EE-2200 Spring-99

Lecture 7

Sampling & Aliasing

23-April-99

Information

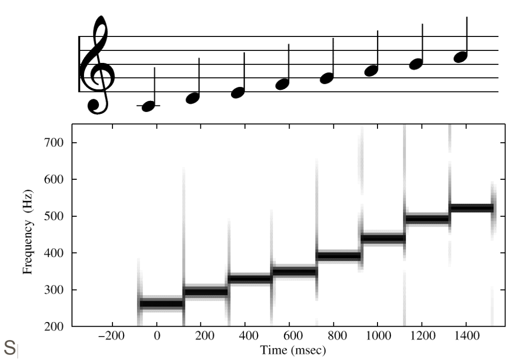
- Check the Bulletin Board for msgs
- Problem Set #3 due today
- Quiz #1 on 26–April (Monday)
  - Calculator; One page hand–written notes
  - Will include Fourier Series
  - Quiz Review on Sunday @ 7PM in ECE Auditorium

READING ASSIGNMENTS

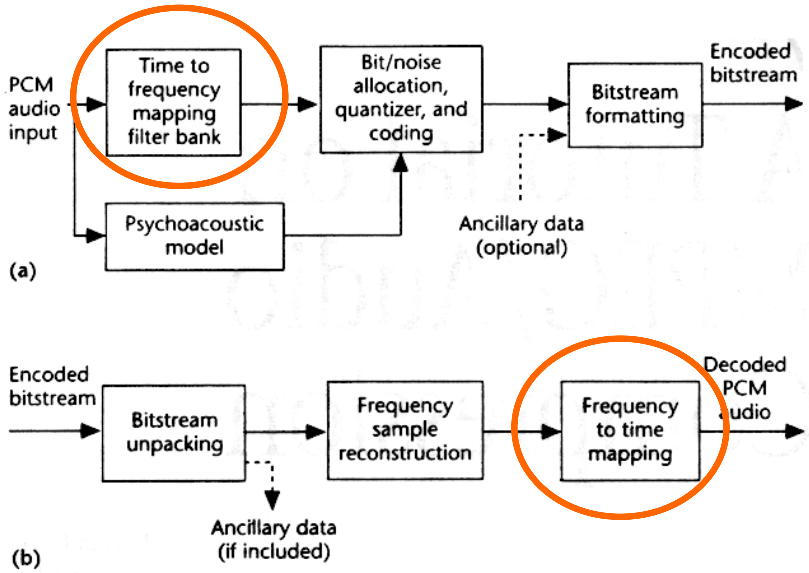
- This Lecture:
  - Chapter 4, pp. 83–94
- Other Reading:
  - Recitation: Chapter 4, pp. 90–100
    - Strobe Demo
  - Next Lecture: Chap. 4, pp. 100–111

CD-ROM DEMOS

- USE THE DEMOS
- Chapter 3: Spectrum
  - DEMOS of SPECTROGRAM
  - BEAT NOTES/AM
  - SPEECH
  - MUSIC



MP-3 Block Diagram



LECTURE OBJECTIVES

- SAMPLING can cause ALIASING
  - Sampling Theorem
  - Sampling Rate > 2(Highest Frequency)
- Spectrum for digital signals, x[n]
  - Normalized Frequency

$$\hat{\omega} = \omega T_s = \frac{2\pi f}{f_s} + 2\pi \ell$$

ALIASING

SYSTEMS Process Signals

- PROCESSING GOALS:
  - Change x(t) into y(t)
    - For example, more BASS
  - Improve x(t), e.g., image deblurring
  - Extract Information from x(t)

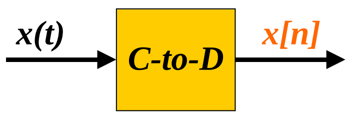
System IMPLEMENTATION

- ANALOG/ELECTRONIC:
  - Circuits: resistors, capacitors, op-amps

- DIGITAL/MICROPROCESSOR
  - Convert x(t) to numbers stored in memory

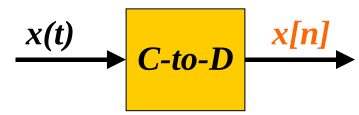
SAMPLING x(t)

