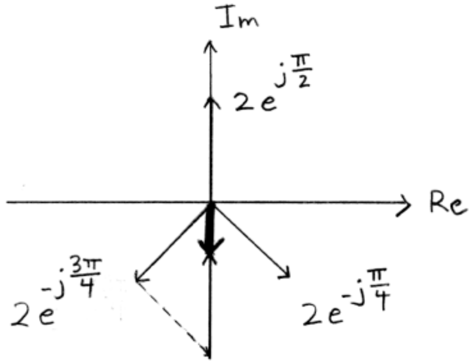
Problem 2.1

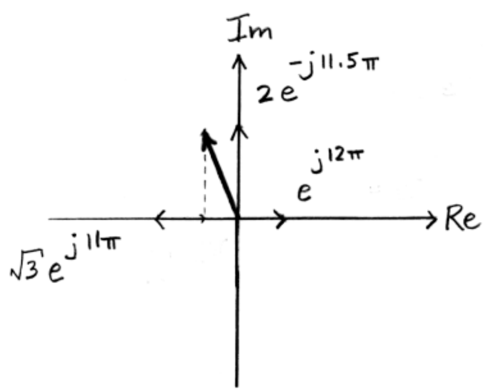
(a)



$$Ae^{j\phi} = 2e^{-j\frac{3\pi}{4}} + 2e^{-j\frac{\pi}{4}} + 2e^{j\frac{\pi}{2}}$$
$$= (-\sqrt{2} - j\sqrt{2}) + (\sqrt{2} - j\sqrt{2}) + (0 + j2)$$
$$= 0 - j(2\sqrt{2} - 2)$$
$$= (2\sqrt{2} - 2)e^{-j\frac{\pi}{2}}$$

$$\therefore \quad x_a(t) = (2\sqrt{2} - 2)\cos(40\pi t - \tfrac{\pi}{2})$$

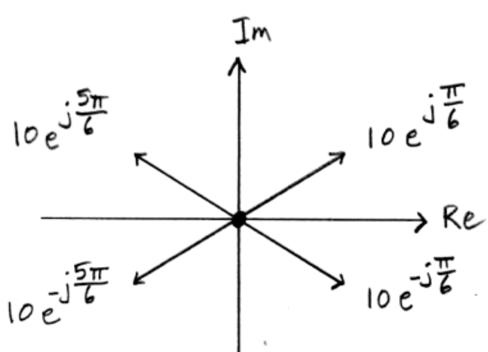
(b)



$$Ae^{j\phi} = \sqrt{3}e^{j11\pi} + 2e^{-j11.5\pi} + e^{j12\pi}$$
$$= \sqrt{3}e^{j\pi} + 2e^{j\frac{\pi}{2}} + e^{j0}$$
$$= (-\sqrt{3} + j0) + (0 + j2) + (1 + j0)$$
$$= -(\sqrt{3} - 1) + j2$$
$$= 2.1298\, e^{j0.6117\pi}$$

$$\therefore \quad x_b(t) = 2.1298 \cos(50\pi t + 0.6117\pi)$$

(c)



$$Ae^{j\phi} = 10e^{j\frac{\pi}{6}} + 10e^{-j\frac{\pi}{6}} + 10e^{j\frac{5\pi}{6}} + 10e^{-j\frac{5\pi}{6}}$$
$$= 10\left(\tfrac{\sqrt{3}}{2} + j\tfrac{1}{2}\right) + 10\left(\tfrac{\sqrt{3}}{2} - j\tfrac{1}{2}\right) + 10\left(-\tfrac{\sqrt{3}}{2} + j\tfrac{1}{2}\right) + 10\left(-\tfrac{\sqrt{3}}{2} - j\tfrac{1}{2}\right)$$
$$= 0$$

$$\therefore \quad x_c(t) = 0$$

Problem 2.2

$$x(t) = 2\cos\left(20\pi t + \frac{\pi}{6}\right) + \underbrace{2\sqrt{3}\sin\left(20\pi\left(t - \frac{1}{120}\right)\right)}$$

