

EE-2200

Spring-99

Lecture 9

FIR Filtering Intro

3-May-99

Information

- Music Listening this Thursday
 - ! Lab Quiz next week !!!!!!!
- Problem Set #4 due Friday
 - ! On-Line HW due on Wed at Noon.
- MATLAB help: Wed @ 6pm, VL-456

- Quiz #2 on 24-May (Monday)

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READING ASSIGNMENTS

- This Lecture:
 - ! Chapter 5, pp. 119–131

- Other Reading:
 - ! Recitation: Ch. 5, pp. 127–133, 142–146
 - ! CONVOLUTION
 - ! Next Lecture: Chapter 5, pp. 133–152

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LECTURE OBJECTIVES

- INTRODUCE FILTERING IDEA
 - ! Weighted Average
 - ! Running Average

- FINITE IMPULSE RESPONSE FILTERS
 - ! FIR Filters
 - ! Show how to compute the output $y[n]$ from the input signal, $x[n]$

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DIGITAL FILTERING



- CONCENTRATE on the COMPUTER
 - PROCESSING ALGORITHMS
 - SOFTWARE (MATLAB)
 - HARDWARE: DSP chips, VLSI
- DSP: DIGITAL SIGNAL PROCESSING

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The TMS32010, 1983



First PC plug-in board from Atlanta Signal Processors Inc.

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Rockland Digital Filter, 1971

**Model 4136
PROGRAMMABLE
DIGITAL
FILTER**

Variable-Order Digital Filter for Realizing All Classical Designs

The Rockland Model 4136 Programmable Digital Filter consists of a second-order digital filter section which is multiplexed four ways to achieve eighth-order filtering. Each of the four sections has fully-programmable coefficients which are stored internally in a read-write memory.

Filter input and output words are in 16-bit parallel form at a maximum sampling rate of 80 KHz while internal computations are made with 24-bit accuracy.

TRANSFER FUNCTION
The transfer function from filter input to filter output in z-transform notation is given by

$$H_n(z) = \prod_{n=1}^N \frac{K_n(1+z^{-1}A1_n+z^{-2}B2_n)}{1-z^{-1}B1_n-z^{-2}B2_n} \quad (1)$$

where $N=0,1,2,3,4$ is one-half the filter order se-

For the price of a small house, you could have one of these.

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DISCRETE-TIME SYSTEM



- OPERATE on $x[n]$ to get $y[n]$
- WANT GENERAL CLASS of SYSTEMS
 - ANALYZE the SYSTEM
 - TOOLS: TIME-DOMAIN & FREQUENCY-DOMAIN
 - SYNTHESIZE the SYSTEM

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D-T SYSTEM EXAMPLES



EXAMPLES:

POINTWISE OPERATORS

SQUARING: $y[n] = (x[n])^2$

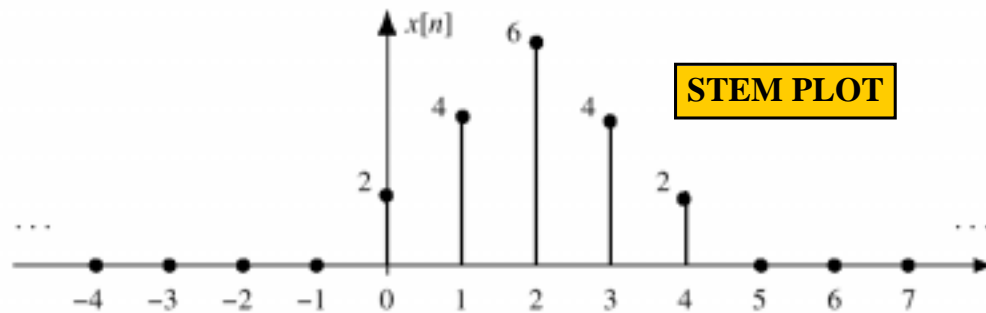
RUNNING AVERAGE

RULE: “the output at time n is the average of three consecutive input values”

DISCRETE-TIME SIGNAL

$x[n]$ is a LIST of NUMBERS

INDEXED by “ n ”



3-PT AVERAGE SYSTEM

ADD 3 CONSECUTIVE NUMBERS

Do this for each “ n ”

the following input–output equation

Make a TABLE

$$y[n] = \frac{1}{3}(x[n] + x[n + 1] + x[n + 2])$$

n	$n < -2$	-2	-1	0	1	2	3	4	5	$n > 5$
$x[n]$	0	0	0	2	4	6	4	2	0	0
$y[n]$	0	$\frac{2}{3}$	2	4	$\frac{14}{3}$	4	2	$\frac{2}{3}$	0	0