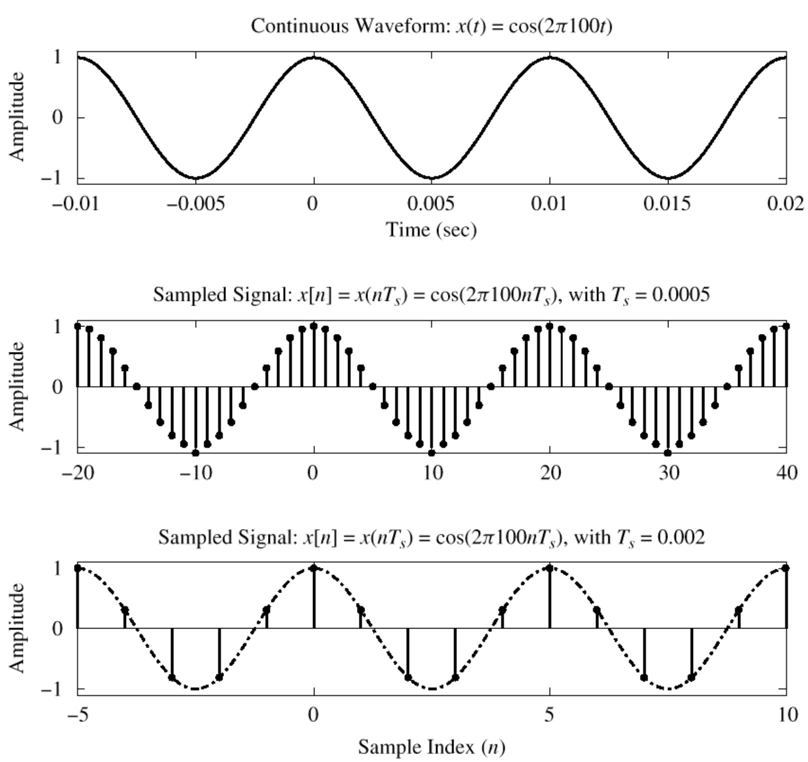
- SAMPLING PROCESS
  - Convert x(t) to numbers x[n]
  - “n” is an integer; x[n] is a sequence
  - “n” is the storage address in memory
- UNIFORM SAMPLING at $t = nT_s$
  - IDEAL: $x[n] = x(nT_s)$



SAMPLING RATE

- SAMPLING RATE (f_s)
  - $1/T_s$ = NUMBER of SAMPLES PER SECOND
  - 125 microsec ---> 8000 samples/sec
    - UNITS ARE HERTZ: 8000 Hz
- UNIFORM SAMPLING at $t = nT_s = n/f_s$
  - IDEAL: $x[n] = x(nT_s)=x(n/f_s)$





SAMPLING A SINUSOID

- HOW OFTEN ?
  - DEPENDS on FREQUENCY of SINUSOID
  - ANSWERED by SHANNON/NYQUIST
  - DEPENDS on “RECONSTRUCTION”

Shannon Sampling Theorem

A continuous-time signal $x(t)$ with frequencies no higher than $f_{\max}$ can be reconstructed exactly from its samples $x[n] = x(nT_s)$, if the samples are taken at a rate $f_s = 1/T_s$ that is greater than $2f_{\max}$.

STORING DIGITAL SOUND

- $x[n]$ is a SAMPLED SINUSOID
  - A list of numbers stored in memory
- CD rate is 44,100 samples per second
- 16–bit samples
- Stereo uses 2 channels
- Number of bytes for 1 minute is
  - 2 X (16/8) X 60 X 44100 = 10.584 Mbytes

DISCRETE-TIME SINUSOID

- Change x(t) into x[n]

$$x(t) = A\cos(\omega t + \varphi)$$

$$x[n] = x(nT_s) = A\cos(\omega n T_s + \varphi)$$

$$x[n] = A\cos((\omega T_s)n + \varphi)$$

$$x[n] = A\cos(\hat{\omega} n + \varphi)$$

$$\hat{\omega} = \omega T_s$$

DIGITAL FREQUENCY

DIGITAL FREQUENCY $\hat{\omega}$

- DIGITAL FREQUENCY is NORMALIZED
- UNITS are radians, not rad/sec
- $\hat{\omega}$ GOES from 0 to 2π, as f goes from 0 to the sampling frequency

$$\hat{\omega} = \omega T_s = \frac{2\pi f}{f_s}$$

ALIASING DERIVATION

- Other Frequencies give the same $\hat{\omega}$

If $x(t) = A\cos(2\pi(f + \ell f_s)t + \varphi)$

and we want : $x[n] = A\cos(\hat{\omega} n + \varphi)$

then : $\hat{\omega} = \dfrac{2\pi(f + \ell f_s)}{f_s} = \dfrac{2\pi f}{f_s} + \dfrac{2\pi \ell f_s}{f_s}$

$$\hat{\omega} = \omega T_s = \frac{2\pi f}{f_s} + 2\pi\ell$$

ALIASING CONCLUSIONS

- ADDING f_s or $2f_s$ TO THE FREQ of x(t) gives exactly the same x[n]
- x[n] = x(n/ f_s) HAS THE SAME VALUES

