EE-2200 Spring-99

Lecture 16

DIGITAL FILTERING of ANALOG SIGNALS

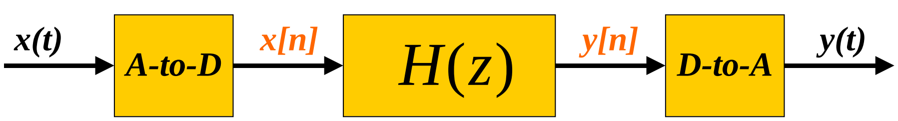
4-June-99

Info: Web-CT, Lab, HW

- Final Exam is Period 13 (8am, Fri)
- Review Session planned (ECE Aud ?)
  - Thursday 6–7:30 PM
- Prob Set #7 is due Today
- All Labs due Today
- Reading: Ch. 8, except for 2nd–order

LECTURE OBJECTIVES

- THREE–DOMAIN APPROACH
  - EXHIBIT BANDPASS FILTERS
- RE–UNIFICATION:
  - How does Frequency Response affect x(t) to produce y(t) ?



THREE DOMAINS

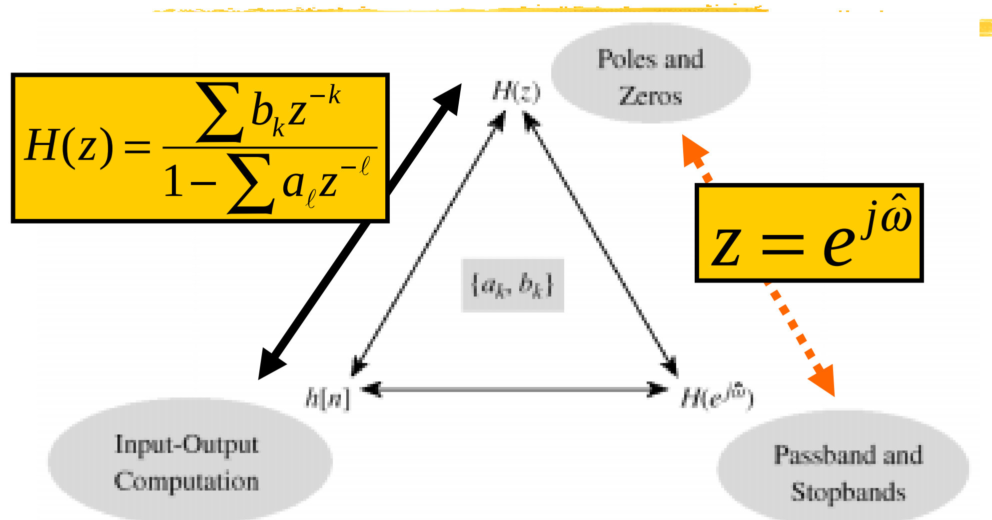
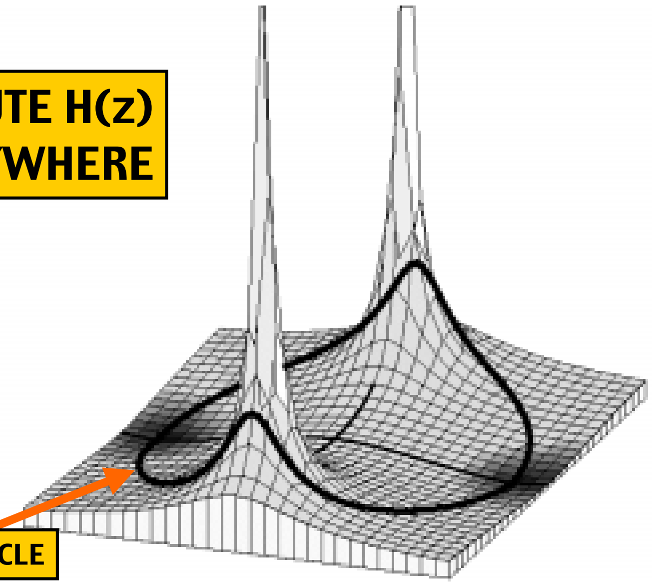


Figure 8.13 Relationship among the n-, z-, and $\hat{\omega}$-domains. The filter coefficients $\{a_k, b_k\}$ play a central role.

