EE-2200 Spring-99

Lecture 15

H(z) & Frequency Response

28-May-99

Info: Web-CT, Lab, HW

- Calendar:
  - Final Exam is Period 13: Friday 8 AM !!!
  - Review Session is planned
    - Thursday evening probably
- Grade Weightings are posted
- Prob Set #7 due on 4–June Friday
  - On–Line HW will up ASAP
- Last Lab #9 is PEZ (on–Line & in–Lab)

READING ASSIGNMENTS

- This Lecture:
  - Chapter 8, pp. 263–279
- Other Reading:
  - Recitation: Ch. 8, pp. 261–272
    - POLES & ZEROS
  - Next Lecture: Chapter 8, pp. 279–300

LECTURE OBJECTIVES

- SYSTEM FUNCTION: H(z)
- H(z) has POLES and ZEROS
- FREQUENCY RESPONSE of IIR
  - Get H(z) first

$$H(e^{j\hat{\omega}}) = H(z)\Big|_{z=e^{j\hat{\omega}}}$$

- THREE–DOMAIN APPROACH

$$h[n] \leftrightarrow H(z) \leftrightarrow H(e^{j\hat{\omega}})$$

IIR FILTER REVIEW

- ADD PREVIOUS OUTPUTS

$$y[n] = 0.8y[n-1] + 3x[n] - 2x[n-1]$$

FEEDBACK COEFFICIENT

SIGN CHANGE

- MATLAB
  - `yy = filter([3,-2],[1,-0.8],xx)`

IMPULSE RESPONSE

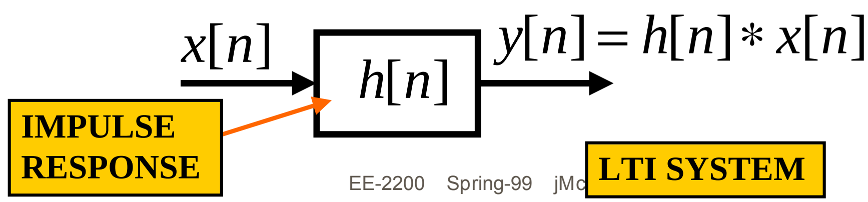
- DIFFERENCE EQUATION:

$$y[n] = 0.8y[n-1] + 3x[n]$$

- Find h[n]

