EE-2025 Fall-2000

Lecture 21
Amplitude Modulation (AM)
13-Nov-00

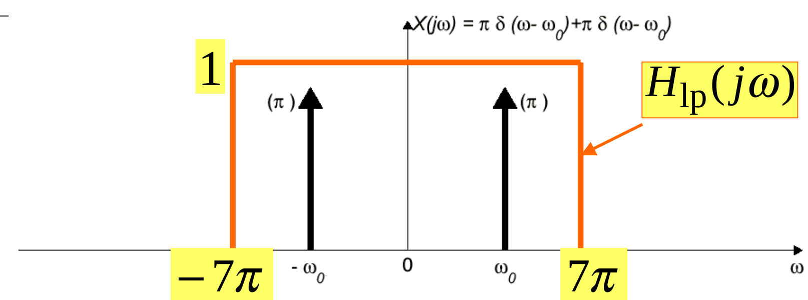
Info: Web-CT, Lab, HW

- Calendar:
  - Quiz #3 is 20-Nov
  - One page hand-written notes
  - Calculator
  - Covers z-Transform, Impulses, Convolution and FT.
- Quiz Review: Sunday (19-Nov) @ 7 pm
- Prob Set #10 is due this week

Info: Final Exam, etc.

- Finals Week:
  - 11am Lecture: exam on Mon, 11-Dec @ 8am
  - Noon Lecture: Wed, 13-Dec @ 11:30am
- Last Day for turning in Lab Reports:
  - Friday, 8-Dec at 5pm.

Pop Quiz



$$x(t) = \cos(3\pi t)$$

$$\omega_0 = ?$$
$$y(t) = ???$$

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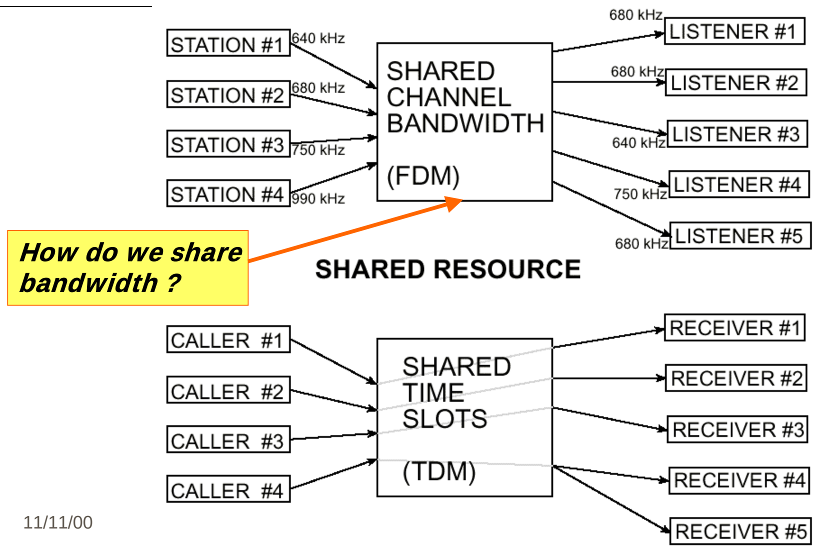
Lecture 21
Amplitude Modulation (AM)

