

EE-2200

Spring-99

Lecture 12

Digital Filtering of Analog Signals

14-May-99

Info: Web-CT, Lab, HW

- **Quiz #2 on 24-May (Monday)**
 - Coverage: HW #4, #5, and #6
- **MATLAB Help on Wednesdays**
 - 6 PM, VL-456
- **Prob Set #5 due TODAY**
- **Lab #7 on Frequency Response**
 - Touch-Tone Phone will be Lab #8

READING ASSIGNMENTS

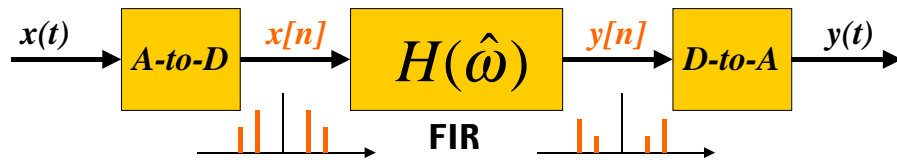
- **This Lecture:**
 - Chapter 6, pp. 188–194
- **Other Reading:**
 - Recitation: Ch. 6, pp. 176–188
 - FREQUENCY RESPONSE EXAMPLES
 - Next Lecture: Chapter 7, start

COURSE OBJECTIVE

- **Students will be able to:**
- Understand **mathematical** descriptions of signal processing **algorithms** and express those algorithms as computer **implementations** (MATLAB)

LECTURE OBJECTIVES

- Track the spectrum of $x[n]$ thru an FIR Filter.
- UNIFICATION:**
 - How does Frequency Response affect $x(t)$ to produce $y(t)$?



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LECTURE #11 REVIEW

- SINUSOIDAL INPUT SIGNAL**
 - OUTPUT has **SAME FREQUENCY**
 - DIFFERENT** Amplitude and Phase
- FREQUENCY RESPONSE** of FIR
 - MAGNITUDE vs. Frequency
 - PHASE vs. Freq
 - PLOTTING:

$$H(\hat{\omega}) = |H(\hat{\omega})| e^{j\phi(\hat{\omega})}$$

MAG

PHASE

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FREQUENCY RESPONSE

- $yy = \text{freqz}(bb, 1, ww)$
 - VECTOR **bb** contains Filter Coefficients
 - DSP-First: $yy = \text{freesz}(bb, 1, ww)$
- FILTER COEFFICIENTS** $\{b_k\}$

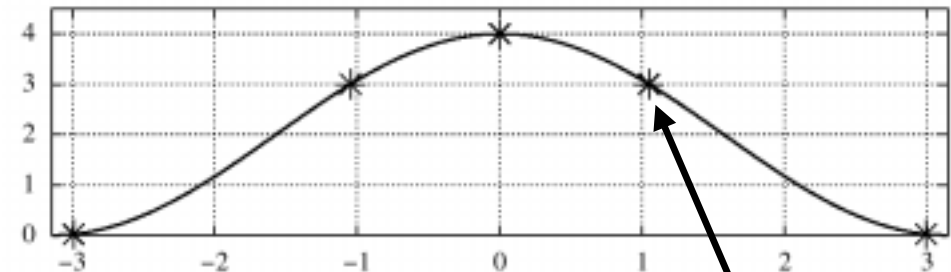
$$H(\hat{\omega}) = \sum_{k=0}^M b_k e^{-j\hat{\omega}k}$$

