HOMEWORK #3
SOLUTIONS

ECE 2025
FALL 2001

3.1) a) The frequency of the DC component is zero (by definition).

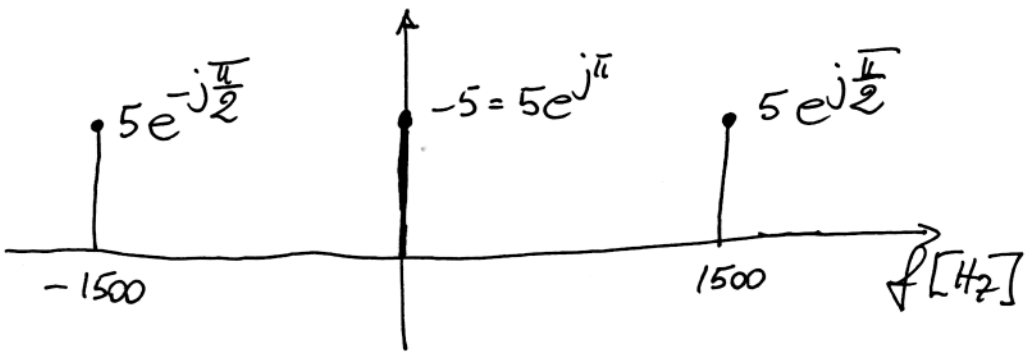
The sinusoid has 3 periods in a 2-ms interval, so:

$$f = \frac{3}{2\times10^{-3}} = 1500 \text{ Hz}$$

b) $x(t) = -5 + 10\cos\left(2\pi\cdot 1500\, t + \frac{\pi}{2}\right)$

c) $x(t) = 5e^{-j\frac{\pi}{2}}e^{-j2\pi\cdot 1500 t} - 5 + 5e^{j\frac{\pi}{2}}e^{j2\pi\cdot 1500 t}$

d)



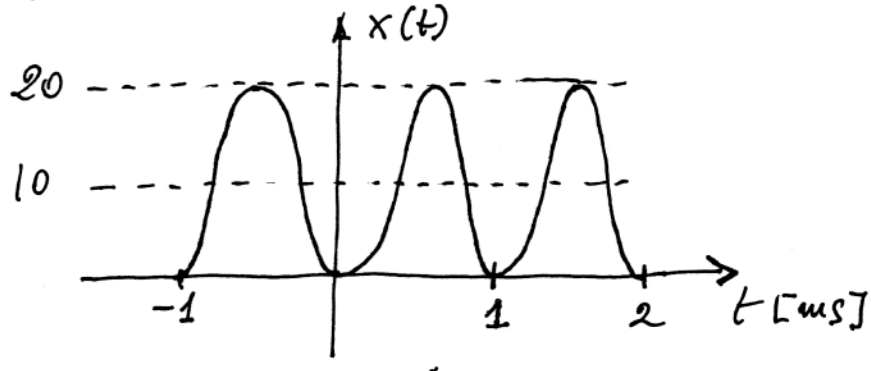
3.2) a) $$20\left(\frac{e^{j1000\pi t} - e^{-j1000\pi t}}{2j}\right)^2 =$$

$$= -5\left(e^{j2000\pi t} - 2 + e^{-j2000\pi t}\right) =$$

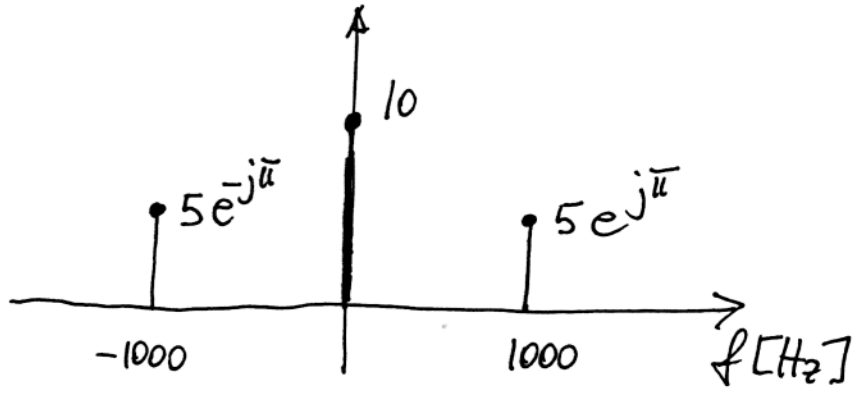
$$= -5e^{j2000\pi t} + 10 - 5e^{-j2000\pi t}$$

b) $x(t) = 10 - 10\cos(2000\pi t) = 10 + 10\cos(2000\pi t + \pi)$

c) $f = 1000\ Hz \Rightarrow T_0 = 1\ ms$



d)



3.3) $x(t) = \frac{j}{2\pi} e^{-j2\pi \cdot 150t} - \frac{1}{2\pi} e^{-j2\pi \cdot 100t} +$

