EE-2025 Fall-2000

Lecture 26
Review: Potpourri
8-Dec-00

LAST LAB This Week

- ALL Lab Reports due by Friday (TODAY)
  - 5pm in VanLeer-475, or directly to TA
    - In PERSON
- Course Evaluations during last week
  - THREE
    - Two for GT: Lecture & Recitation
    - ******** One on Web-CT ***********
- HW #12 solution has been posted

Final Exam Info

- Calendar: Final Exam(s)
  - Period 1, Monday, 11-Dec @ 8 am
  - Period 8, Wed, 13-Dec @ 11:30am
- **ID check will be done at Final Exam**
- **Report CONFLICTS immediately !!!!**
  - E.g., 3 exams in one day
- Reviews will be held on Sunday & Tuesday
  - 7pm in ECE Auditorium

FINAL EXAM

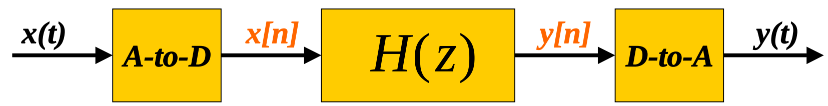
- FORMULA PAGES ?
  - Students bring **ONE** page **HAND-WRITTEN**
    - Tables 12.1 & 12.2 will be supplied with the exam.
- COVERAGE / EMPHASIS?
  - Fourier Transform
    - Sampling, Filtering & Spectrum
  - Digital Filters: IIR & FIR & H(z)
  - Hard problems from Quizzes #2, #3.
  - Homework & Old Quizzes

EE-2025 Georgia Tech

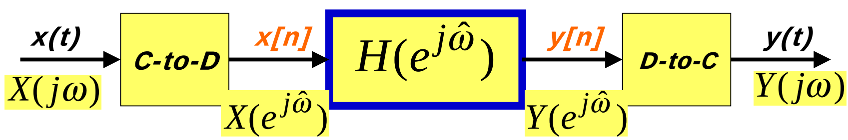
Lecture 26
Review: Digital Filtering
Frequency Response
& Sampling

LECTURE OBJECTIVES

- THREE-DOMAIN APPROACH
  - EXHIBIT BANDPASS FILTERS
- RE-UNIFICATION:
  - How does Frequency Response affect x(t) to produce y(t) ?



DT Filtering of CT Signals

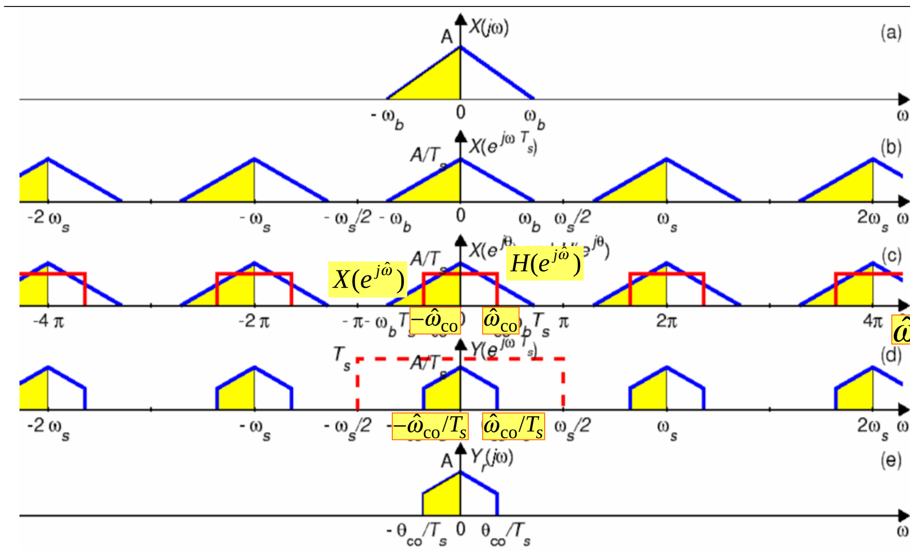


If no aliasing occurs in sampling x(t), then it follows that

$$Y(j\omega) = H_{\text{eff}}(j\omega) X(j\omega)$$

$$H_{\text{eff}}(j\omega) = \begin{cases} H(e^{j\omega T_s}) & |\omega| < \frac{1}{2}\omega_s \\ 0 & |\omega| > \frac{1}{2}\omega_s \end{cases}$$