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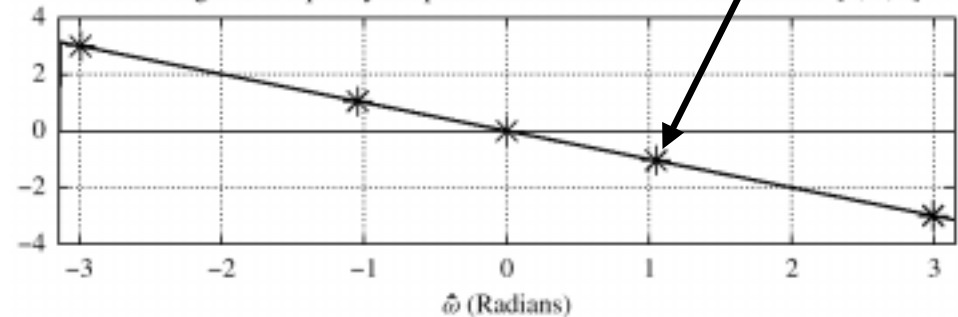
Magnitude of Frequency Response of FIR Filter with Coefficients [1, 2, 1]



$$H(\hat{\omega}) = (2 + 2 \cos \hat{\omega}) e^{-j\hat{\omega}}$$

RESPONSE at $\pi/3$

Phase Angle of Frequency Response of FIR Filter with Coefficients [1, 2, 1]



LTI SYSTEMS

- **LTI:**
 - Linear & Time-Invariant
- **COMPLETELY CHARACTERIZED** by:
 - FREQUENCY RESPONSE
 - IMPULSE RESPONSE $h[n]$
- **Two DOMAINS:** time & frequency
 - Go back and forth

TIME & FREQUENCY

$$y[n] = \sum_{k=0}^M b_k x[n - k] = \sum_{k=0}^M h[k] x[n - k]$$

FIR DIFFERENCE EQUATION

The frequency response of an LTI system

$$\mathcal{H}(\hat{\omega}) = \sum_{k=0}^M b_k e^{-j\hat{\omega}k} = \sum_{k=0}^M h[k] e^{-j\hat{\omega}k} \quad (6.1.4)$$

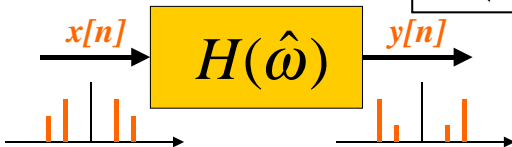
IMPULSE RESPONSE

FREQ DOMAIN --> TIME ??

- **START** with $H(\hat{\omega})$ and find $h[n]$ or b_k



$$H(\hat{\omega}) = 7e^{-j2\hat{\omega}} \cos(\hat{\omega})$$



FREQ DOMAIN --> TIME

$$H(\hat{\omega}) = 7e^{-j2\hat{\omega}} \cos(\hat{\omega}) \quad \text{EULER'S FORMULA}$$

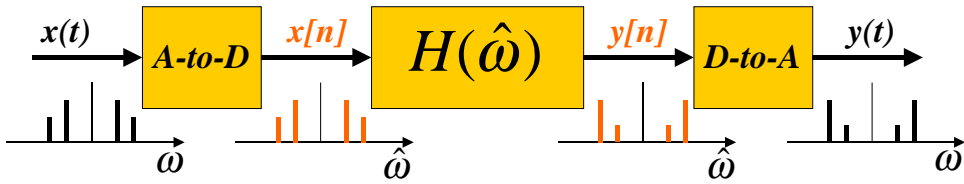
$$= 7e^{-j2\hat{\omega}} (0.5e^{j\hat{\omega}} + 0.5e^{-j\hat{\omega}})$$

$$= (3.5e^{-j\hat{\omega}} + 3.5e^{-j3\hat{\omega}})$$

$$h[n] = 3.5\delta[n - 1] + 3.5\delta[n - 3]$$

$$b_k = \{0, 3.5, 0, 3.5\}$$

DIGITAL "FILTERING"



ω | SPECTRUM of $x(t)$ (SUM of SINUSOIDS)

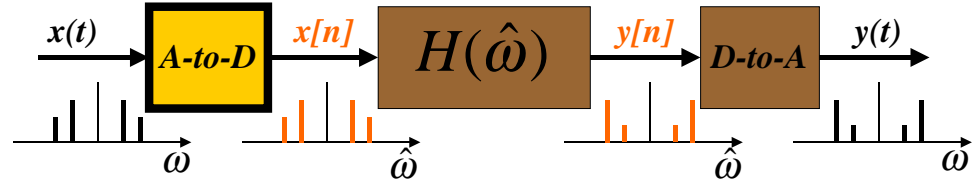
ω | SPECTRUM of $x[n]$

| Is ALIASING a PROBLEM?

