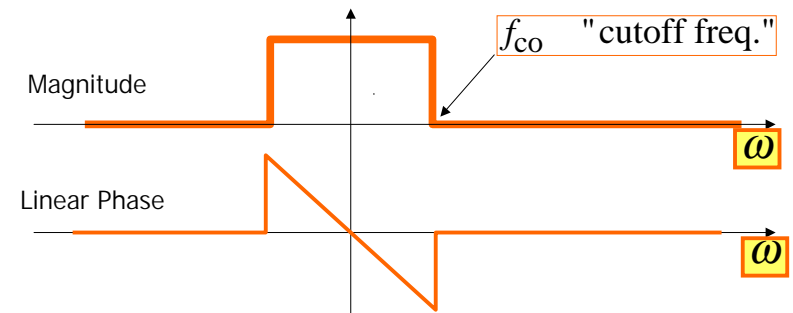
$$x(t) = e^{j\omega t} \mapsto H(j\omega)$$

$$y(t) = e^{j\omega(t-t_d)} = e^{-j\omega t_d} e^{j\omega t}$$

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Ideal Lowpass Filter

$$H_{LP}(j\omega) = \begin{cases} e^{-j\omega t_d} & |\omega| < \omega_{co} \\ 0 & |\omega| > \omega_{co} \end{cases}$$



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Example: Ideal Low Pass

$$H_{LP}(j\omega) = \begin{cases} e^{-j3\omega} & |\omega| < 2 \\ 0 & |\omega| > 2 \end{cases}$$

$$x(t) = 10e^{j\pi/3} e^{j5t} \mapsto y(t) = H(j5)10e^{j\pi/3} e^{j5t}$$

$$y(t) = (e^{-j15}) 10e^{j\pi/3} e^{j5t} = 10e^{j\pi/3} e^{j5(t-3)}$$

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Cosine Input

$$x(t) = A \cos(\omega_0 t + \phi) = \frac{A}{2} e^{j\phi} e^{j\omega_0 t} + \frac{A}{2} e^{-j\phi} e^{-j\omega_0 t}$$

$$y(t) = H(j\omega_0) \frac{A}{2} e^{j\phi} e^{j\omega_0 t} + H(-j\omega_0) \frac{A}{2} e^{-j\phi} e^{-j\omega_0 t}$$

Since $H(-j\omega_0) = H^*(j\omega_0)$

$$y(t) = A |H(j\omega_0)| \cos(\omega_0 t + \phi + \angle H(j\omega_0))$$

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Review Fourier Series

ANALYSIS

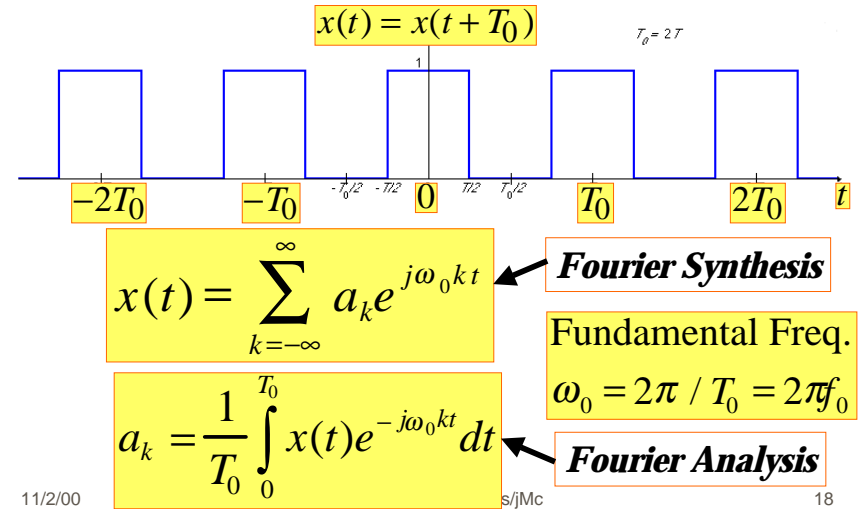
- Get representation from the signal
- Works for PERIODIC Signals