Illustration of DT Filtering of a CT Signal



EFFECTIVE Freq. Response

- Assume NO Aliasing, then
  - ANALOG FREQ <--> DIGITAL FREQ

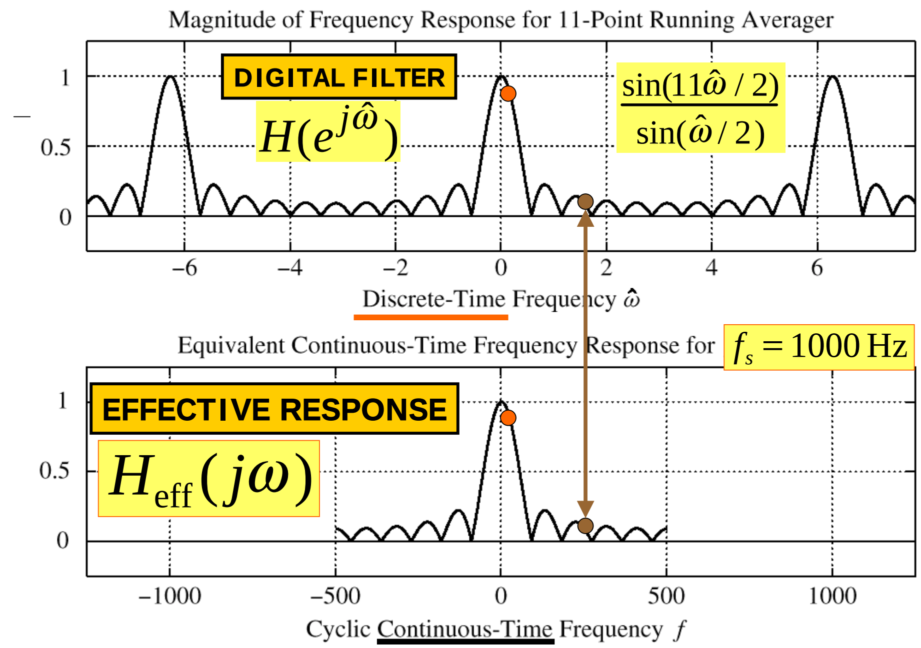
$$\hat{\omega} = \omega T_s = \frac{\omega}{f_s}$$

DIGITAL FILTER

- So, we can plot:
- Scaled Freq. Axis

$$H(e^{j\omega T_s}) \text{ vs. } \omega$$

ANALOG FREQUENCY



Universities

- Universities are centers of knowledge; freshman bring a little in; seniors take none away. Therefore, it accumulates.

THREE DOMAINS

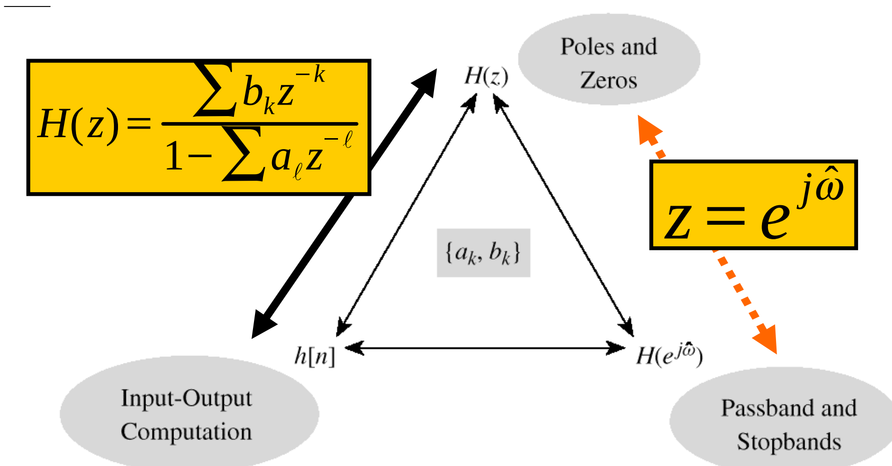
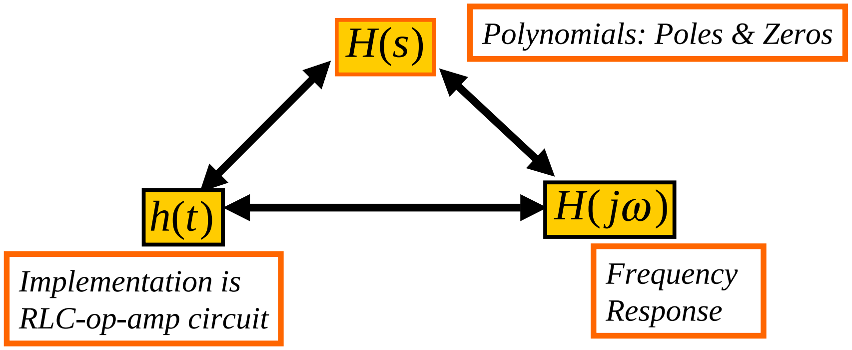