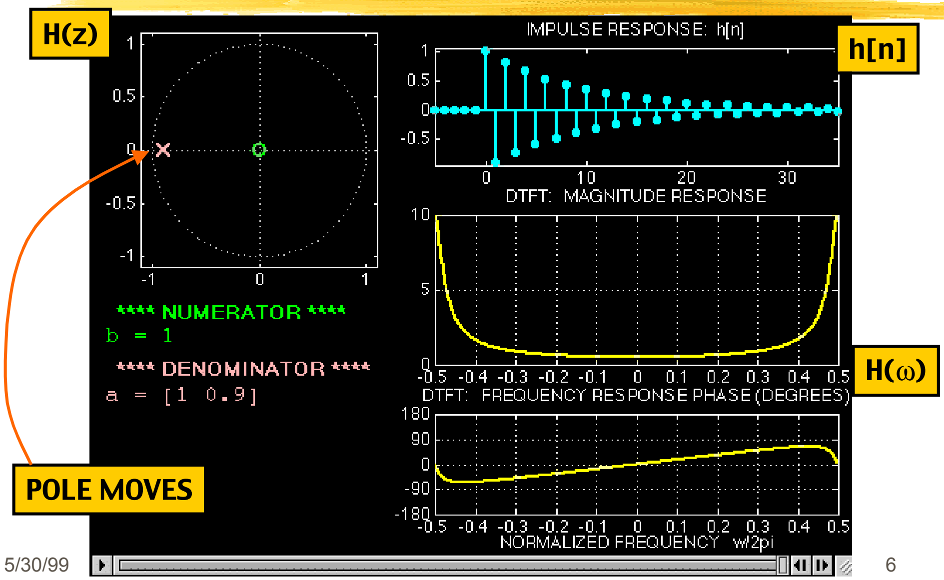
3-D VIEW

EVALUTE H(z) EVERYWHERE



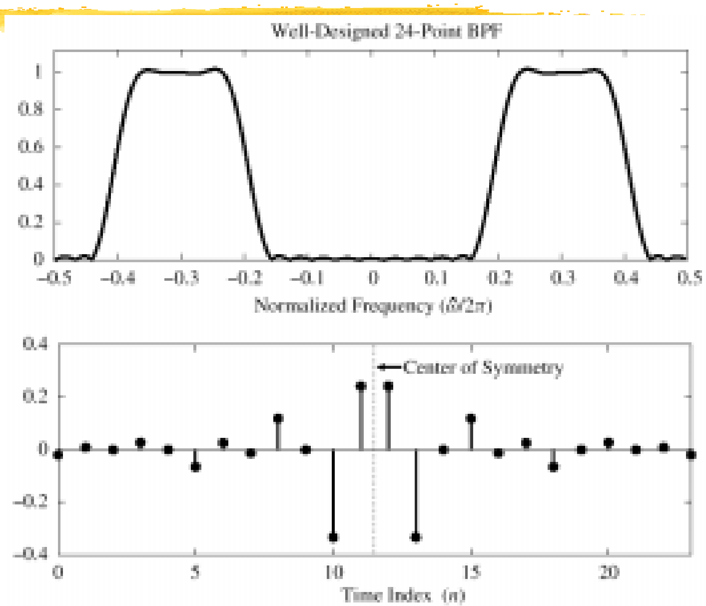
The poles are at $z = 0.85e^{\pm j\pi/2}$ and the zeros at $z = \pm 1$.