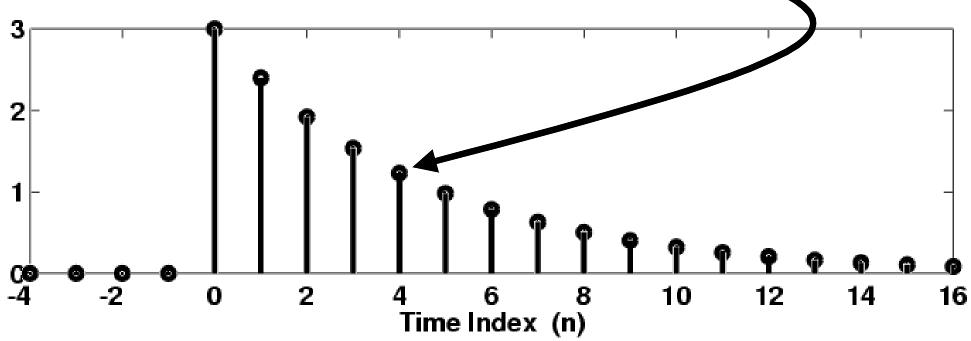
$$h[n] = 3(0.8)^n u[n]$$

- CONVOLUTION in TIME-DOMAIN



PLOT IMPULSE RESPONSE

$$h[n] = b_0 (a_1)^n u[n] = 3(0.8)^n u[n]$$



THREE DOMAINS

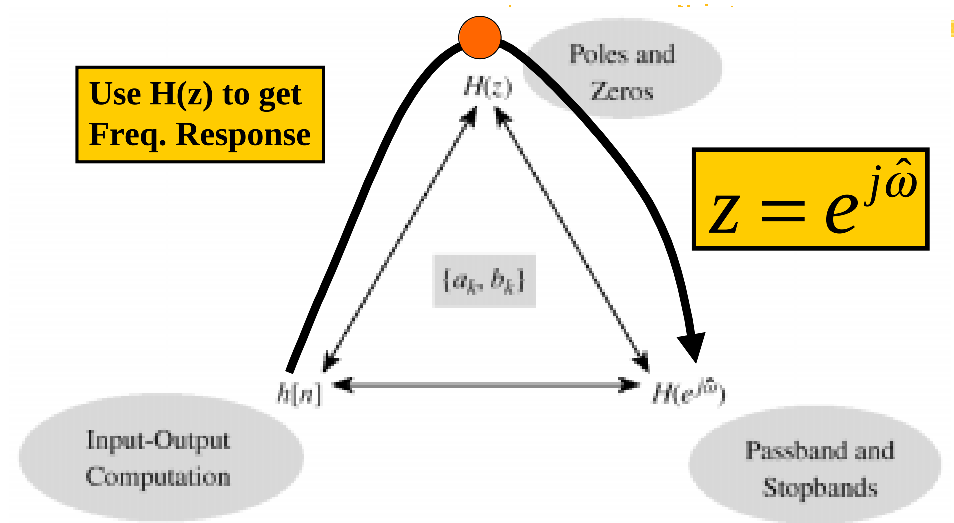


Figure 8.13 Relationship among the n-, z-, and $\hat{\omega}$-domains. The filter coefficients $\{a_k, b_k\}$ play a central role.

Z-Transform of h[n]

- POLYNOMIAL Representation

$$H(z)=\sum_{n} h[n]z^{-n}$$

APPLIES to Any SIGNAL

- IIR EXAMPLE?

$$h[n]=b_0(a_1)^n u[n]=3(0.8)^n u[n]$$

$$H(z)=\sum_{n} 3(0.8)^n u[n]z^{-n}$$

$$=\sum_{n=0}^{\infty} 3(0.8)^n z^{-n}=\frac{3}{1-0.8z^{-1}}$$

Z-Transform of IIR Filter

- DERIVE the SYSTEM FUNCTION *H(z)*
  - Use DELAY PROPERTY

$$y[n] = a_1 y[n-1] + b_0 x[n] + b_1 x[n-1]$$

$$Y(z) = a_1 z^{-1} Y(z) + b_0 X(z) + b_1 z^{-1} X(z)$$

EASIER with DELAY PROPERTY

Time delay of n_0 samples multiplies the z-transform by z^{-n_0}

$$x[n-n_0] \iff z^{-n_0} X(z)$$

SYSTEM FUNCTION of IIR

- NOTE the FILTER COEFFICIENTS

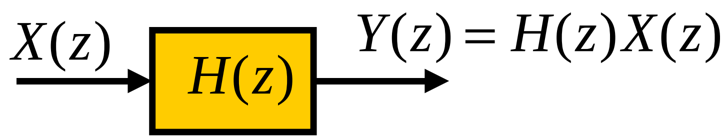
$$Y(z) - a_1 z^{-1} Y(z) = b_0 X(z) + b_1 z^{-1} X(z)$$

$$(1 - a_1 z^{-1}) Y(z) = (b_0 + b_1 z^{-1}) X(z)$$

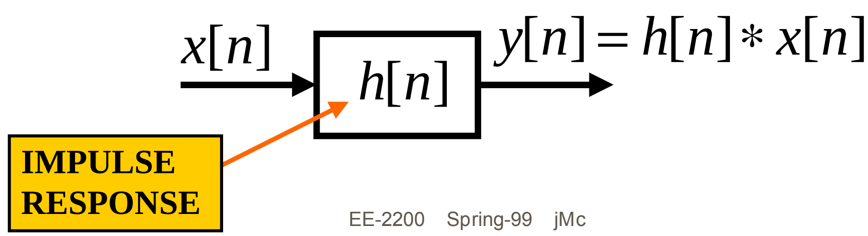
$$H(z) = \frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1}}{1 - a_1 z^{-1}} = \frac{B(z)}{A(z)}$$