LECTURE OBJECTIVES

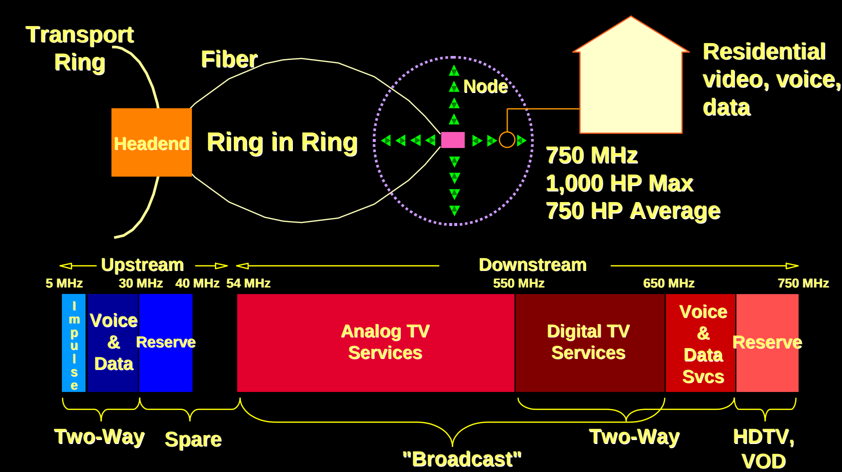
- Review of FT properties
  - Convolution <--> multiplication
  - Frequency shifting
- Sinewave Amplitude Modulation
  - AM radio
- Frequency-division multiplexing
  - FDM

The way communication systems work



From a Network Perspective,,,

Cox Cable Strategy: Broadband



HFC Full Service Network in Residential Areas

Table of Easy FT Properties

Linearity Property

$$ax_1(t)+bx_2(t) \Leftrightarrow aX_1(j\omega)+bX_2(j\omega)$$

Delay Property

$$x(t-t_d) \Leftrightarrow e^{-j\omega t_d}X(j\omega)$$

Frequency Shifting

$$x(t)e^{j\omega_0 t} \Leftrightarrow X(j(\omega-\omega_0))$$

Scaling

$$x(at) \Leftrightarrow \frac{1}{|a|}X(j(\frac{\omega}{a}))$$

Table of FT Properties

$$x(t)*h(t) \Leftrightarrow H(j\omega)X(j\omega)$$

$$x(t)p(t) \Leftrightarrow \frac{1}{2\pi}X(j\omega)*P(j\omega)$$

$$x(t)e^{j\omega_0 t} \Leftrightarrow X(j(\omega-\omega_0))$$

Differentiation Property

$$\frac{dx(t)}{dt} \Leftrightarrow (j\omega)X(j\omega)$$

Cosine Input to LTI System

$$Y(j\omega)=H(j\omega)X(j\omega)$$

$$=H(j\omega)[\pi\delta(\omega-\omega_0)+\pi\delta(\omega+\omega_0)]$$

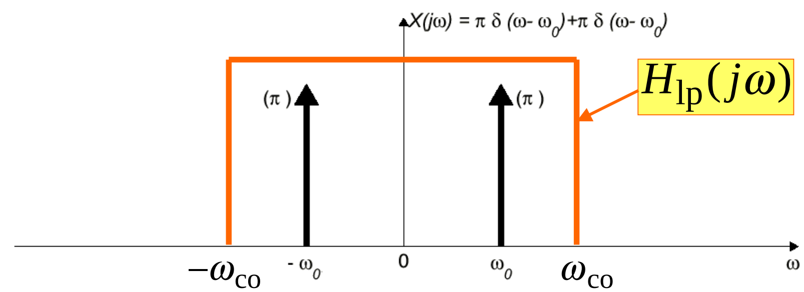
$$=H(j\omega_0)\pi\delta(\omega-\omega_0)+H(-j\omega_0)\pi\delta(\omega+\omega_0)$$

$$y(t) = H(j\omega_0)\tfrac{1}{2}e^{j\omega_0 t}+H(-j\omega_0)\tfrac{1}{2}e^{-j\omega_0 t}$$

$$= H(j\omega_0)\tfrac{1}{2}e^{j\omega_0 t}+H^*(j\omega_0)\tfrac{1}{2}e^{-j\omega_0 t}$$