$\hat{\omega}$ | SPECTRUM $y[n]$ (FIR Gain or Nulls)

ω | Then, OUTPUT $y(t)$ = SUM of SINUSOIDS

FREQUENCY SCALING



■ TIME SAMPLING:

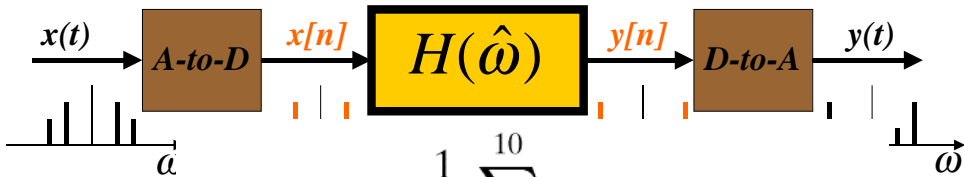
$$t = nT_s$$

■ IF NO ALIASING:

■ FREQUENCY SCALING

$$\hat{\omega} = \omega T_s = \frac{\omega}{f_s}$$

11-pt AVERAGER Example

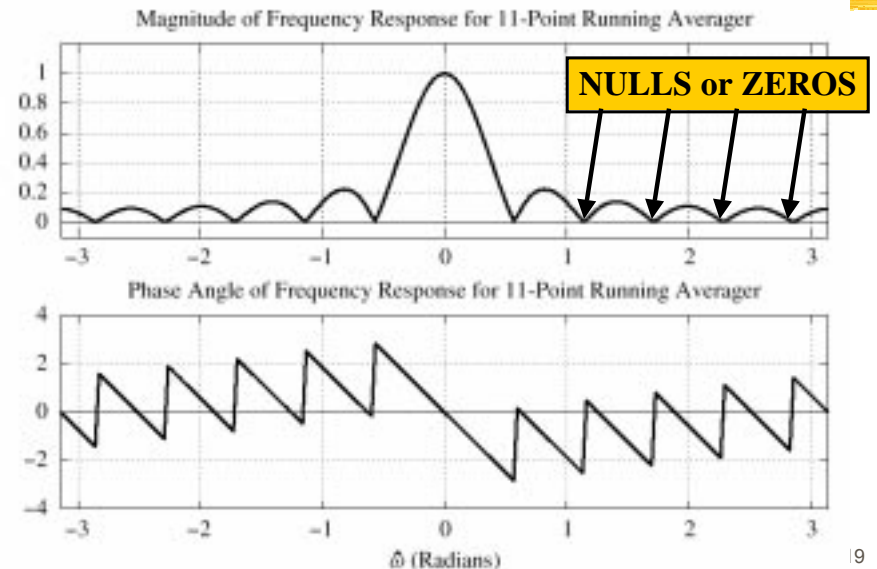


$$y[n] = \frac{1}{11} \sum_{k=0}^{10} x[n - k]$$

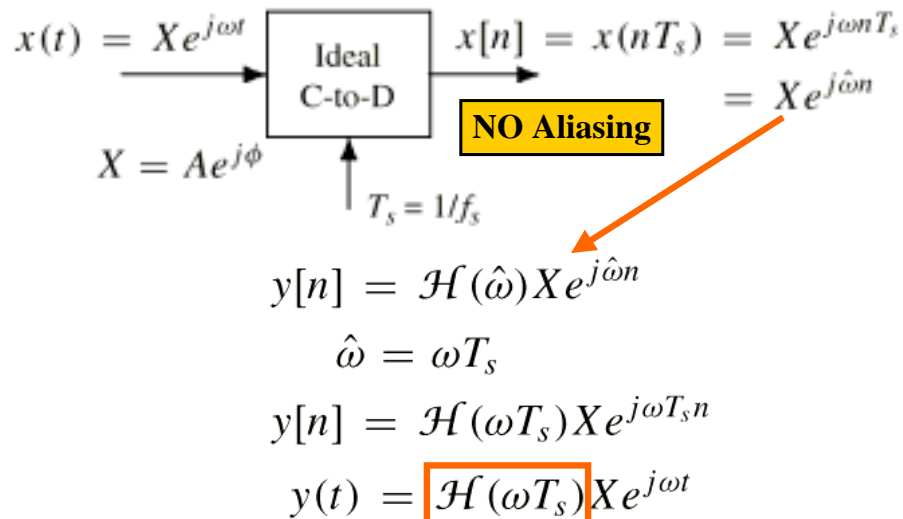
$$\mathcal{H}(\hat{\omega}) = \frac{\sin(\hat{\omega}11/2)}{11 \sin(\hat{\omega}/2)} e^{-j\hat{\omega}5}$$

$$x(t) = \cos(2\pi(25)t) + \sin(2\pi(250)t)$$

11-pt AVERAGER



11-pt AVG Example (2)



SINUSOID thru FIR

$$\begin{aligned}
 x[n] &= X_0 + \sum_{k=1}^N \left(\frac{X_k}{2} e^{j\hat{\omega}_k n} + \frac{X_k^*}{2} e^{-j\hat{\omega}_k n} \right) \\
 &= X_0 + \sum_{k=1}^N |X_k| \cos(\hat{\omega}_k n + \angle X_k)
 \end{aligned}$$

if $\mathcal{H}(-\hat{\omega}) = \mathcal{H}^*(\hat{\omega})$, the corresponding output is