Fourier Series

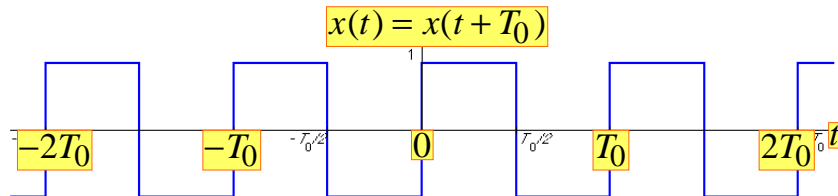
- INTEGRAL over one period

$$a_k = \frac{1}{T_0} \int_0^{T_0} x(t) e^{-j\omega_0 kt} dt$$

General Periodic Signals



Square Wave Signal

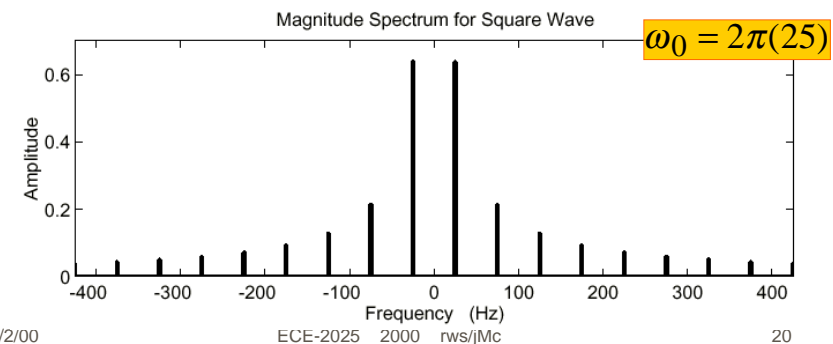


$$a_k = \frac{1}{T_0} \int_0^{T_0/2} (1) e^{-j\omega_0 kt} dt + \frac{1}{T_0} \int_{T_0/2}^{T_0} (-1) e^{-j\omega_0 kt} dt$$

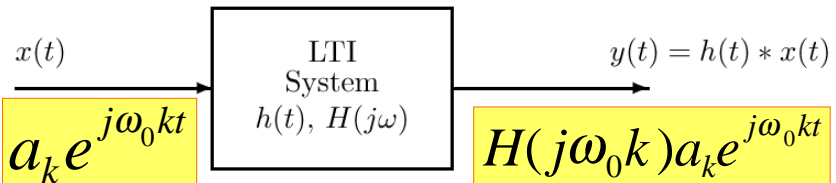
$$a_k = \frac{e^{-j\omega_0 kt} \Big|_0^{T_0/2}}{-j\omega_0 k T_0} - \frac{e^{-j\omega_0 kt} \Big|_{T_0/2}^{T_0}}{-j\omega_0 k T_0} = \frac{1 - e^{-j\pi k}}{j\pi k}$$

Spectrum from Fourier Series

$$a_k = \frac{1 - e^{-j\pi k}}{j\pi k} = \begin{cases} \frac{2}{j\pi k} & k = \pm 1, \pm 3, \dots \\ 0 & k = 0, \pm 2, \pm 4, \dots \end{cases}$$