- GIVEN x[n], WE CAN'T DSITINGUISH f_o FROM ($f_o + f_s$) or ($f_o + 2f_s$)
- CALLED ALIASING

NORMALIZED FREQUENCY

- DIGITAL FREQUENCY

Normalized Radian Frequency

$$\hat{\omega} = \omega T_s = \frac{2\pi f}{f_s} + 2\pi\ell$$

Normalized Cyclic Frequency

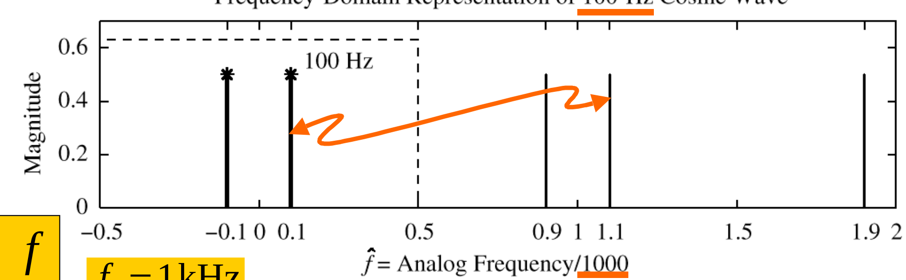
$$\hat{f} = \hat{\omega}/(2\pi) = f T_s = f/f_s$$

SPECTRUM for x[n]

- PLOT versus NORMALIZED FREQUENCY
- INCLUDE ALL SPECTRUM LINES
  - ALIASES
    - MULTIPLES of 2π
  - FOLDED ALIASES
    - (discussed later)
    - ALIASES of NEGATIVE FREQS

SPECTRUM (DIGITAL)

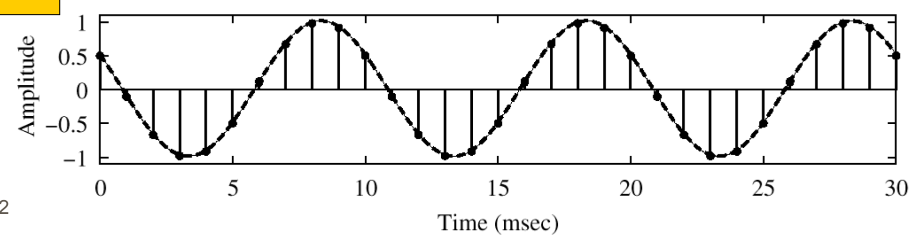
Frequency-Domain Representation of 100-Hz Cosine Wave



$$\hat{f} = \frac{f}{f_s}$$

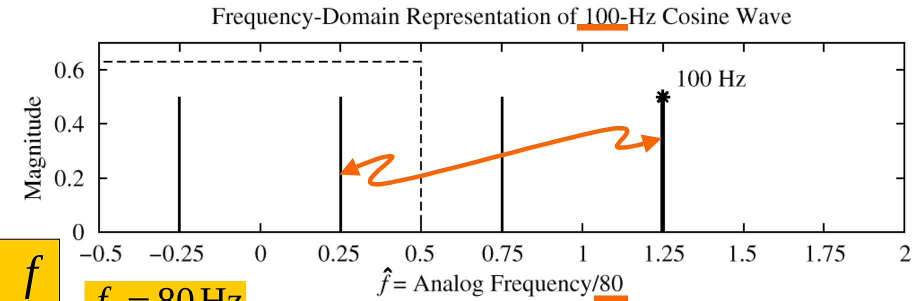
$f_s = 1\text{kHz}$

100-Hz Cosine Wave: Sampled with T_s = 1 msec (1000 Hz)