$$= 2\sqrt{3}\sin\left(20\pi t - \frac{\pi}{6}\right)$$

$$= 2\sqrt{3}\cos\left(20\pi t - \frac{\pi}{6} - \frac{\pi}{2}\right)$$

(a)

$$z(t) = 2e^{j\frac{\pi}{6}}e^{j20\pi t} + 2\sqrt{3}\,e^{-j\frac{\pi}{6}}e^{-j\frac{\pi}{2}}e^{j20\pi t}$$

$$= \left(2e^{j\frac{\pi}{6}} + 2\sqrt{3}\,e^{-j\frac{2\pi}{3}}\right)e^{j20\pi t}$$

$$= \left(\sqrt{3} + j1 - \sqrt{3} - j3\right)e^{j20\pi t}$$

$$= -j2e^{j20\pi t}$$

$$= 2e^{-j\frac{\pi}{2}}e^{j20\pi t}$$

$$\therefore\ x(t) = \operatorname{Re}\{z(t)\} = 2\cos\left(20\pi t - \frac{\pi}{2}\right)$$

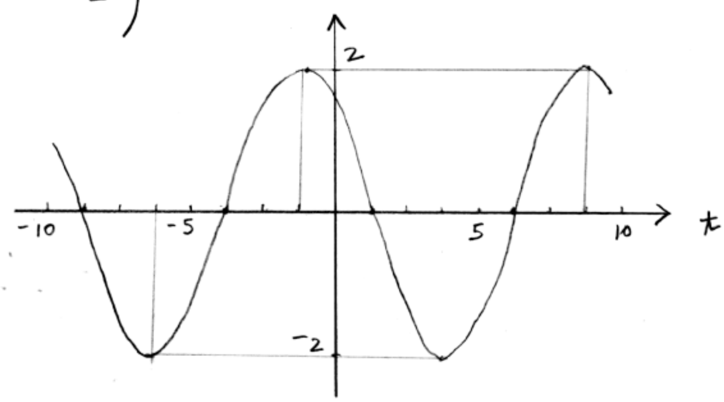
(b)

$$\operatorname{Re}\left\{\sqrt{2}(1+j)e^{j0.2\pi t}\right\} = \operatorname{Re}\left\{\sqrt{2}\cdot\sqrt{2}\,e^{j\frac{\pi}{4}}e^{j0.2\pi t}\right\}$$

$$= \operatorname{Re}\left\{2e^{j\frac{\pi}{4}}e^{j0.2\pi t}\right\}$$

$$= 2\cos\left(0.2\pi t + \frac{\pi}{4}\right), \quad T = \frac{2\pi}{0.2\pi} = 10$$

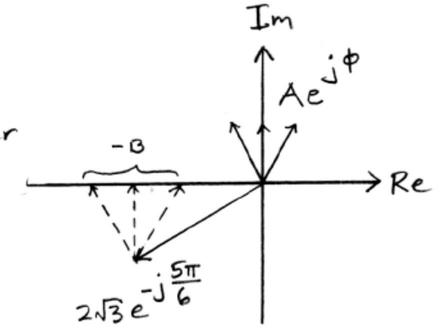
$$\left(\#\text{ periods} = \frac{20}{T} = 2\right)$$



Problem 2.3

$$x(t) = 2\sqrt{3}\cos\left(10\pi t - \frac{5\pi}{6}\right) + A\cos(10\pi t + \phi) = B\cos(10\pi t + \pi)$$

$$\Rightarrow \quad 2\sqrt{3}\, e^{-j\frac{5\pi}{6}} + A e^{j\phi} = B e^{j\pi} = \underbrace{-B}_{\text{negative real number}}$$



$$\Rightarrow \quad (-3 - j\sqrt{3}) + (A\cos\phi + jA\sin\phi) = -B$$

$$\Rightarrow \quad \begin{cases} -3 + A\cos\phi = -B & \text{(real part)} \\ -\sqrt{3} + A\sin\phi = 0 & \text{(imag part)} \end{cases}$$