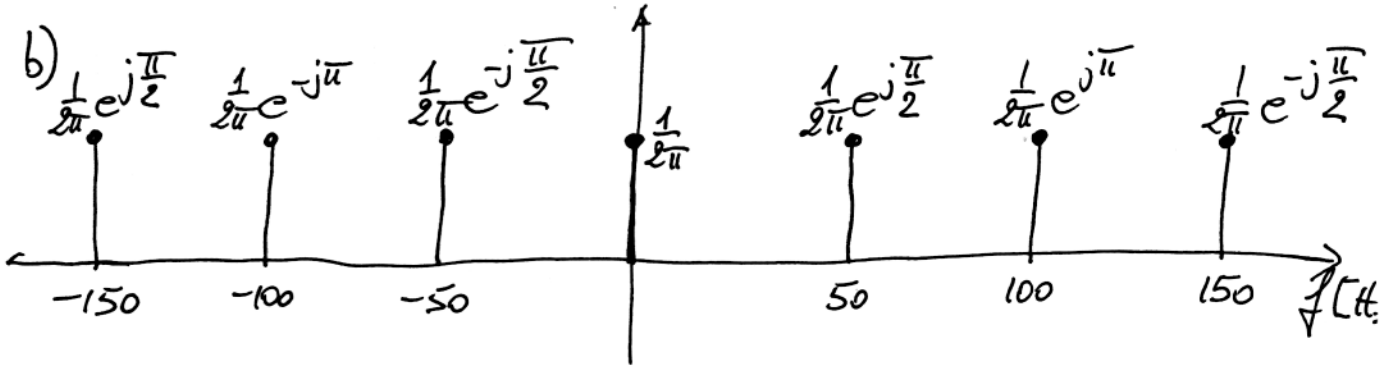
$- \frac{j}{2\pi} e^{-j2\pi \cdot 50t} + \frac{1}{2\pi} + \frac{j}{2\pi} e^{j2\pi \cdot 50t} +$

$- \frac{1}{2\pi} e^{j2\pi \cdot 100t} - \frac{j}{2\pi} e^{j2\pi \cdot 150t}$

a) $x(t) = \frac{1}{2\pi} + \frac{1}{\pi}\cos\left(2\pi \cdot 50t + \frac{\pi}{2}\right) + \frac{1}{\pi}\cos\left(2\pi \cdot 100t + \pi\right) +$

$+ \frac{1}{\pi}\cos\left(2\pi \cdot 150t - \frac{\pi}{2}\right)$

b)



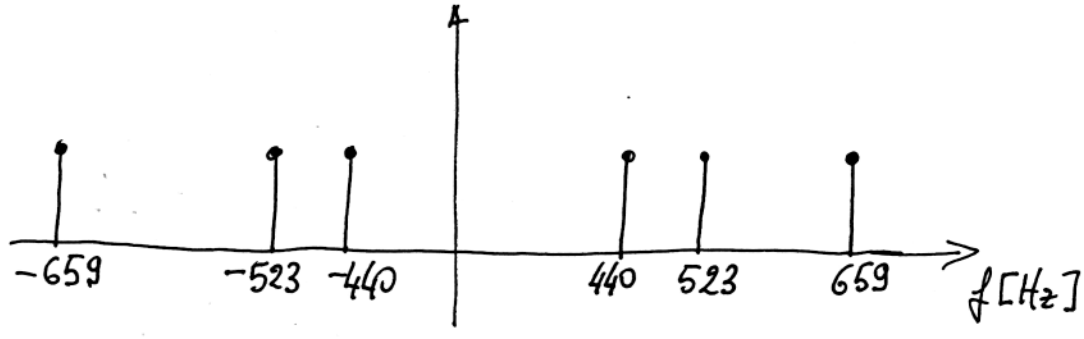
c) $f_0 = 50\,Hz \Rightarrow T_0 = \frac{1}{f_0} = 20\,ms$

3.4) a) The ratio between the frequencies of consecutive notes is constant, and it is equal to $\sqrt[12]{2} \cong 1.0595$

	A	B^b	B	C	$C^\#$	D	E^b	E	F	$F^\#$	G	$G^\#$	A
	49	50	51	52	53	54	55	56	57	58	59	60	61
f[Hz]	440	466	494	523	554	587	622	659	698	740	784	831	880

b) $f_m = 440\left(\sqrt[12]{2}\right)^{m-49} = 440 \cdot 2^{\frac{m-49}{12}}$

c)



3.5) $a \leftrightarrow 2 \quad a(t) = 4\cos(3\pi t + \frac{\pi}{2})$

$b \leftrightarrow 4 \quad b(t) = 4\cos(3\pi t) + 2.4\cos(4\pi t - \frac{\pi}{10})$