3 DOMAINS MOVIE: IIR

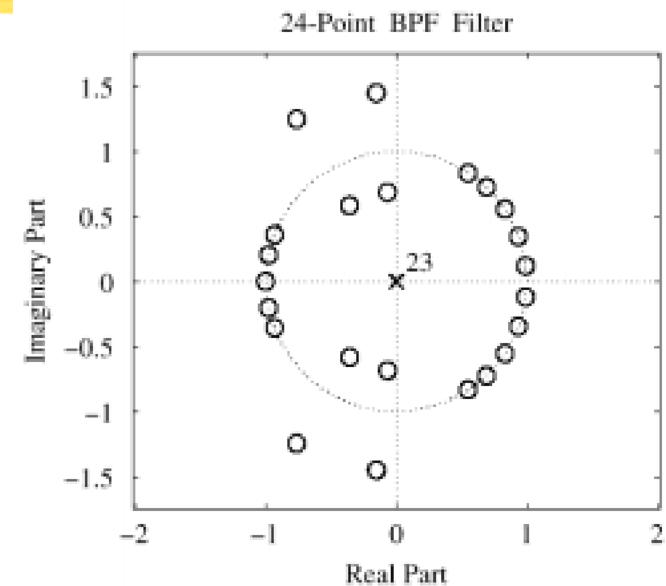


REALISTIC FIR BANDPASS

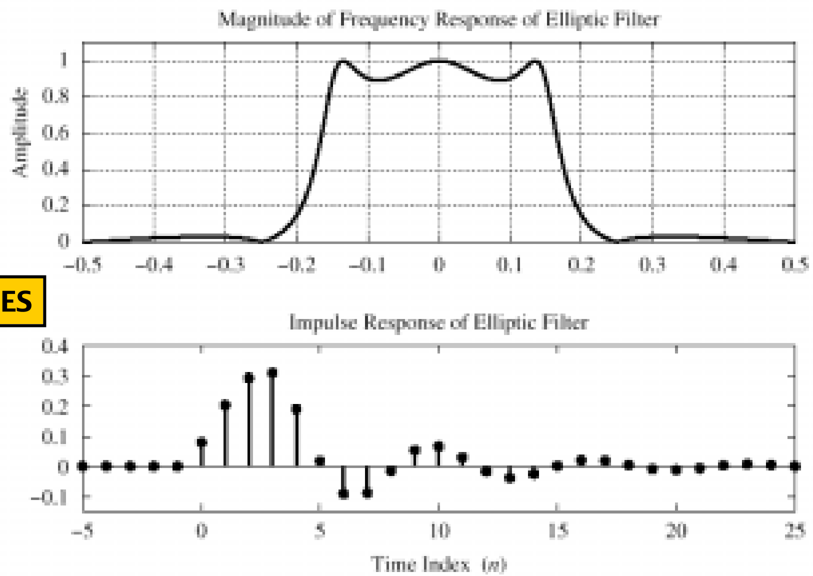
- FIR
- L = 24
- M = 23
- 23 zeros



FIR BPF: 23 ZEROS

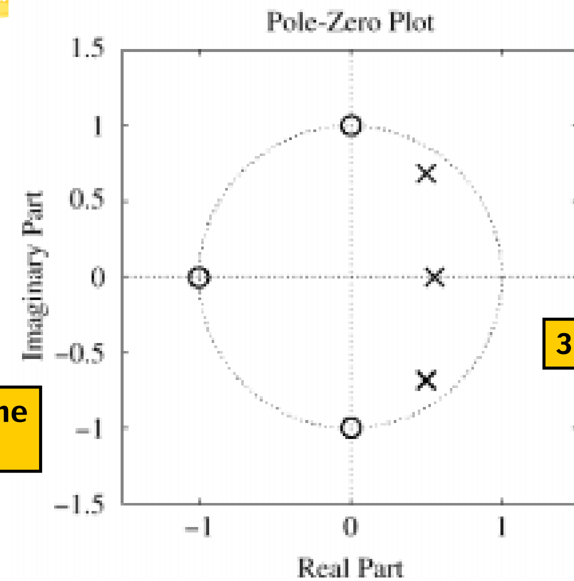


IIR Elliptic LPF (N=3)



3 POLES

POLES & ZEROS of IIR

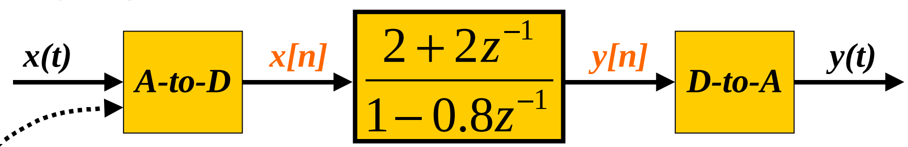


Zeros on the Unit Circle

3 POLES

POP QUIZ

- Given:



- Find the output, y(t)
  - When $x(t) = \cos(2000\pi t)$

$f_s = 5000\,\text{Hz}$

POP QUIZ BECOMES

- Given: $H(z) = \frac{2+2z^{-1}}{1-0.8z^{-1}}$
- Find the output, y[n]
  - When $x[n] = \cos(0.4\pi n)$
  - Because $\omega T_s = 2000\pi / 5000 = 0.4\pi$

NO Aliasing

SINUSOIDAL RESPONSE

- x[n] = SINUSOID => y[n] is SINUSOID
- Get MAGNITUDE & PHASE from H(z)

$$\text{if } x[n] = e^{j\hat{\omega}n}, \text{ then}$$

$$y[n] = \mathcal{H}(\hat{\omega})e^{j\hat{\omega}n}$$

$$\mathcal{H}(\hat{\omega}) = H(e^{j\hat{\omega}}) = H(z)\Big|_{z=e^{j\hat{\omega}}}$$