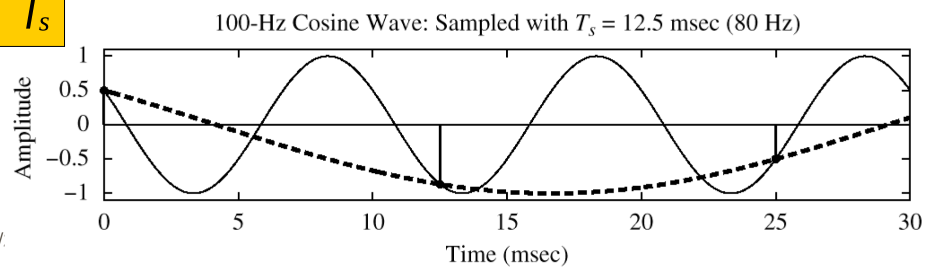
SPECTRUM of x[n]

ALIASING CASE



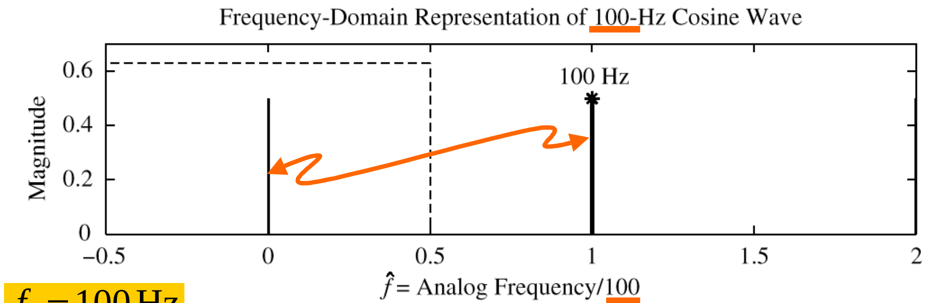
$$\hat{f} = \frac{f}{f_s}$$

$f_s = 80\,\text{Hz}$

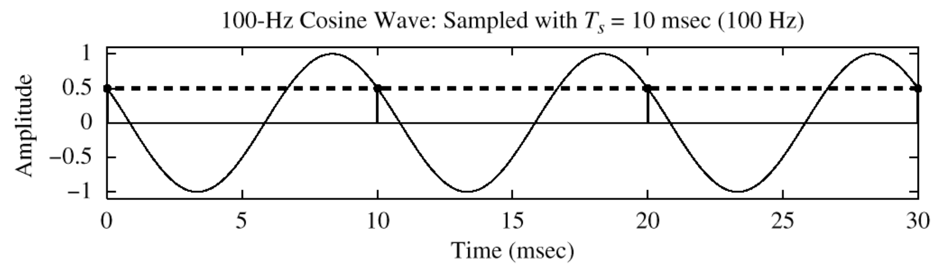


SPECTRUM of x[n]

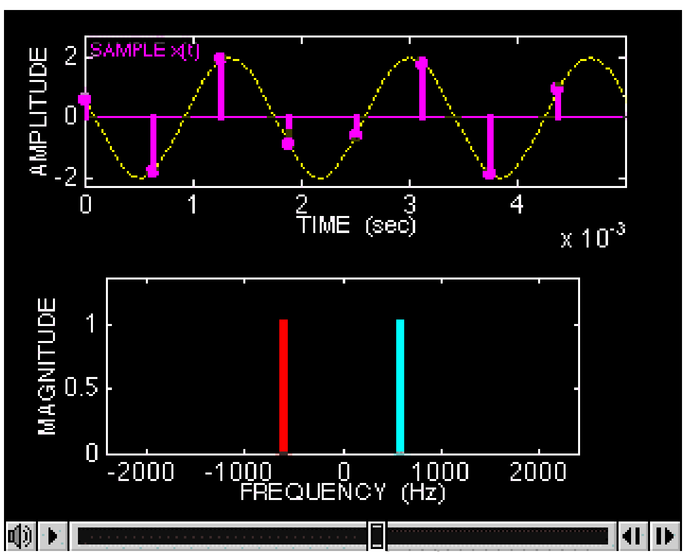
ALIASING to ZERO FREQ



$f_s = 100\,\text{Hz}$



SAMPLING DEMO (Chap. 4)



FOLDING DERIVATION

- Negative Frequencies give the same $\hat{\omega}$

$$x(t) = A\cos(2\pi(-f + \ell f_s)t - \varphi)$$

$$x[n] = x(nT_s) = A\cos(2\pi(-f + \ell f_s)nT_s - \varphi)$$

$$x[n] = A\cos((-2\pi f T_s)n + (2\pi \ell f_s T_s)n - \varphi)$$

$$x[n] = A\cos((2\pi f T_s)n - 2\pi \ell n + \varphi)$$

$\cos(-\theta) = \cos\theta$

$$x[n] = A\cos(\hat{\omega} n + \varphi)$$ ← **SAME DIGITAL SIGNAL**

FOLDING (a type of ALIASING)