$c \leftrightarrow 5 \quad c(t) = 1.5 + 4\cos(3\pi t - \frac{2\pi}{5})$

$d \leftrightarrow 3 \quad d(t) = 4\cos(3\pi t - \frac{2\pi}{5})$

$e \leftrightarrow 1 \quad e(t) = 4\cos(3\pi t + \frac{\pi}{2}) +$

$+ 2.4\cos(4,8\pi t + \pi)$

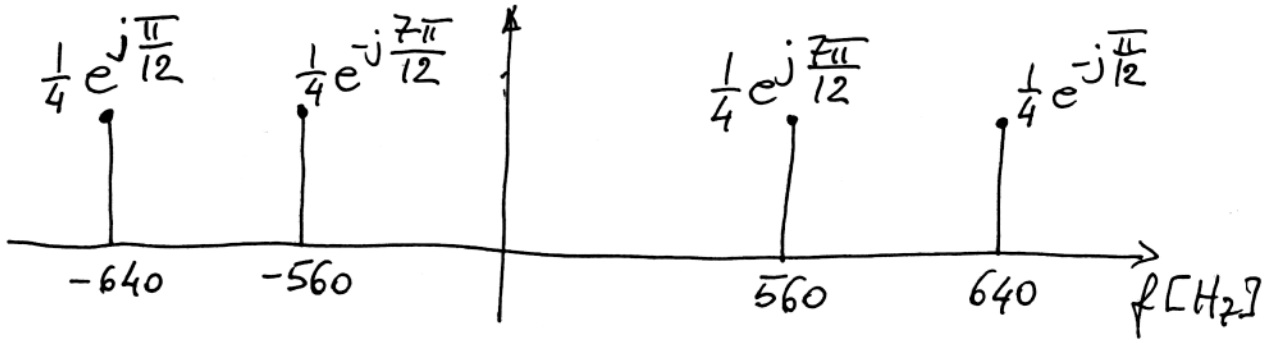
3.6) a) $x(t) = \frac{e^{j(2\pi \cdot 40t - \frac{\pi}{3})} + e^{-j(2\pi \cdot 40t - \frac{\pi}{3})}}{2} \times$

$\frac{e^{j(2\pi 600t + \frac{\pi}{4})} + e^{-j(2\pi 600t + \frac{\pi}{4})}}{2} =$

$= \frac{1}{4} e^{j(2\pi \cdot 640t - \frac{\pi}{12})} + \frac{1}{4} e^{-j(2\pi \cdot 560t + \frac{7}{12}\pi)} +$

$+ \frac{1}{4} e^{j(2\pi \cdot 560t + \frac{7}{12}\pi)} + \frac{1}{4} e^{-j(2\pi \cdot 640t - \frac{\pi}{12})}$

b)



c) $z(t) = \frac{1}{2} e^{j(2\pi \cdot 640 t - \frac{\pi}{12})} + \frac{1}{2} e^{j(2\pi \cdot 560 t + \frac{7}{12}\pi)}$

d) $x(t) = \frac{1}{2} \cos(2\pi \cdot 560 t + \frac{7}{12}\pi) +$

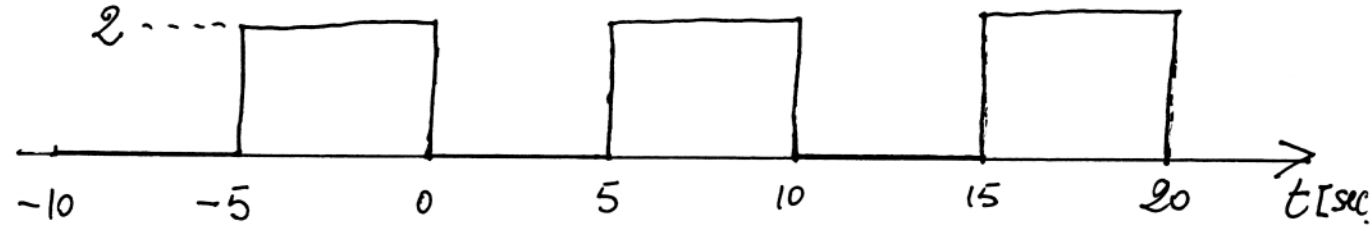
$+ \frac{1}{2} \cos(2\pi \cdot 640 t - \frac{\pi}{12})$

$A = B = \frac{1}{2} \quad f_c = 600 Hz \quad \Delta = 40 Hz$

$\phi_1 = \frac{7\pi}{12} \quad \phi_2 = -\frac{\pi}{12}$

e) $f_0 = 80 Hz \Rightarrow T_0 = 12.5 ms$

3.7) a)



b) $a_0 = \frac{1}{T_0} \int_0^{T_0} x(t)\,dt = \frac{1}{10} \int_5^{10} 2 \cdot dt = 1$

c) $a_1 = \frac{1}{10} \int_{-5}^{5} x(t) e^{-j\omega_0 t} dt = \frac{1}{10} \int_{-5}^{0} 2 e^{-j\omega_0 t} dt =$

$= \frac{1}{5} \left[\frac{e^{-j\omega_0 t}}{-j\omega_0} \right]_{t=-5}^{t=0} = \frac{j}{5\frac{2\pi}{10}} \left(1 - e^{j\frac{2\pi}{10}5}\right) = \frac{j}{\pi}\left(1 - e^{j\pi}\right) =$

$= \frac{2j}{\pi}$

d) Adding a constant value to $x(t)$ changes only its DC coefficient; all the other coefficients remain unchanged.

$$b_0 = a_0 + 1 = 2$$

$$b_1 = a_1 = \frac{2j}{\pi}$$