$n=0$ $y[0] = \frac{1}{3}(x[0] + x[1] + x[2])$

$n=1$ $y[1] = \frac{1}{3}(x[1] + x[2] + x[3])$

INPUT SIGNAL

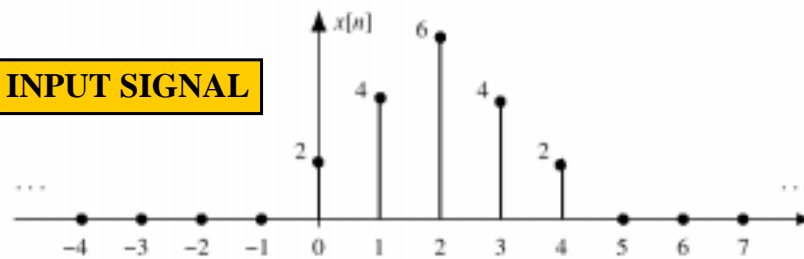


Figure 5.2 Finite-length input signal, $x[n]$.

$$y[n] = \frac{1}{3}(x[n] + x[n + 1] + x[n + 2])$$

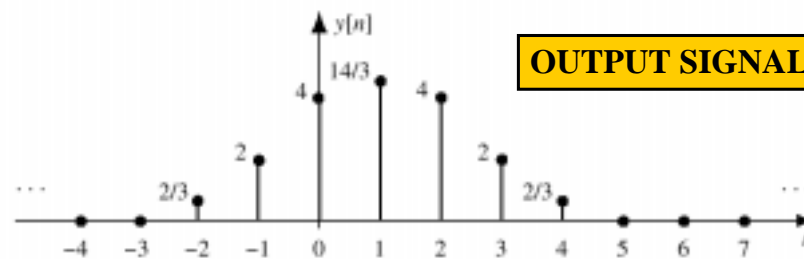


Figure 5.3 Output of running average, $y[n]$.

PAST, PRESENT, FUTURE

Sec. 5.2 The Running Average Filter 123

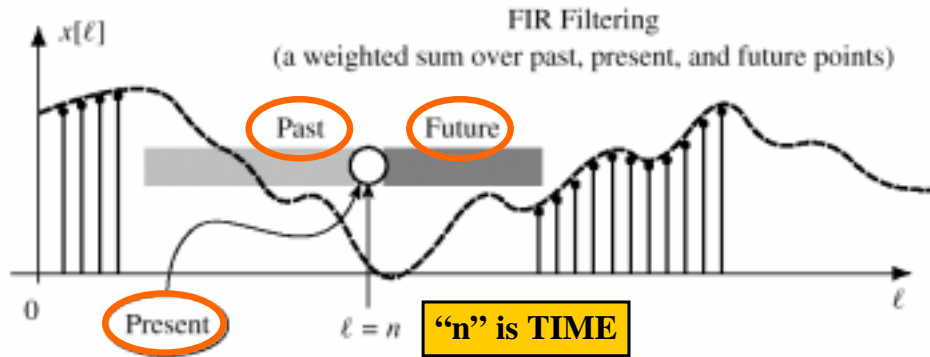


Figure 5.4 The running-average filter calculation at time index n uses values within a sliding window (shaded). Dark shading indicates the future ($\ell > n$); light shading, the past ($\ell < n$).

ANOTHER 3-pt AVERAGER

- Uses "PAST" VALUES of $x[n]$
- IMPORTANT IF "n" represents REAL TIME
- WHEN $x[n]$ & $y[n]$ ARE STREAMS

$$y[n] = \frac{1}{3}(x[n] + x[n-1] + x[n-2])$$

n	$n < -2$	-2	-1	0	1	2	3	4	5	6	7	$n > 7$
$x[n]$	0	0	0	2	4	6	4	2	0	0	0	0
$y[n]$	0	0	0	$\frac{2}{3}$	2	4	$\frac{14}{3}$	4	2	$\frac{2}{3}$	0	0