$$= |H(j\omega_0)|\cos(\omega_0 t+\angle H(j\omega_0))$$

Ideal Lowpass Filter

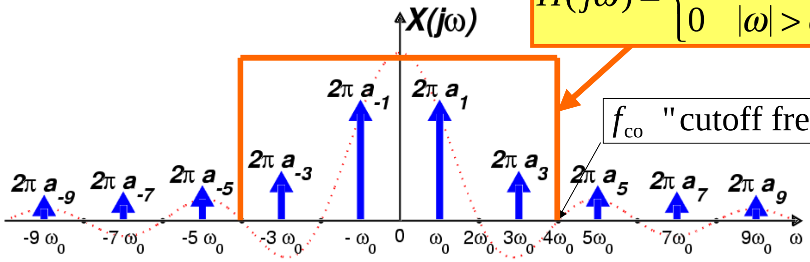


$$y(t)=x(t) \quad \text{if } \omega_0<\omega_{co}$$

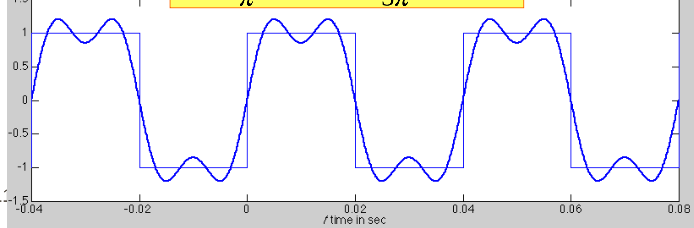
$$y(t)=0 \quad \text{if } \omega_0>\omega_{co}$$

Ideal Lowpass Filter

$$H(j\omega) = \begin{cases} 1 & |\omega| < \omega_{co} \\ 0 & |\omega| > \omega_{co} \end{cases}$$



$$y(t) = \frac{4}{\pi}\sin(50\pi t) + \frac{4}{3\pi}\sin(150\pi t)$$



Frequency Shifting Property

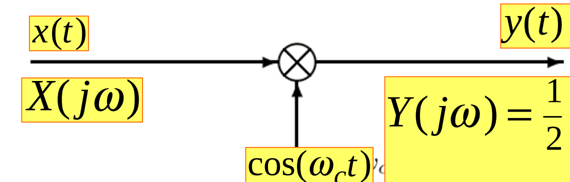
$$x(t)e^{j\omega_0 t} \Leftrightarrow X(j(\omega - \omega_0))$$

$$\int_{-\infty}^{\infty} e^{-j\omega_0 t} x(t) e^{-j\omega t} dt = \int_{-\infty}^{\infty} x(t) e^{-j(\omega-\omega_0)t} dt$$

$$= X(j(\omega - \omega_0))$$

$$y(t) = \frac{\sin 7t}{\pi t} e^{j\omega_0 t} \Leftrightarrow Y(j\omega) = \begin{cases} 1 & \omega_0 - 7 < \omega < \omega_0 + 7 \\ 0 & elsewhere \end{cases}$$

Amplitude Modulator



$$Y(j\omega) = \frac{1}{2}X(j(\omega - \omega_c)) + \frac{1}{2}X(j(\omega + \omega_c))$$

- *x(t)* modulates the amplitude of the cosine wave. The result in the frequency-domain is two shifted copies of *X(jω).*