- ANOTHER x[n] THAT IS IDENTICAL
- CAN'T TELL f_o FROM $(f_s - f_o)$
  - Or, $(2f_s - f_o)$ or, $(3f_s - f_o)$
- EXAMPLE:
  - y(t) has 1000 Hz component
  - SAMPLING FREQ = 1500 Hz
  - WHAT is the "FOLDED" ALIAS ?

DIGITAL FREQ $\hat{\omega}$ AGAIN

Normalized Radian Frequency

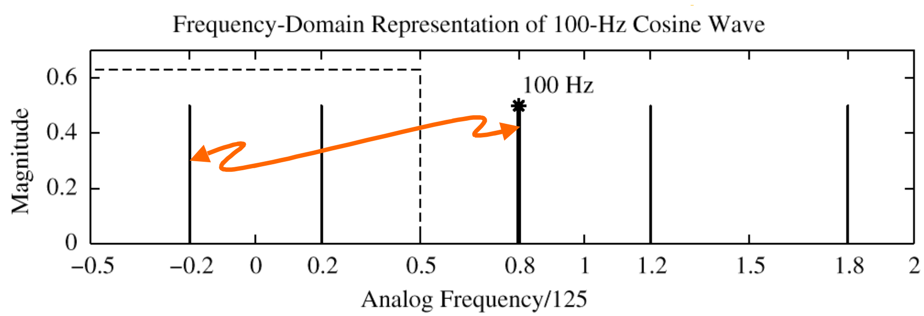
$$\hat{\omega} = \omega T_s = \frac{2\pi f}{f_s} + 2\pi\ell$$

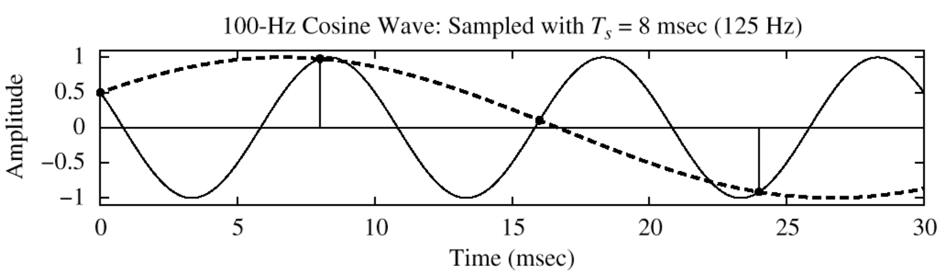
ALIASING

$$\hat{\omega} = \omega T_s = -\frac{2\pi f}{f_s} + 2\pi\ell$$

FOLDED ALIAS

SPECTRUM of x[n] FOLDING CASE





FOLDING DIAGRAM

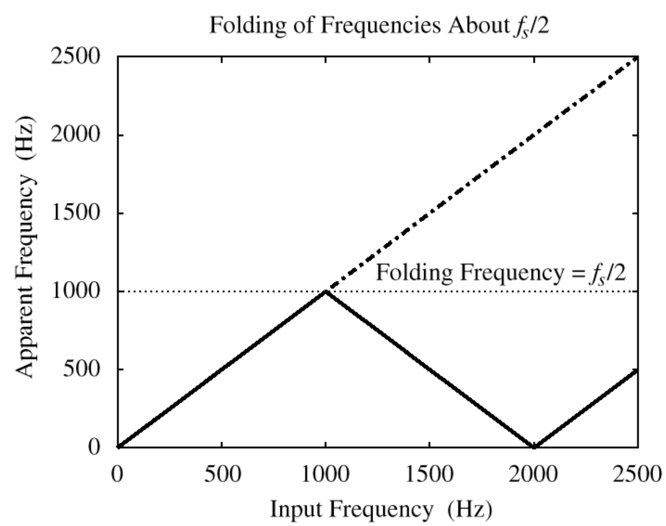


Figure 4.4 Folding of a sinusoid sampled at $f_s = 2000$ samples/sec. The apparent frequency is the lowest frequency of a sinusoid that has exactly the same samples as the input sinusoid.