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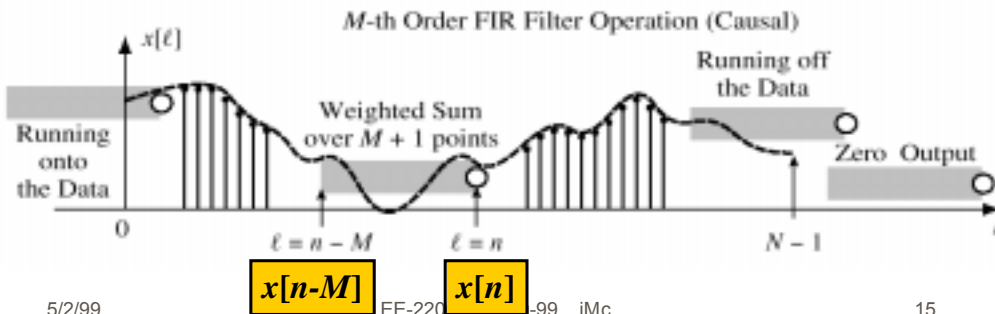
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GENERAL FIR FILTER

- SLIDE a WINDOW across $x[n]$

$$y[n] = \sum_{k=0}^M b_k x[n-k]$$



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GENERAL FIR FILTER

- FILTER COEFFICIENTS $\{b_k\}$

- DEFINE THE FILTER

$$y[n] = \sum_{k=0}^M b_k x[n-k]$$

- For example, $\{b_k\} = \{3, -1, 2, 1\}$

$$y[n] = \sum_{k=0}^3 b_k x[n-k]$$

$$= 3x[n] - x[n-1] + 2x[n-2] + x[n-3]$$

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GENERAL FIR FILTER

FILTER COEFFICIENTS $\{b_k\}$

$$y[n] = \sum_{k=0}^M b_k x[n - k]$$

FILTER ORDER is M

FILTER LENGTH is $L = M+1$

NUMBER of FILTER COEFFS is L

FILTERING EXAMPLE

7-point AVERAGER $y_7[n] = \frac{1}{7} \left(\sum_{k=0}^6 x[n - k] \right)$

Removes cosine

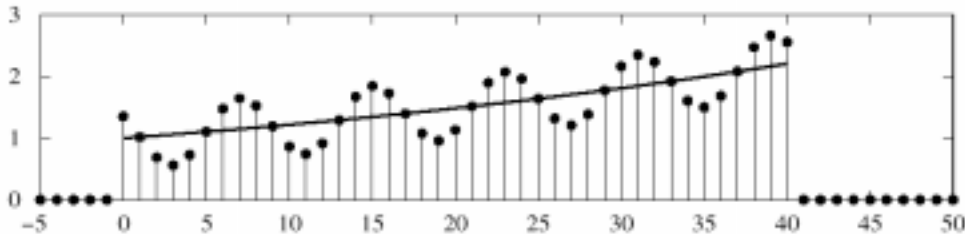
By making its amplitude (A) smaller

3-point AVERAGER $y_3[n] = \frac{1}{3} \left(\sum_{k=0}^2 x[n - k] \right)$

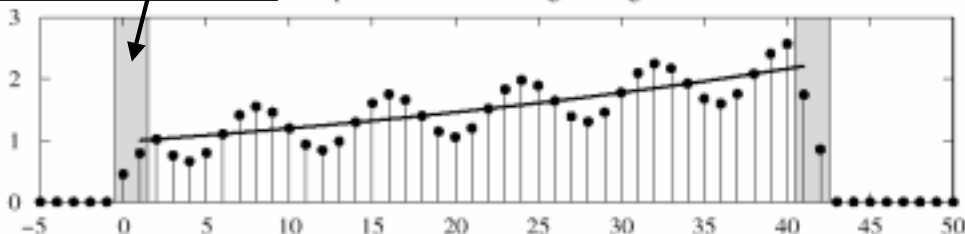
Changes A slightly

3-pt AVG EXAMPLE

Input Signal: $x[n] = (1.02)^n + \cos(2\pi n/8 + \pi/4)$ for $0 \leq n \leq 40$