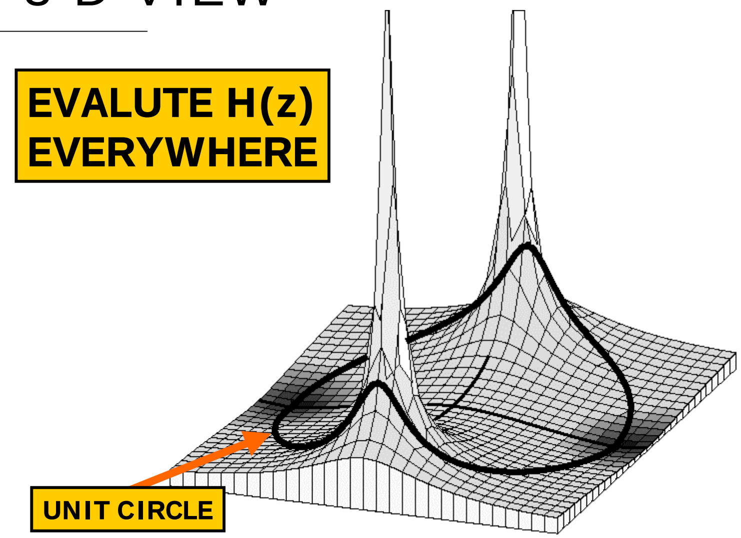
Figure 8.13 Relationship among the n-, z-, and $\hat{\omega}$-domains. The filter coefficients $\{a_k,\ b_k\}$ play a central role.

THE FUTURE

- Circuits & Laplace Transforms



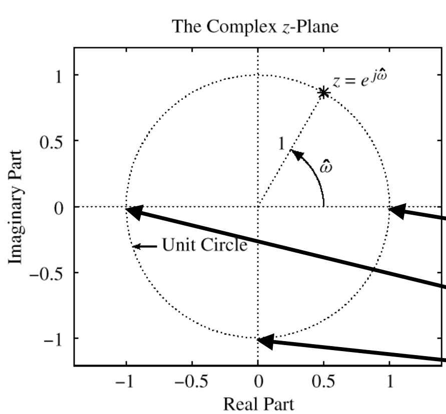
3-D VIEW



The poles are at $z = 0.85e^{\pm j\pi/2}$ and the zeros at $z = \pm 1$.

UNIT CIRCLE

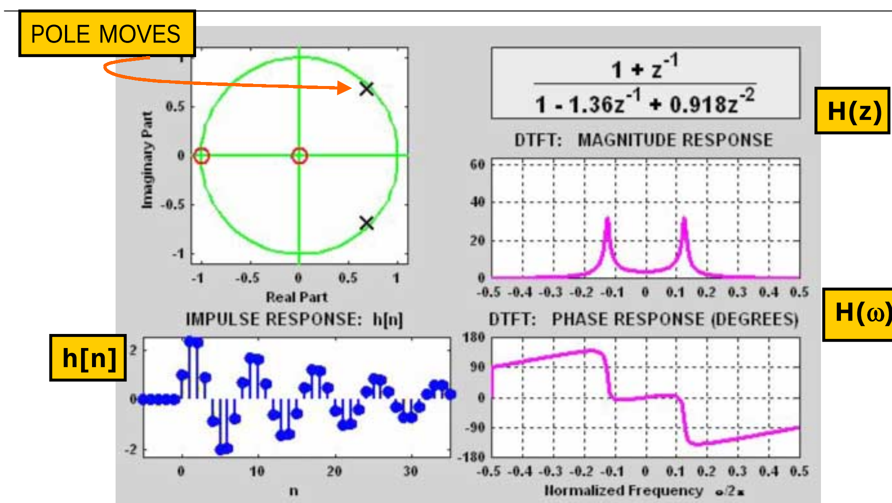
- MAPPING BETWEEN z and $\hat{\omega}$



$$z = e^{j\hat{\omega}}$$

$z = 1$	$\leftrightarrow$	$\hat{\omega} = 0$
$z = -1$	$\leftrightarrow$	$\hat{\omega} = \pm\pi$
$z = \pm j$	$\leftrightarrow$	$\hat{\omega} = \pm\frac{1}{2}\pi$