(a) $\boxed{A\sin\phi = \sqrt{3}}$ $\boxed{A\cos\phi < 3}$

(b) $B = 4 \Rightarrow \begin{cases} A\cos\phi = -1 \\ A\sin\phi = \sqrt{3} \end{cases} \Rightarrow \boxed{\begin{array}{l} \phi = \frac{2\pi}{3} \\ A = 2 \end{array}}$

(c) A is minimized if $\phi = \frac{\pi}{2}$ (see figure)

$$\boxed{\begin{array}{l} \phi = \frac{\pi}{2} \\ A = \sqrt{3} \\ B = 3 \end{array}}$$

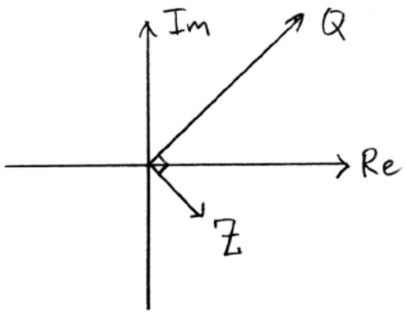
Problem 2.4

$$z(t) = Z e^{j\pi t}, \quad Z = e^{-j\frac{\pi}{4}}$$

(a) $$\frac{d}{dt} z(t) = \frac{d}{dt}\left(Z e^{j\pi t}\right) = Z(j\pi)e^{j\pi t} = Q e^{j\pi t}$$

where $$Q = Z \pi e^{j\pi/2} = \pi e^{j\pi/4}$$

(b)



Q leads Z by $\frac{\pi}{2}$ radians

(c)
$$\text{Re}\left\{\frac{d}{dt} z(t)\right\} = \text{Re}\left\{\pi e^{j\frac{\pi}{4}} e^{j\pi t}\right\} = \pi\cos\left(\pi t + \frac{\pi}{4}\right)$$

$$\frac{d}{dt}\text{Re}\{z(t)\} = \frac{d}{dt}\text{Re}\left\{e^{-j\frac{\pi}{4}} e^{j\pi t}\right\} = \frac{d}{dt}\cos\left(\pi t - \frac{\pi}{4}\right)$$
$$= -\pi\sin\left(\pi t - \frac{\pi}{4}\right)$$
$$= -\pi\cos\left(\pi t - \frac{3\pi}{4}\right)$$
$$= \pi\cos\left(\pi t + \frac{\pi}{4}\right)$$

agree

(d)
$$\int_{-\frac{1}{2}}^{\frac{1}{2}} z(t)\,dt = \int_{-\frac{1}{2}}^{\frac{1}{2}} e^{j(\pi t - \frac{\pi}{4})}\,dt$$
$$= \left.\frac{e^{j(\pi t - \frac{\pi}{4})}}{j\pi}\right|_{-\frac{1}{2}}^{\frac{1}{2}}$$
$$= \frac{1}{\pi} e^{-j\frac{\pi}{2}}\left(e^{j\frac{\pi}{4}} - e^{-j\frac{3\pi}{4}}\right) = \frac{2}{\pi} e^{-j\frac{\pi}{4}}$$

(e)
$$z(t - t_d) = e^{j(\pi(t - t_d) - \frac{\pi}{4})}$$
$$= e^{-j(\pi t_d + \frac{\pi}{4})} e^{j\pi t}$$
$$= Q e^{j\pi t}, \quad Q = e^{-j(\pi t_d + \frac{\pi}{4})}$$

Problem 2.5

$$x(t) = 2 + 2\cos\left(5000\pi t + \frac{3\pi}{7}\right) + 3\cos\left(7000\pi t - \frac{\pi}{5}\right)$$