$$y(t) = x(t)\cos(\omega_c t) \Leftrightarrow$$

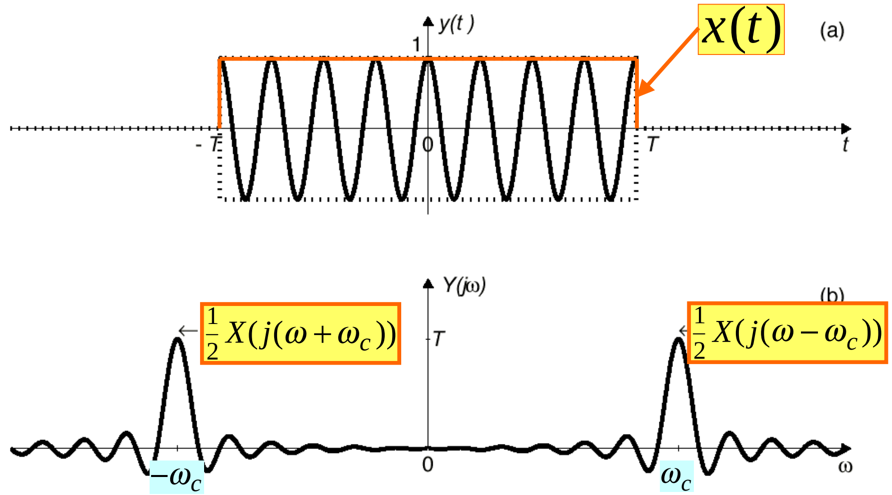
$$Y(j\omega) = \frac{1}{2}X(j(\omega - \omega_c)) + \frac{1}{2}X(j(\omega + \omega_c))$$

$$x(t) = \begin{cases} 1 & |t| < T \\ 0 & |t| > T \end{cases} \Leftrightarrow X(j\omega) = 2\frac{\sin(\omega T)}{(\omega)}$$

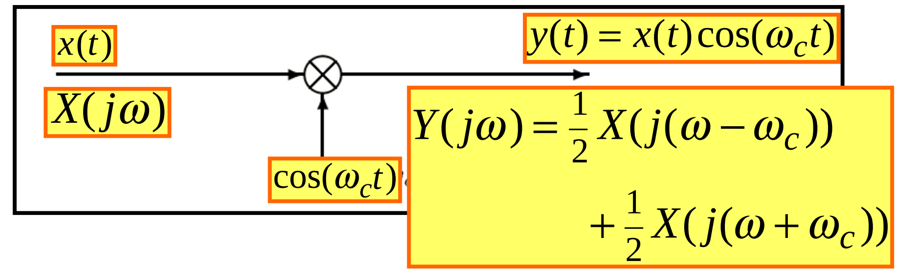
$$y(t) = x(t)\cos(\omega_c t) \Leftrightarrow$$

$$Y(j\omega) = \frac{\sin((\omega - \omega_c))}{(\omega - \omega_c)} + \frac{\sin((\omega + \omega_c))}{(\omega + \omega_c)}$$

$$y(t) = x(t)\cos(\omega_c t) \Leftrightarrow$$
$$Y(j\omega) = \frac{1}{2}X(j(\omega-\omega_c)) + \frac{1}{2}X(j(\omega+\omega_c))$$