3 DOMAINS MOVIE: IIR

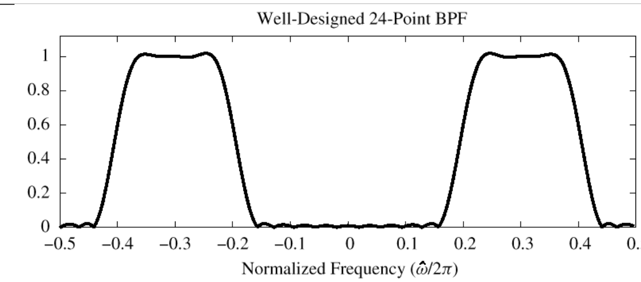


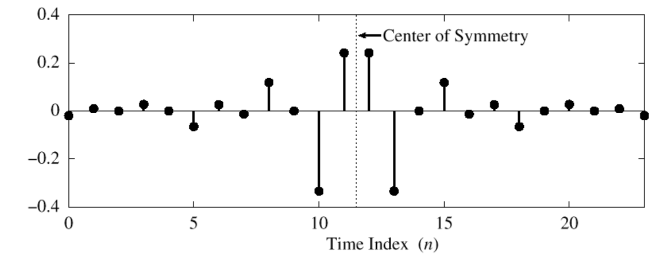
DIGITAL FILTER DESIGN

- Find the COEFFICIENTS to satisfy
  - PASSBAND & STOPBAND specifications
- FIR FILTERS
  - High Order: Many ZEROS. e.g., L=100
- IIR FILTERS
  - Poles & Zeros: 8–10 poles for a good filter
    - Implementation tricky with finite-precision