$$\begin{aligned}
 y[n] &= \mathcal{H}(0) X_0 + \sum_{k=1}^N \left(\mathcal{H}(\hat{\omega}_k) \frac{X_k}{2} e^{j\hat{\omega}_k n} + \mathcal{H}(-\hat{\omega}_k) \frac{X_k^*}{2} e^{-j\hat{\omega}_k n} \right) \\
 &= \mathcal{H}(0) X_0 + \sum_{k=1}^N |\mathcal{H}(\hat{\omega}_k)| |X_k| \cos(\hat{\omega}_k n + \angle X_k + \angle \mathcal{H}(\hat{\omega}_k))
 \end{aligned}$$

EVALUATE Freq. Response

$$x(t) = \cos(2\pi(25)t) + \sin(2\pi(250)t)$$

evaluating at 25 and 250 Hz.

$$\mathcal{H}(2\pi(25)/1000) = \frac{\sin(\pi(25)(11)/1000)}{11 \sin(\pi(25)/1000)} e^{-j2\pi(25)(5)/1000}$$

$$= 0.8811 e^{-j\pi/4}$$

f_s = 1000 **MAG SCALE** **PHASE CHANGE**

$$\mathcal{H}(2\pi(250)/1000) = \frac{\sin(\pi(250)(11)/1000)}{11 \sin(\pi(250)/1000)} e^{-j2\pi(250)(5)/1000}$$

$$= 0.0909 e^{-j\pi/2}$$

$$y(t) = \underline{0.8811} \cos(2\pi(25)t - \underline{\pi/4}) + \underline{0.0909} \sin(2\pi(250)t - \underline{\pi/2})$$