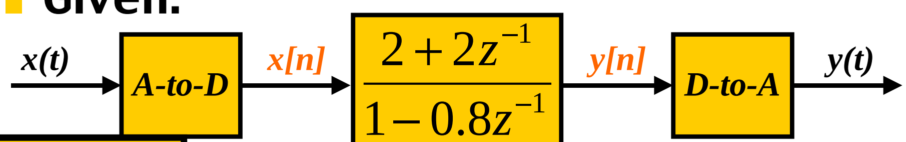
POP QUIZ INSIDE ANSWER

- Given: $H(z) = \dfrac{2+2z^{-1}}{1-0.8z^{-1}}$
- The input: $x[n] = \cos(0.4\pi n)$
- Then y[n] $y[n] = M\cos(0.4\pi n + \psi)$

$$H(e^{j0.4\pi}) = \frac{2+2e^{-j0.4\pi}}{1-0.8e^{-j0.4\pi}} = 3.02e^{-j0.452\pi}$$

POP QUIZ ANSWER

- Given:



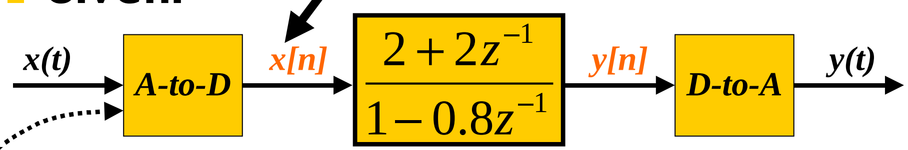
$f_s = 5000\,\text{Hz}$

- When $x(t) = \cos(2000\pi t)$
- The output is

$$y(t) = 3.02\cos(2000\pi t - 0.452\pi)$$

ANOTHER POP QUIZ

- Given: $\hat{\omega} = ?$



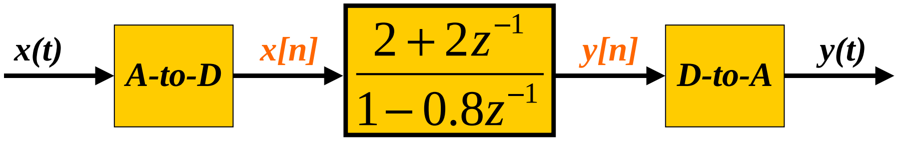
- Find the output, y(t)
  - When $x(t) = \cos(2\pi(7500)t)$

$f_s = 5000\,\text{Hz}$

$$\hat{\omega} = ?$$

2nd POP QUIZ ANSWER

- **Given:**

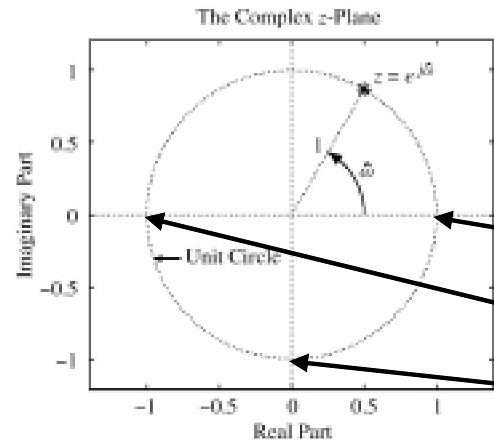


- **When** $x(t)=\cos(2\pi(7500)t)$

$f_s = 5000\,\text{Hz} \rightarrow \hat{\omega} = 3\pi \rightarrow y(t) = 0$

UNIT CIRCLE

- **MAPPING BETWEEN** z **and** $\hat{\omega}$



$$z = e^{j\hat{\omega}}$$

$z = 1 \leftrightarrow \hat{\omega} = 0$

$z = -1 \leftrightarrow \hat{\omega} = \pm\pi$

$z = \pm j \leftrightarrow \hat{\omega} = \pm\frac{1}{2}\pi$

IMPORTANT CONCEPTS

- **ALL Signals have Frequency Content**
  - **Sum of Sinusoids**
  - **Complex Exponentials**
- **FILTERS alter the Frequency Content**
  - **Image Processing Example: Blur**
  - **Linear Time-Invariant Processing**
- **3 Domains for Analysis**

THE FUTURE

- **Circuits & Laplace Transforms**