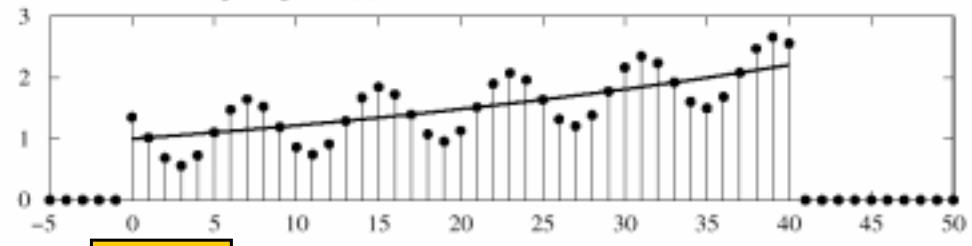


Output of 3-Point Running-Average Filter



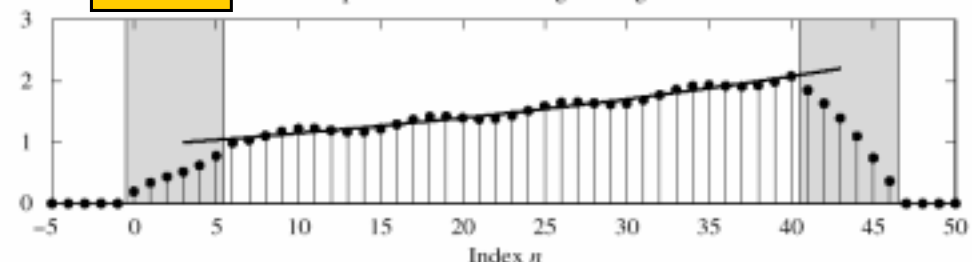
7-pt AVG EXAMPLE

Input Signal: $x[n] = (1.02)^n + \cos(2\pi n/8 + \pi/4)$ for $0 \leq n \leq 40$



CAUSAL

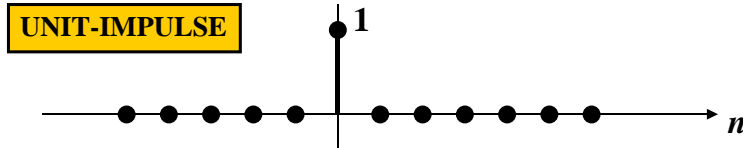
Output of 7-Point Running-Average Filter



SPECIAL INPUT SIGNALS

- $x[n] = \text{SINUSOID}$
- $x[n]$ has only one **NON-ZERO VALUE**

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$



UNIT IMPULSE SIGNAL $\delta[n]$

n	...	-2	-1	0	1	2	3	4	5	6	...
$\delta[n]$	0	0	0	1	0	0	0	0	0	0	0
$\delta[n-3]$	0	0	0	0	0	0	1	0	0	0	0

NON-ZERO
When its argument
is equal to ZERO

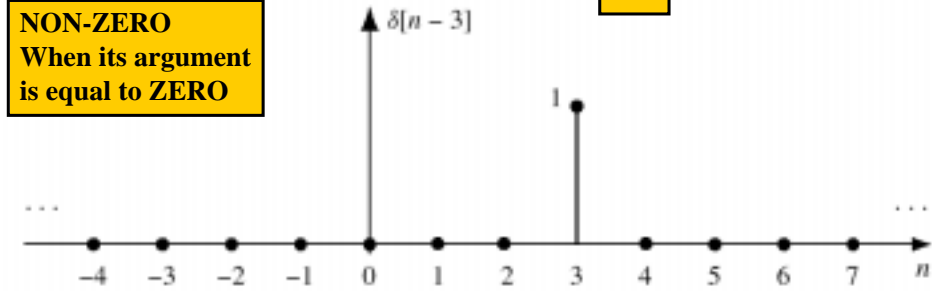
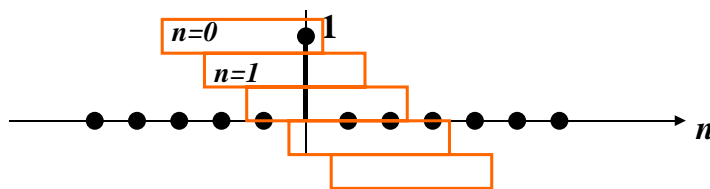


Figure 5.7 Shifted impulse sequence, $\delta[n-3]$.

4-pt AVERAGER

- **CAUSAL SYSTEM: USE PAST VALUES**
- **INPUT = UNIT IMPULSE SIGNAL**
- **OUTPUT is called "IMPULSE RESPONSE"**



4-pt Avg Impulse Response

- $y[n] = 0.25(x[n]+x[n-1]+x[n-2]+x[n-3])$
- **"READS OUT" the FILTER COEFFICIENTS**
- $y[n] = \{..., 0, 0, 0, 0.25, 0.25, 0.25, 0.25, 0, 0, \dots\}$