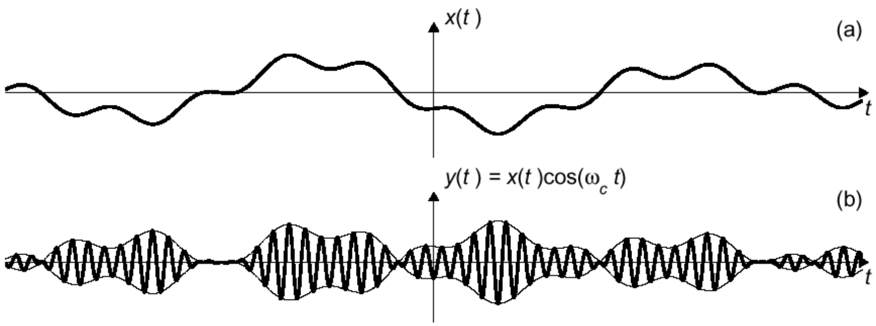
DSBAM Modulator



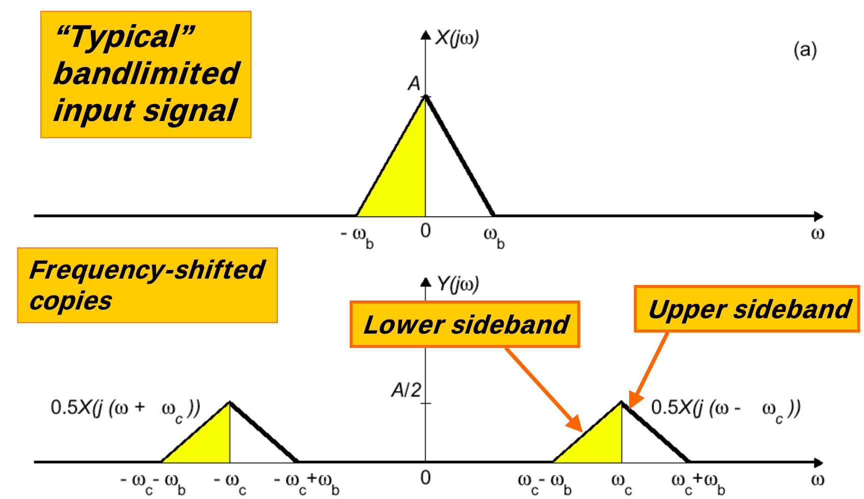
- If *X(jω)*=0 for $|\omega| > \omega_b$ and $\omega_c > \omega_b$, the result in the frequency-domain is two shifted and scaled exact copies of *X(jω).*

DSBAM Waveform

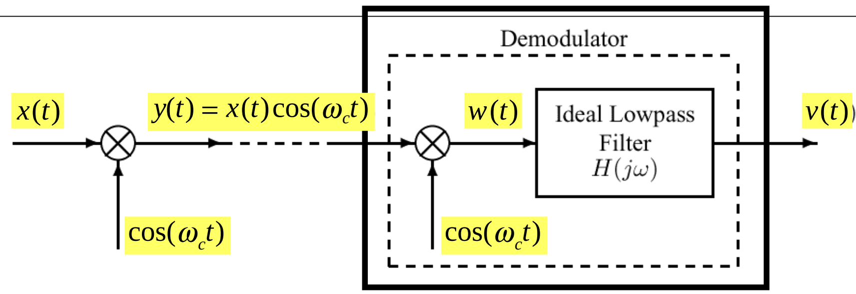
- In the time-domain, the envelope of sinewave peaks follows $|x(t)|$



Double Sideband AM (DSBAM)



DSBAM Demodulator

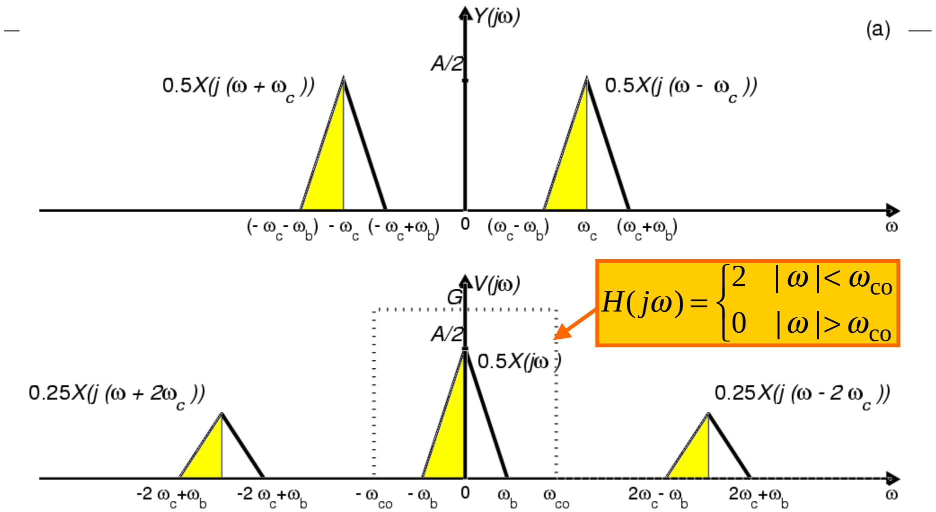


$$w(t) = x(t)[\cos(\omega_c t)]^2 = \frac{1}{2}x(t) + \frac{1}{2}x(t)\cos(2\omega_c t)$$