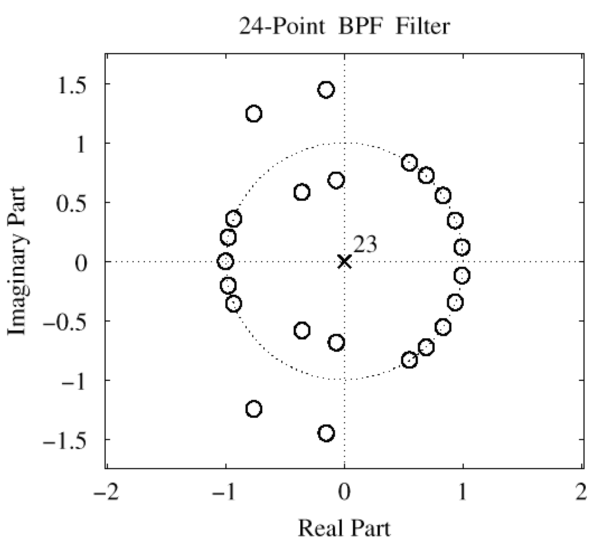
REALISTIC FIR BANDPASS

- FIR
- L = 24
- M=23
- 23 zeros

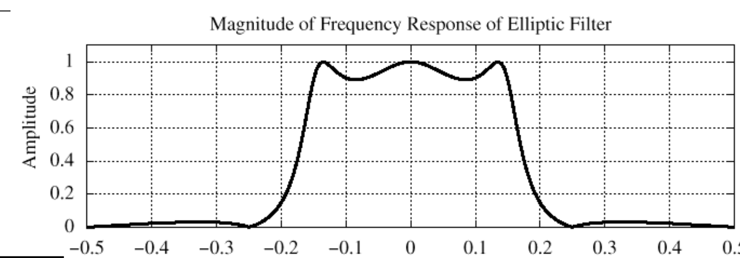




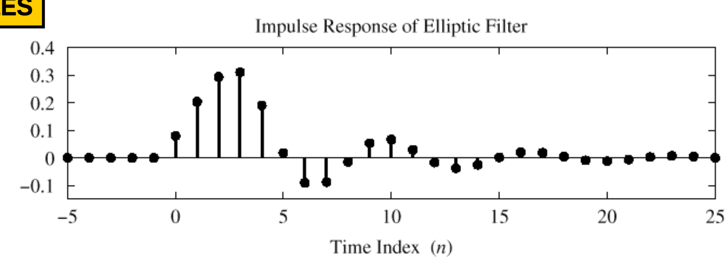
FIR BPF: 23 ZEROS



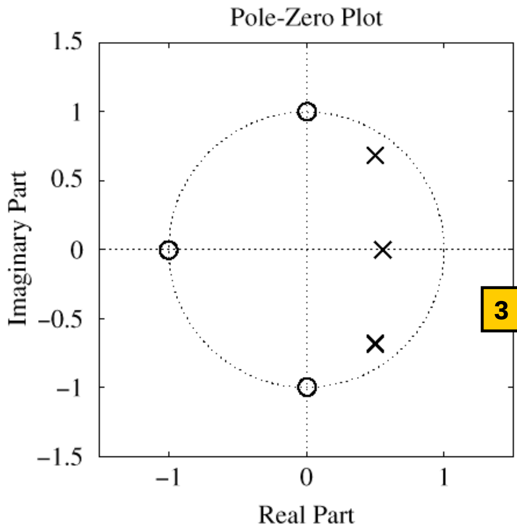
IIR Elliptic LPF (N=3)



3 POLES



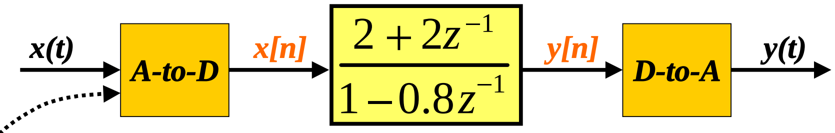
POLES & ZEROS of IIR



3 POLES

POP QUIZ

- Given:



$$\frac{2+2z^{-1}}{1-0.8z^{-1}}$$

- Find the output, y(t)
  - When $x(t)=\cos(2000\pi t)$

$$f_s = 5000\,\text{Hz}$$

POP QUIZ BECOMES

- Given: $H(z)=\dfrac{2+2z^{-1}}{1-0.8z^{-1}}$
- Find the output, y[n]
  - When $x[n]=\cos(0.4\pi n)$
  - Because $\omega T_s = 2000\pi/5000 = 0.4\pi$

NO Aliasing

SINUSOIDAL RESPONSE

- x[n] = SINUSOID => y[n] is SINUSOID
- Get MAGNITUDE & PHASE from H(z)

$$\text{if } \; x[n]=e^{j\hat{\omega}n} \; \text{ then}$$

$$y[n]=H(e^{j\hat{\omega}})e^{j\hat{\omega}n}$$

$$\text{where } \; H(e^{j\hat{\omega}})=H(z)\Big|_{z=e^{j\hat{\omega}}}$$

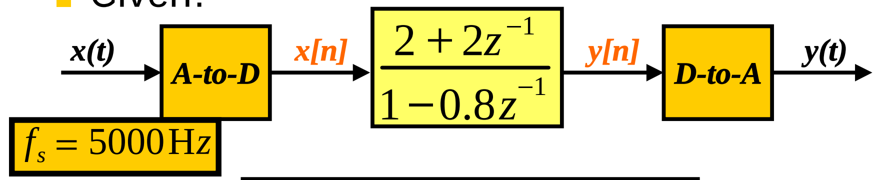
POP QUIZ INSIDE ANSWER

- Given: $H(z)=\dfrac{2+2z^{-1}}{1-0.8z^{-1}}$
- The input: $x[n]=\cos(0.4\pi n)$
- Then y[n] $y[n]=M\cos(0.4\pi n+\psi)$

$$H(e^{j0.4\pi})=\frac{2+2e^{-j0.4\pi}}{1-0.8e^{-j0.4\pi}}=3.02e^{-j0.452\pi}$$

POP QUIZ ANSWER

- Given:

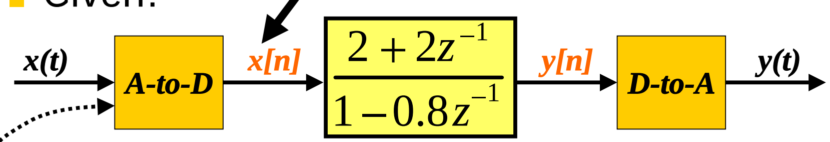


- When $x(t)=\cos(2000\pi t)$
- The output is

$$y(t)=3.02\cos(2000\pi t-0.452\pi)$$

ANOTHER INPUT FREQ

- Given: $\hat{\omega}=?$



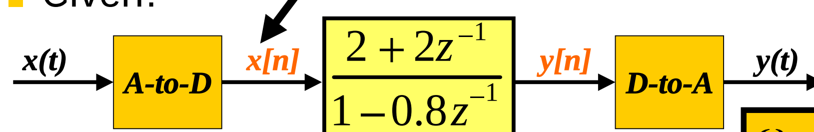
- Find the output, y(t)
  - When $x(t)=\cos(2\pi(7500)t)$

$f_s = 5000\,\text{Hz}$

$$\hat{\omega}=?$$

2nd POP QUIZ ANSWER

- Given: $\hat{\omega}=\cos(2\pi(7500)/5000=2\pi(1.5)$



$\omega=?$

- When $x(t)=\cos(2\pi(7500)t)$

$f_s = 5000\,\text{Hz}$ → $\hat{\omega}=3\pi$ → $y(t)=?$

POP QUIZ-2

- Given: $H(z) = \dfrac{2 + 2z^{-1}}{1 - 0.8z^{-1}}$
- Find the Impulse Response, h[n]
- Find the output, y[n]
  - When $x[n] = \cos(0.25\pi n)$

Z-TRANSFORM TABLES

SHORT TABLE OF z-TRANSFORMS

	$x[n]$	$\Longleftrightarrow$	$X(z)$
1.	$ax_1[n] + bx_2[n]$	$\Longleftrightarrow$	$aX_1(z) + bX_2(z)$
2.	$x[n - n_0]$	$\Longleftrightarrow$	$z^{-n_0}X(z)$
3.	$y[n] = x[n] * h[n]$	$\Longleftrightarrow$	$Y(z) = H(z)X(z)$
4.	$\delta[n]$	$\Longleftrightarrow$	1
5.	$\delta[n - n_0]$	$\Longleftrightarrow$	z^{-n_0}
6.	$a^n u[n]$	$\Longleftrightarrow$	$\dfrac{1}{1 - az^{-1}}$

POP QUIZ: Invert Z

- Given:

$$H(z) = \frac{2 + 2z^{-1}}{1 - 0.8z^{-1}} = \frac{2}{1 - 0.8z^{-1}} + \frac{2z^{-1}}{1 - 0.8z^{-1}}$$

- Find the Impulse Response, h[n]
  - Use the DELAY PROPERTY

$$h[n] = 2(0.8)^n u[n] + 2(0.8)^{n-1} u[n-1]$$

Cosine Input

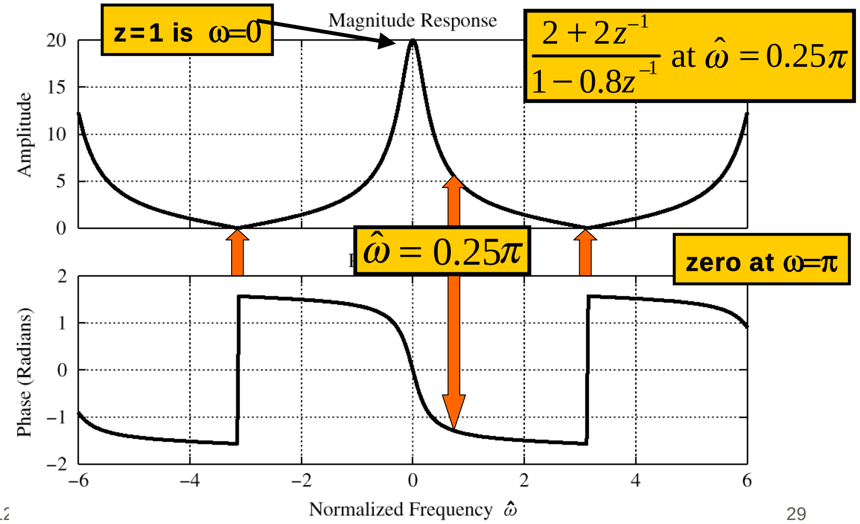
- x[n] = SINUSOID => y[n] is SINUSOID
- Get MAGNITUDE & PHASE from H(z)