EFFECTIVE Freq. Response

- Assume NO Aliasing, then
 - ANALOG FREQ <--> DIGITAL FREQ

$$\hat{\omega} = \omega T_s = \frac{\omega}{f_s}$$

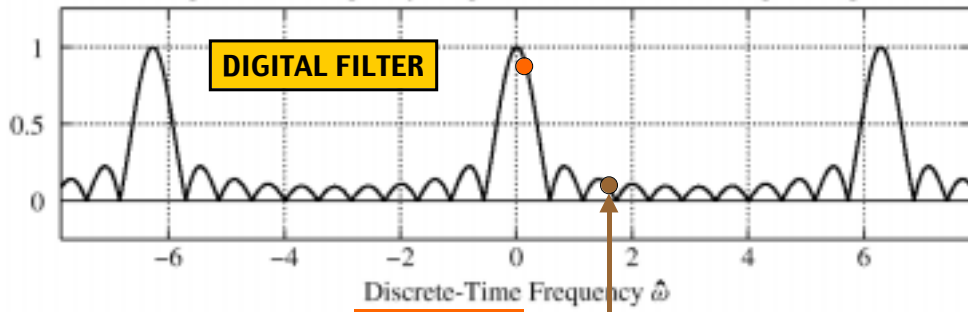
DIGITAL FILTER

- So, we can plot:
- Scaled Freq. Axis

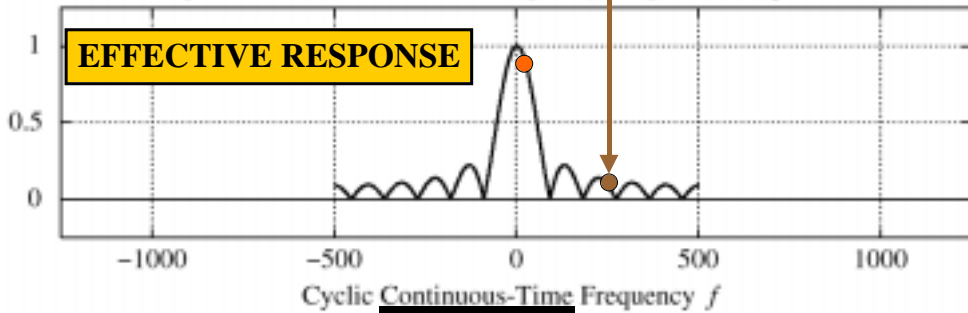
$$H(\omega T_s) \text{ vs. } \omega$$

ANALOG FREQUENCY

Magnitude of Frequency Response for 11-Point Running Averager



Equivalent Continuous-Time Frequency Response for $f_s = 1000$

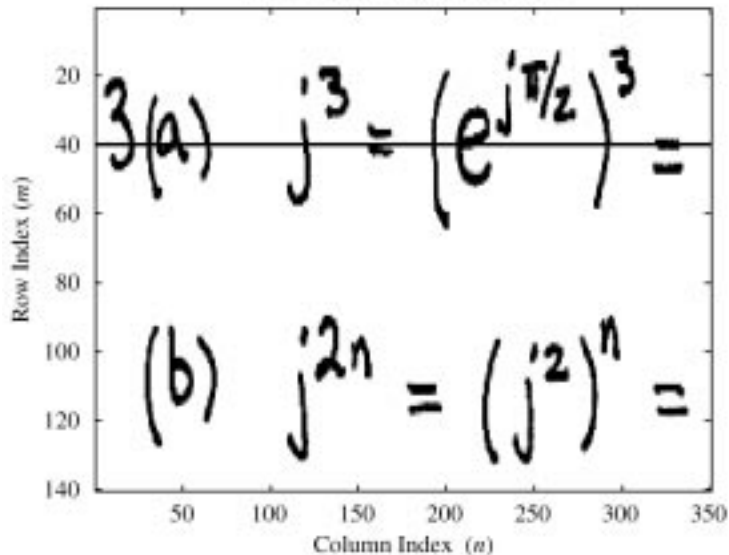


FILTER TYPES

- **LOW-PASS FILTER (LPF)**
 - BLURRING
 - ATTENUATES HIGH FREQUENCIES
- **HIGH-PASS FILTER (HPF)**
 - SHARPENING for IMAGES
 - BOOSTS THE HIGHS
 - REMOVES DC
- **BAND-PASS FILTER (BPF)**