$$W(j\omega) = \frac{1}{2}X(j\omega) + \frac{1}{4}X(j(\omega - 2\omega_c)) + \frac{1}{4}X(j(\omega + 2\omega_c))$$

$$V(j\omega) = H(j\omega)W(j\omega)$$

DSBAM Demodulation



$$H(j\omega) = \begin{cases} 2 & |\omega| < \omega_{co} \\ 0 & |\omega| > \omega_{co} \end{cases}$$

$$V(j\omega) = H(j\omega)W(j\omega) = X(j\omega) \text{ if } \omega_b < \omega_{co} < 2\omega_c - \omega_b$$

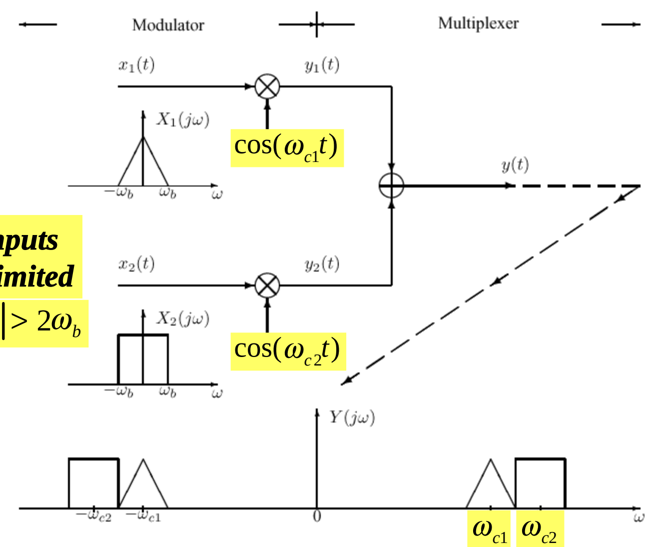
Frequency-Division Multiplexing (FDM)

- Shifting spectrum of signal to higher frequency:
  - Permits transmission of low-frequency signals with high-frequency EM waves
  - By allocating a frequency band to each signal multiple bandlimited signals can share the same channel
  - AM radio: 530-1620 kHz (10 kHz bands)
  - FM radio: 88.1-107.9 MHz (200 kHz bands)

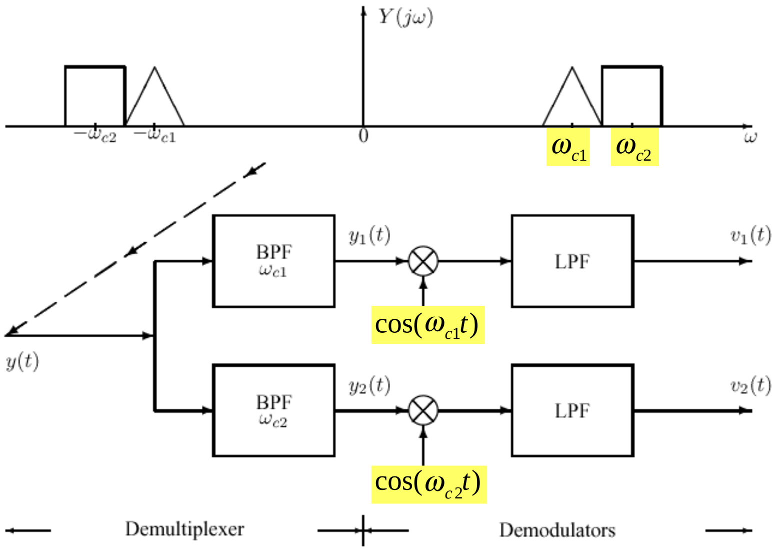
FDM Block Diagram

Spectrum of inputs must be bandlimited

Need $|\omega_{c2} - \omega_{c1}| > 2\omega_b$



Frequency-Division De-Mux



Bandpass Filters for De-Mux