(a)
$$x(t) = 2 + 2\left(\frac{e^{j(5000\pi t + \frac{3\pi}{7})} + e^{-j(5000\pi t + \frac{3\pi}{7})}}{2}\right) + 3\left(\frac{e^{j(7000\pi t - \frac{\pi}{5})} + e^{-j(7000\pi t - \frac{\pi}{5})}}{2}\right)$$

(b) frequencies present: $0, \pm 2500, \pm 3500$ Hz

(c)

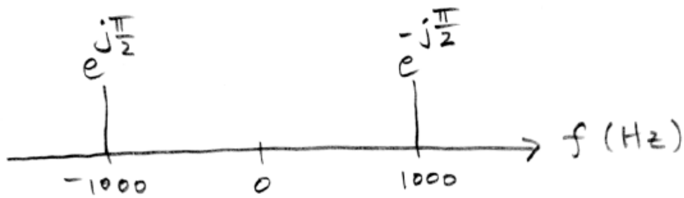
frequency (Hz)	complex amplitude
0	2
2500	$e^{j3\pi/7}$
-2500	$e^{-j3\pi/7}$
3500	$\frac{3}{2}e^{-j\pi/5}$
-3500	$\frac{3}{2}e^{j\pi/5}$

Problem 2.6

(a) $v(t) = 2\sin(2000\pi t)$

$$= 2\left(\frac{e^{j2000\pi t} - e^{-j2000\pi t}}{2j}\right)$$

$$= e^{-j\frac{\pi}{2}} e^{j2000\pi t} + e^{j\frac{\pi}{2}} e^{-j2000\pi t}$$



(b) and (c)

$$\cos(\omega_1 t)\sin(\omega_2 t) = \left(\frac{e^{j\omega_1 t} + e^{-j\omega_1 t}}{2}\right)\left(\frac{e^{j\omega_2 t} - e^{-j\omega_2 t}}{2j}\right)$$

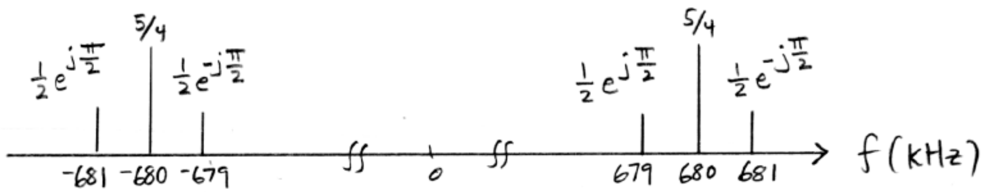
$$= \frac{1}{4}e^{-j\frac{\pi}{2}} e^{j(\omega_1+\omega_2)t} + \frac{1}{4}e^{j\frac{\pi}{2}} e^{-j(\omega_1+\omega_2)t}$$

$$+ \frac{1}{4}e^{j\frac{\pi}{2}} e^{j(\omega_1-\omega_2)t} + \frac{1}{4}e^{-j\frac{\pi}{2}} e^{-j(\omega_1-\omega_2)t}$$

$$\Rightarrow x(t) = 2\cos(\omega_1 t)\sin(\omega_2 t) + 2.5\cos(\omega_1 t)$$

$$\left.\begin{aligned} &= \frac{1}{2}e^{-j\frac{\pi}{2}} e^{j(\omega_1+\omega_2)t} + \frac{1}{2}e^{j\frac{\pi}{2}} e^{-j(\omega_1+\omega_2)t} \\ &\quad + \frac{1}{2}e^{j\frac{\pi}{2}} e^{j(\omega_1-\omega_2)t} + \frac{1}{2}e^{-j\frac{\pi}{2}} e^{-j(\omega_1-\omega_2)t} \\ &\quad + \frac{5}{4}e^{j\omega_1 t} + \frac{5}{4}e^{-j\omega_1 t} \end{aligned}\right\} \text{complex amplitudes not influenced by } \omega_1 \text{ and } \omega_2$$

(b)



(c)