CONVOLUTION PROPERTY

- MULTIPLICATION of z-TRANSFORMS



- CONVOLUTION in TIME-DOMAIN



SYSTEM FUNCTION

- **DIFFERENCE EQUATION:**

$$y[n] = 0.8y[n-1] + 3x[n] - 2x[n-1]$$

- **SYSTEM FUNCTION:**

H(z)

$$Y(z) = \left(\frac{3 - 2z^{-1}}{1 - 0.8z^{-1}} \right) X(z)$$

H(z) = z-Transform{ h[n] }

- **FIRST-ORDER CASE:**

$$b_0 = 1, a_1 = a$$

$$h[n] = a^n u[n]$$

$$H(z) = \sum_{n=0}^{\infty} a^n z^{-n} = \frac{1}{1 - az^{-1}}$$

$$|a| < z.$$

$$a^n u[n] \Longleftrightarrow \frac{1}{1 - az^{-1}}$$

POLES & ZEROS

- **ROOTS of NUMERATOR & DENOMINATOR**

$$H(z) = \frac{b_0 + b_1 z^{-1}}{1 - a_1 z^{-1}} = \frac{b_0 z + b_1}{z - a_1}$$

$$b_0 z + b_1 = 0 \Longrightarrow z = -\frac{b_1}{b_0}$$ ZERO

$$z - a_1 = 0 \Longrightarrow z = a_1$$ POLE

INTERPRET ROOTS

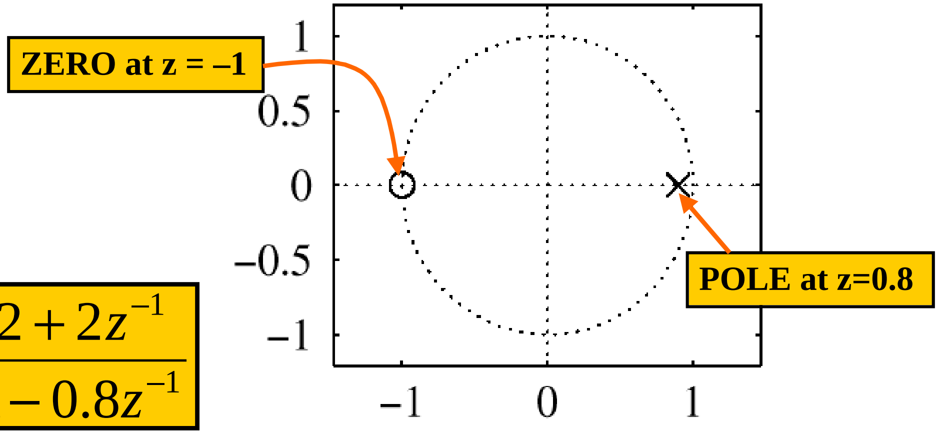
- **VALUE of H(z) at POLES is INFINITE**

$$H(z) = \frac{b_0 + b_1 z^{-1}}{1 - a_1 z^{-1}} = \frac{b_0 z + b_1}{z - a_1}$$

$$H(z)\Big|_{z=-(b_1/b_0)} = 0$$ ZERO

$$H(z)\Big|_{z=a_1} \rightarrow \infty$$ POLE

POLE-ZERO PLOT



$$\frac{2+2z^{-1}}{1-0.8z^{-1}}$$

FREQUENCY RESPONSE

- SYSTEM FUNCTION: **H(z)**
- H(z) has **DENOMINATOR**
- FREQUENCY RESPONSE of IIR
  - We have H(z)