$$\text{if} \quad x[n] = e^{j\hat{\omega}n} \quad \text{then}$$

$$y[n] = H(e^{j\hat{\omega}})e^{j\hat{\omega}n}$$

$$\text{where} \quad H(e^{j\hat{\omega}}) = H(z)\Big|_{z=e^{j\hat{\omega}}}$$

Evaluate FREQ. RESPONSE



POP QUIZ: Eval Freq. Resp.

- Given: $H(z) = \dfrac{2+2z^{-1}}{1-0.8z^{-1}}$
- Find output, y[n], when $x[n] = \cos(0.25\pi n)$
  - Evaluate at $z = e^{j0.25\pi}$

$$H(z) = \frac{2+2(\frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2})}{1-0.8e^{-j0.25\pi}} = 5.182e^{-j1.309}$$

$$y[n] = 5.182\cos(0.25\pi n - 0.417\pi)$$

THREE INPUTS

- Given: $H(z) = \dfrac{5}{1+0.8z^{-1}}$
- Find the output, y[n]
  - When

$$x[n] = \cos(0.2\pi n)$$

$$x[n] = a^n u[n]$$

$$x[n] = \cos(0.2\pi n)u[n]$$

SINUSOID ANSWER

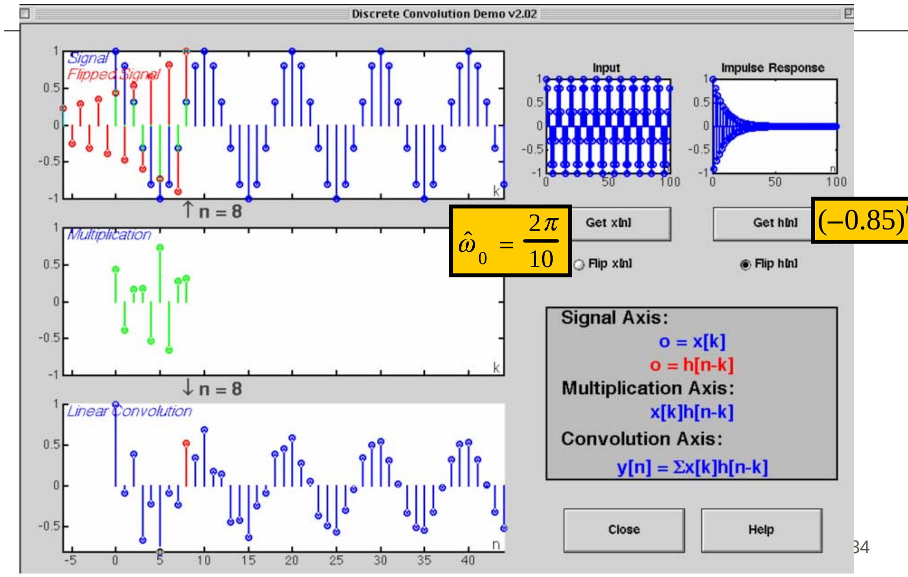
- Given: $H(z) = \dfrac{5}{1+0.8z^{-1}}$
- The input: $x[n] = \cos(0.2\pi n)$
- Then y[n] $y[n] = M\cos(0.2\pi n + \psi)$