LTI Systems with Periodic Inputs



By superposition,

Output has *same* frequencies

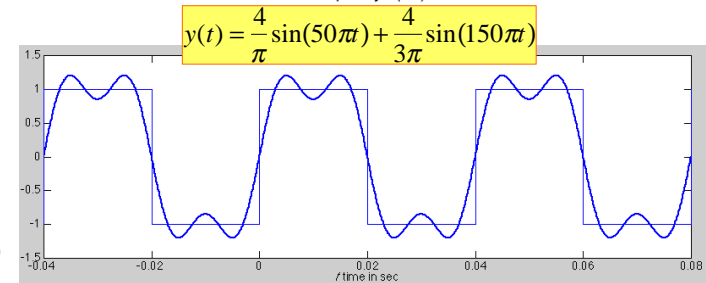
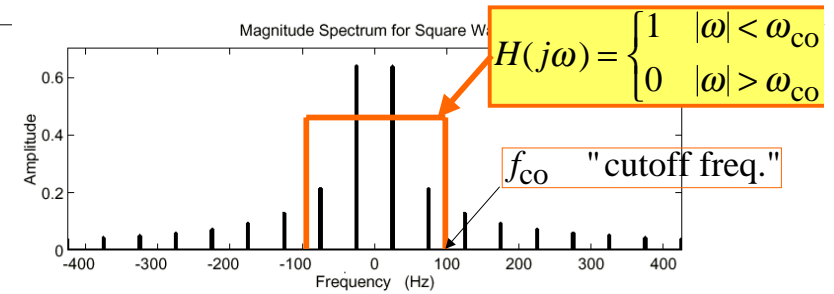
$$y(t) = \sum_{k=-\infty}^{\infty} a_k H(j\omega_0 k) e^{j\omega_0 k t} = \sum_{k=-\infty}^{\infty} b_k e^{j\omega_0 k t}$$

$$b_k = a_k H(j\omega_0 k)$$

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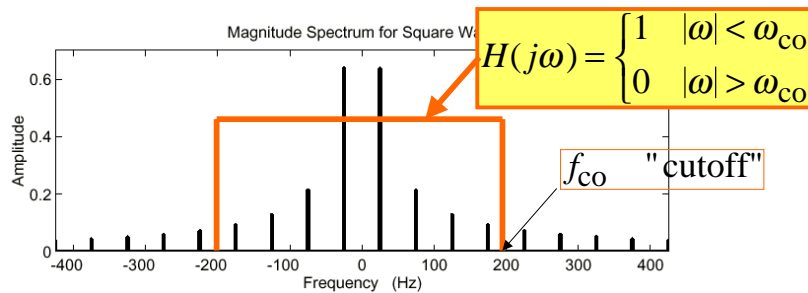
Ideal Lowpass Filter



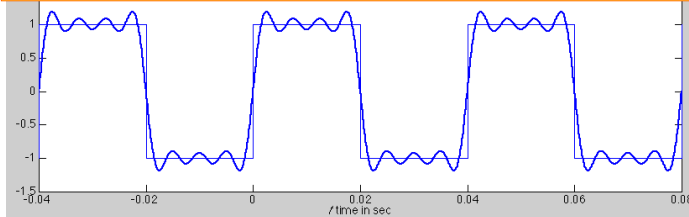
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Ideal Lowpass Filter



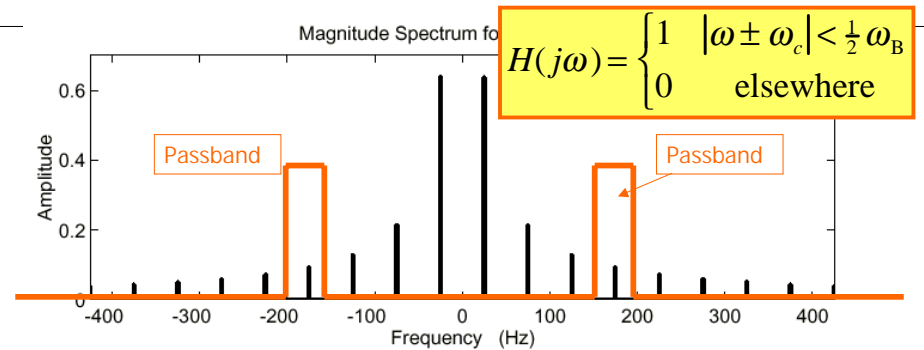
$$y(t) = \frac{4}{\pi} \sin(50\pi t) + \frac{4}{3\pi} \sin(150\pi t) + \frac{4}{5\pi} \sin(250\pi t) + \frac{4}{7\pi} \sin(350\pi t)$$



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Ideal Bandpass Filter



What is the output signal ?

$$y(t) = \frac{2}{j7\pi} e^{j2\pi(175)t} - \frac{2}{j7\pi} e^{-j2\pi(175)t} = \frac{4}{7\pi} \cos(2\pi(175)t - \frac{1}{2}\pi)$$

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