$$H(e^{j\hat{\omega}}) = H(z)\big|_{z=e^{j\hat{\omega}}}$$

- THREE–DOMAIN APPROACH

$$h[n] \leftrightarrow H(z) \leftrightarrow H(e^{j\hat{\omega}})$$

FREQUENCY RESPONSE

- EVALUATE on the UNIT CIRCLE

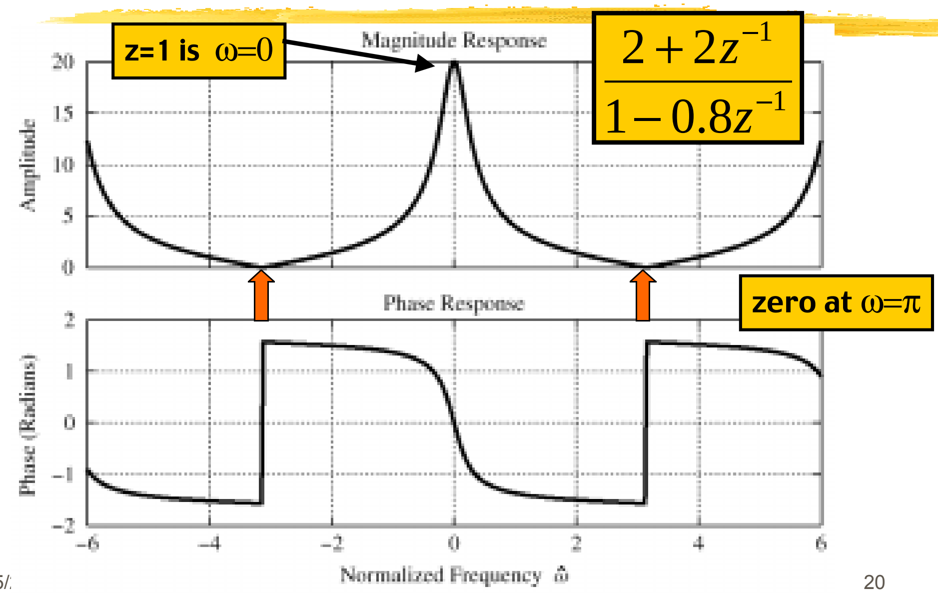
$$H(e^{j\hat{\omega}}) = H(z)\big|_{z=e^{j\hat{\omega}}}$$

$$H(z) = \frac{b_0 + b_1 z^{-1}}{1 - a_1 z^{-1}}$$

$$H(e^{j\hat{\omega}}) = H(z)\big|_{z=e^{j\hat{\omega}}} = \frac{b_0 + b_1 e^{-j\hat{\omega}}}{1 - a_1 e^{-j\hat{\omega}}}$$

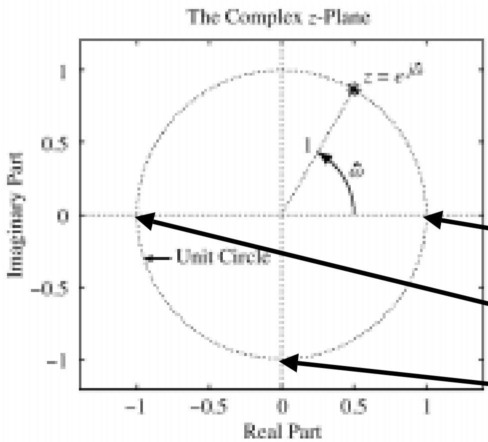
PLOT FREQ. RESPONSE

$$\frac{2+2z^{-1}}{1-0.8z^{-1}}$$



UNIT CIRCLE

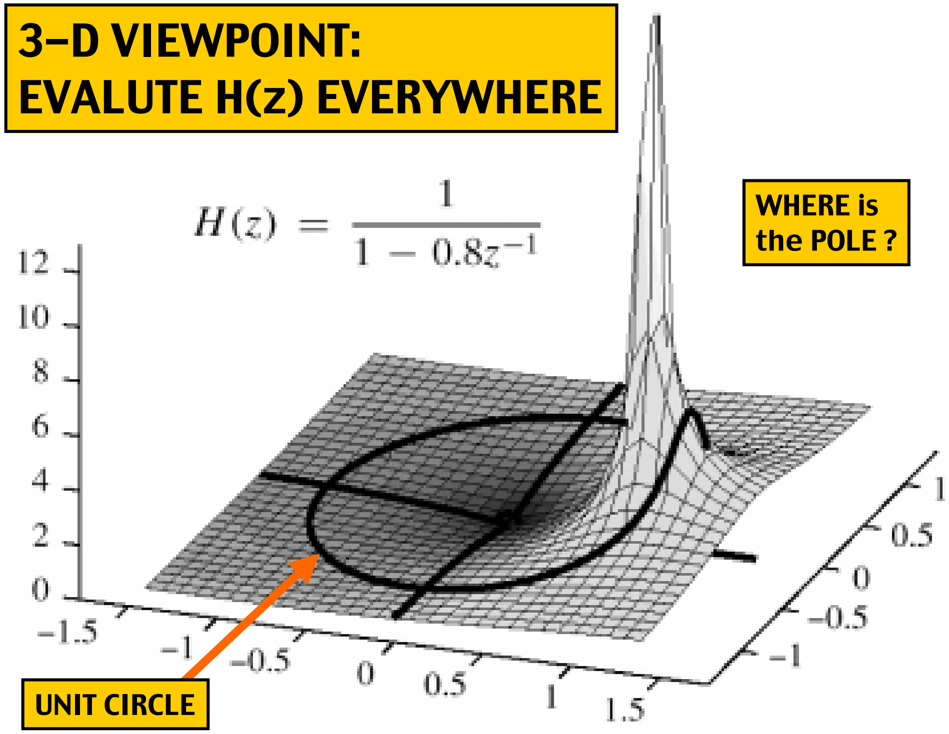
- **MAPPING BETWEEN** z and $\hat{\omega}$