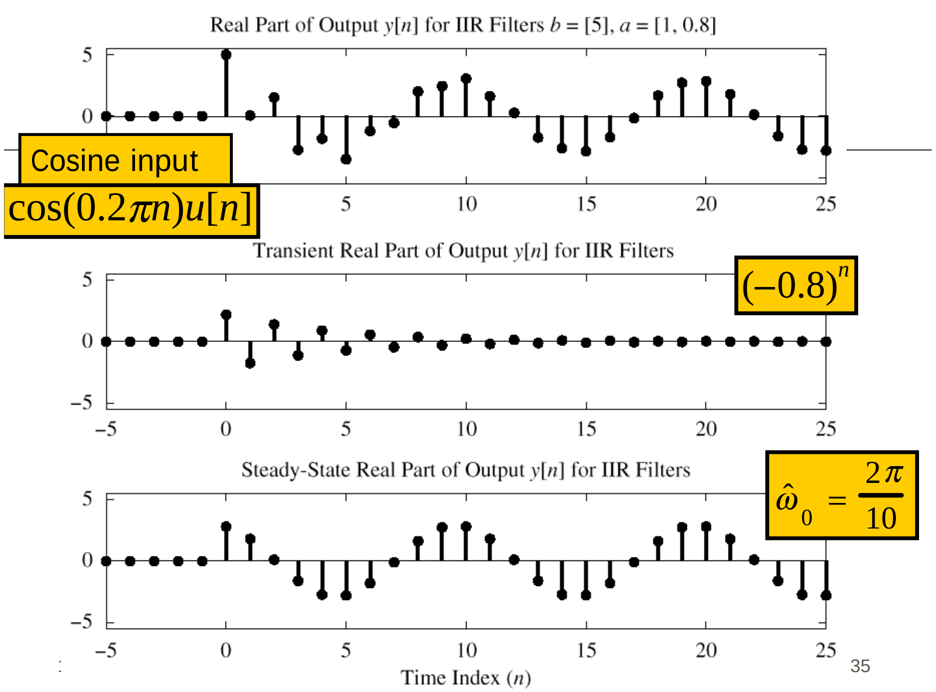
$$H(e^{j0.25\pi}) = \frac{5}{1+0.8e^{-j0.2\pi}} = 2.919e^{j0.089\pi}$$

SINUSOID starting at n=0

- We'll look at an example in MATLAB
  - $\cos(0.2\pi n)$
  - Pole at –0.8, so a^n is $(-0.8)^n$
- There are two components:
  - TRANSIENT
    - Start-up region just after n=0; **$(-0.8)^n$**
  - STEADY-STATE
    - Eventually, y[n] looks sinusoidal.
    - **Magnitude & Phase from Frequency Response**

Transient & Steady State





STABILITY

- When Does the TRANSIENT DIE OUT ?

STEADY-STATE RESPONSE AND STABILITY

A stable system is one that does not "blow up." This intuitive statement can be formalized by saying that the output of a stable system can always be bounded ($|y[n]| < M_y$) whenever the input is bounded ($|x[n]| < M_x$).[3]

$$y[n] = a_1 y[n-1] + b_0 x[n]$$

$$H(z) = \frac{b_0}{1 - a_1 z^{-1}}$$

$$h[n] = b_0 a_1^n u[n]$$

need $|a_1| < 1$

STABILITY CONDITION

- ALL POLES INSIDE the UNIT CIRCLE
- UNSTABLE EXAMPLE:

POLE @ z=1.1

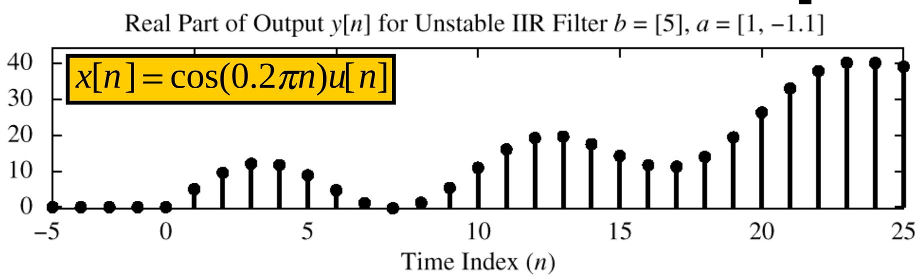


Figure 8.15 Illustration of an unstable IIR system. Pole is at $z = 1.1$.

Superficial Knowledge

- It depends how carefully you think about it. If you don’t think very carefully it’s obvious; but if you think about it in depth, you’ll get confused and it won’t be obvious.

Mathematical Elegance

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}d\omega$$

Fourier S
(Inverse

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

Fourier Analysis
(Forward Transform)

$$\text{Time-domain} \Leftrightarrow \text{Frequency-domain}$$
$$x(t) \Leftrightarrow X(j\omega)$$

IMPORTANT CONCEPTS

- ALL Signals have Frequency Content
  - Sum of Sinusoids
  - Complex Exponentials
  - Impulses, Square Pulses
- FILTERS alter the Frequency Content
  - Image Processing Example: Blur
  - Linear Time-Invariant Processing
- 3 Domains for Analysis