B & W IMAGE

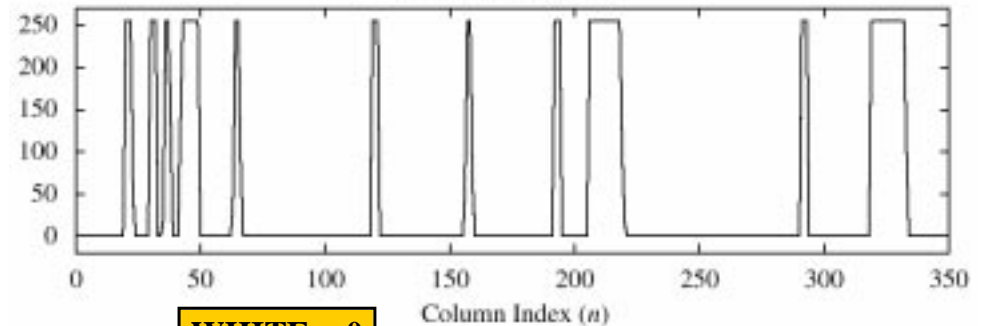
Original Black and White Image



ROW of B&W IMAGE

BLACK = 255

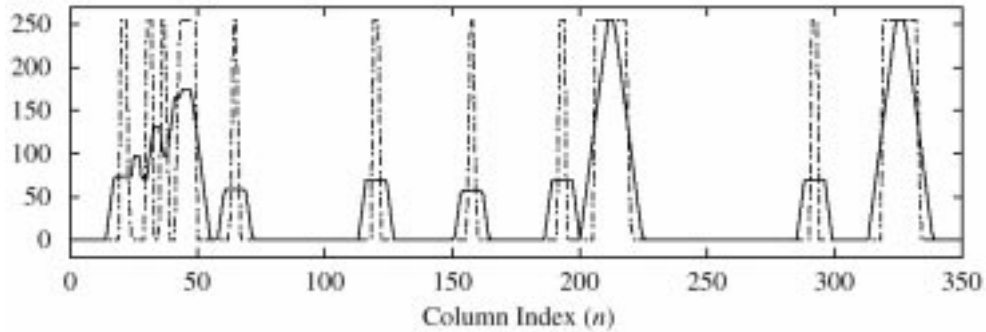
Row 40 of the Image



WHITE = 0

FILTERED ROW of IMAGE

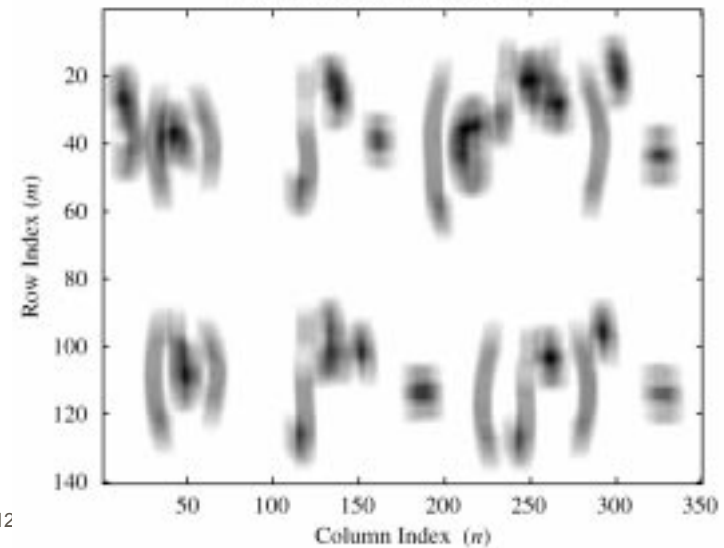
11-Point Averaging: 5-Sample Delay Equalization



ADJUST DELAY by 5 samples

FILTERED B&W IMAGE

Row and Column Filtered Image



LPF:
BLUR

B&W IMAGE with COSINE

FILTERED: 11-pt AVG

Homework plus Cosine

Remove Cosine Stripe with Averaging Fi