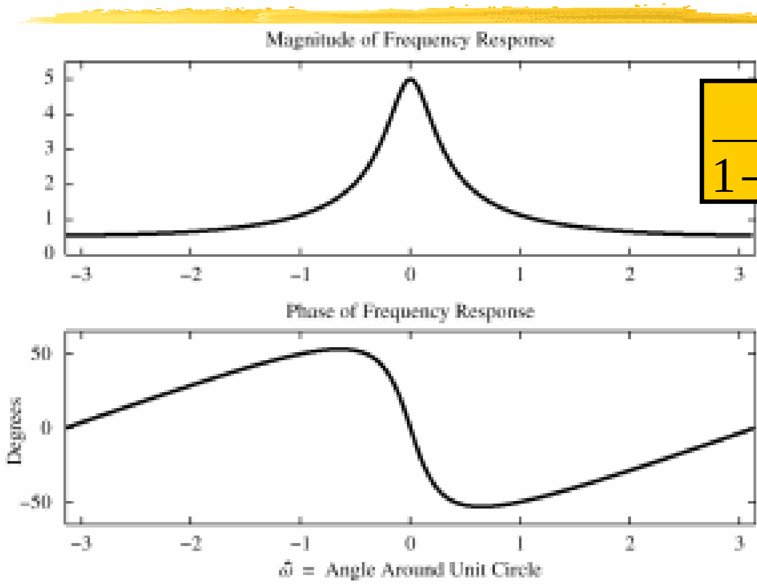
$$z = e^{j\hat{\omega}}$$

$z = 1$	$\leftrightarrow$	$\hat{\omega} = 0$
$z = -1$	$\leftrightarrow$	$\hat{\omega} = \pm\pi$
$z = \pm j$	$\leftrightarrow$	$\hat{\omega} = \pm\frac{1}{2}\pi$

3-D VIEWPOINT: EVALUTE H(z) EVERYWHERE

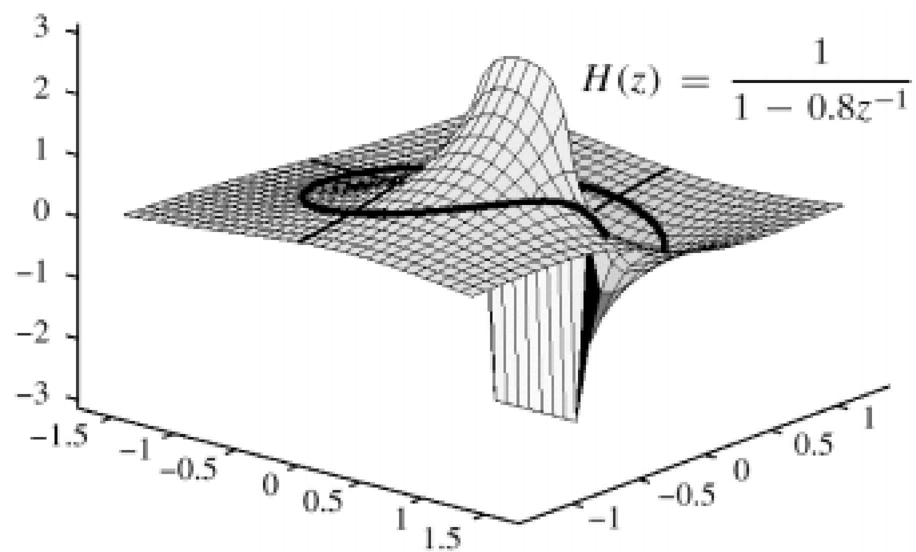


FREQ. RESPONSE from 3-D



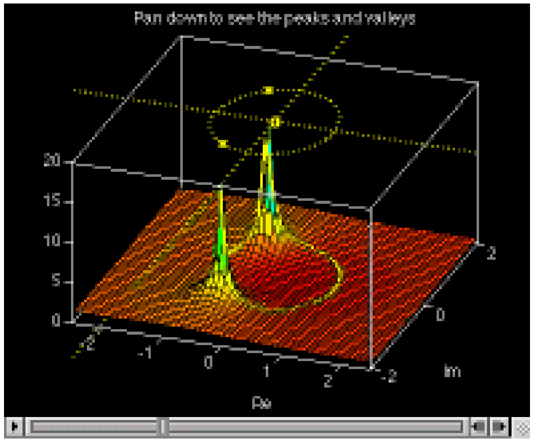
$$\frac{1}{1 - 0.8z^{-1}}$$

PHASE from 3-D PLOT



MOVIE for H(z) in 3-D

- POLES to H(z) to Frequency Reponse
  - TWO POLES SHOWN



THREE DOMAINS

Use H(z) to get Freq. Response

$$H(z) = \sum_n h[n] z^{-n}$$

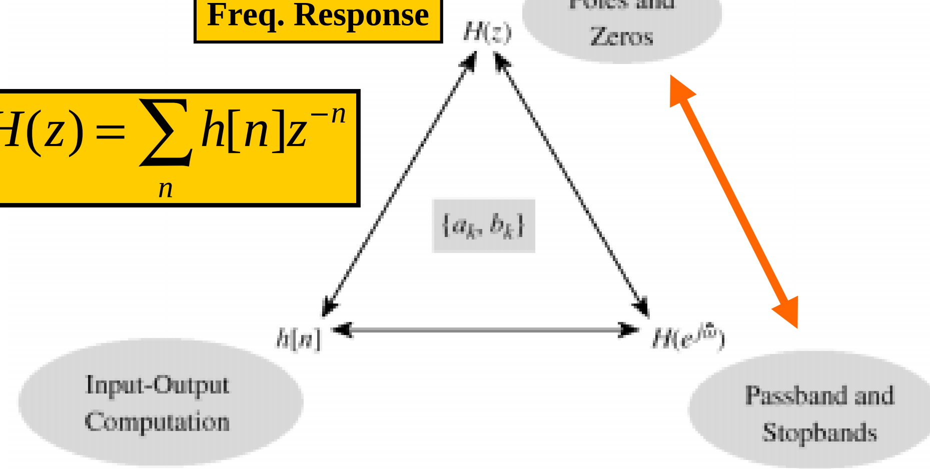
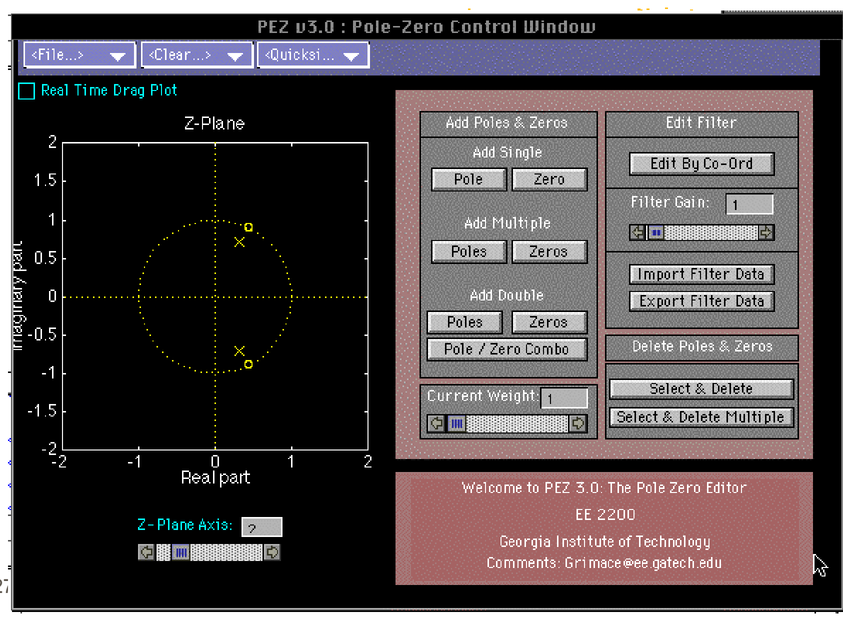
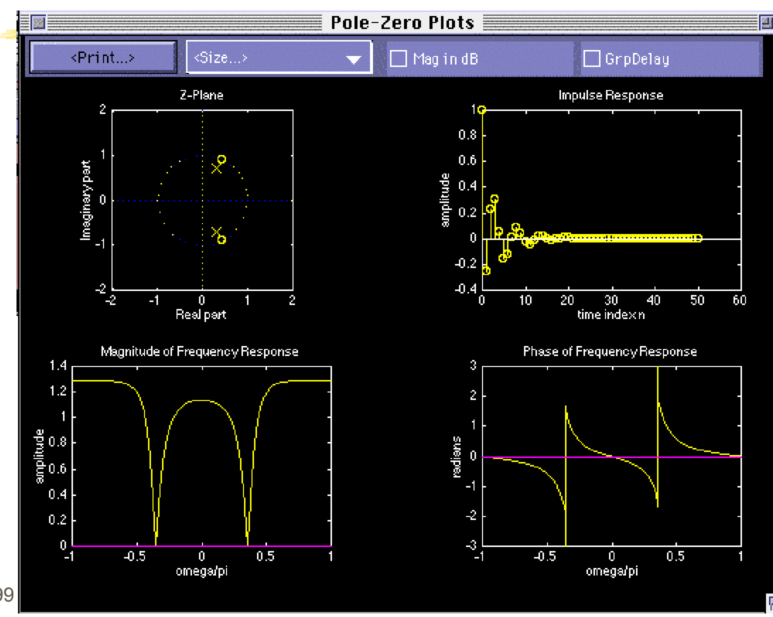


Figure 8.13 Relationship among the n-, z-, and $\hat{\omega}$-domains. The filter coefficients $\{a_k, b_k\}$ play a central role.

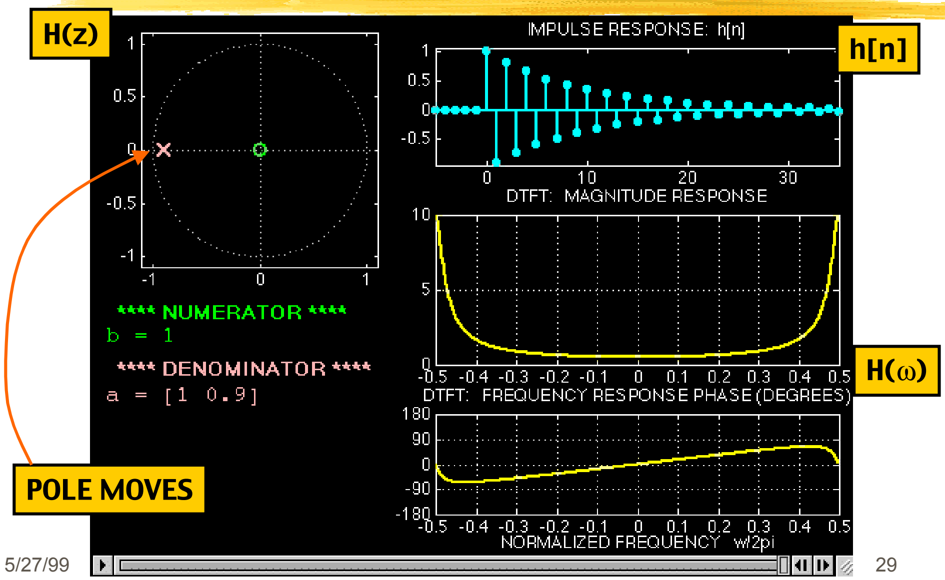
PEZ for Lab #9



PEZ shows 3 DOMAINS



3 DOMAINS MOVIE: IIR



SINUSOIDAL RESPONSE

- x[n] = SINUSOID => y[n] is SINUSOID
- Get MAGNITUDE & PHASE from H(z)

$$\text{if } x[n] = e^{j\hat{\omega}n}, \text{ then}$$

$$y[n] = \mathcal{H}(\hat{\omega})e^{j\hat{\omega}n}$$

$$\mathcal{H}(\hat{\omega}) = H(e^{j\hat{\omega}}) = H(z)\big|_{z=e^{j\hat{\omega}}}$$

POP QUIZ

- Given: $H(z) = \dfrac{2+2z^{-1}}{1-0.8z^{-1}}$
- Find the Impulse Response, h[n]
- Find the output, y[n]
  - When $x[n] = \cos(0.25\pi n)$