1. sol:

$$\cos(1000\pi t) = \frac{1}{2}\left(e^{j1000\pi t} + e^{-j1000\pi t}\right)$$

$$\sin(250\pi t) = \frac{1}{2j}\left(e^{j250\pi t} - e^{-j250\pi t}\right)$$

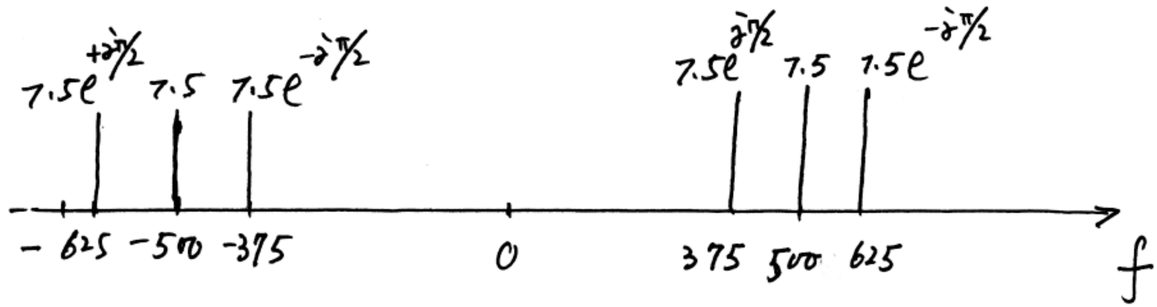
$$x(t) = 7.5e^{j1000\pi t} + 7.5e^{-j1000\pi t}$$

$$+ 7.5(-j)\left(e^{j1250\pi t} + e^{-j750\pi t} - e^{j750\pi t} - e^{-j1250\pi t}\right)$$

$$= 7.5\left[e^{j1000\pi t} + e^{-j1000\pi t} + e^{-j\pi/2}\left(e^{j1250\pi t} + e^{-j750\pi t} - e^{j750\pi t} - e^{-j1250\pi t}\right)\right]$$

$$= 7.5\left[e^{j1000\pi t} + e^{-j1000\pi t} + e^{-j\pi/2}e^{j1250\pi t} + e^{-j\pi/2}e^{-j750\pi t} + e^{j\pi/2}e^{j750\pi t} + e^{j\pi/2}e^{-j1250\pi t}\right]$$

(a) Only need to sketch the complex amplitudes at the corresponding frequencies with the carrier freq. being 1000π, and freq. from modulated signal being 250π.



(b) The waveform is periodic with a period T :

$$T = \frac{1}{\gcd(375, 500, 625)} = \frac{1}{125} \text{ sec.}$$

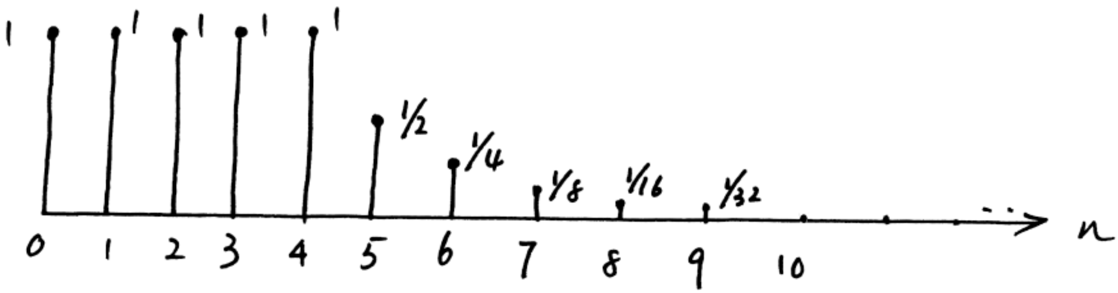
(c) $f_{max} = 625$ Hz

Need $f_s = 2 f_{max}$

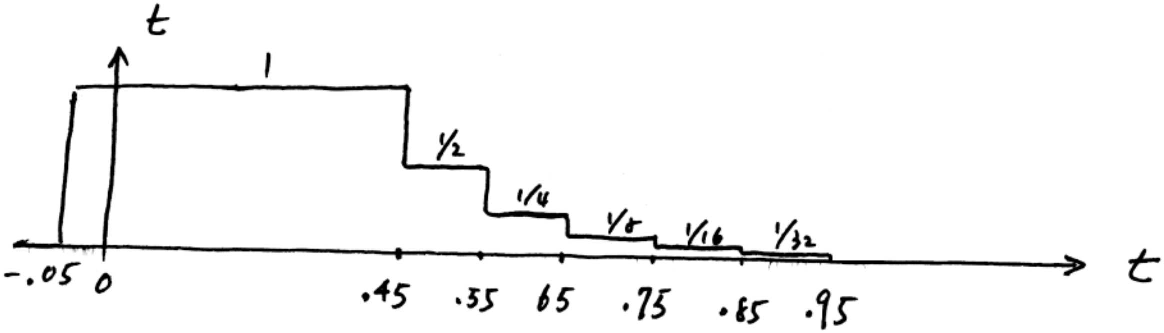
$= 1250$ Hz to recover the original $x(t)$.

2. sol:

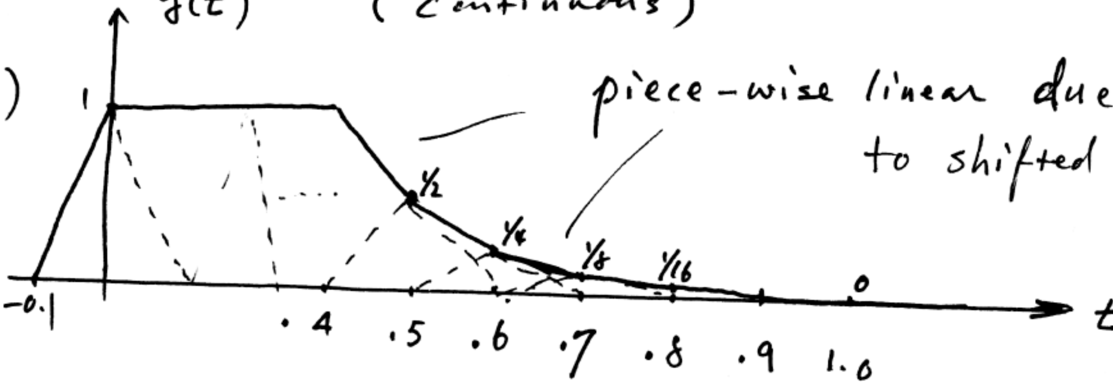
(a) $y[n]$ (discrete)



(b) $y(t)$ (continuous)

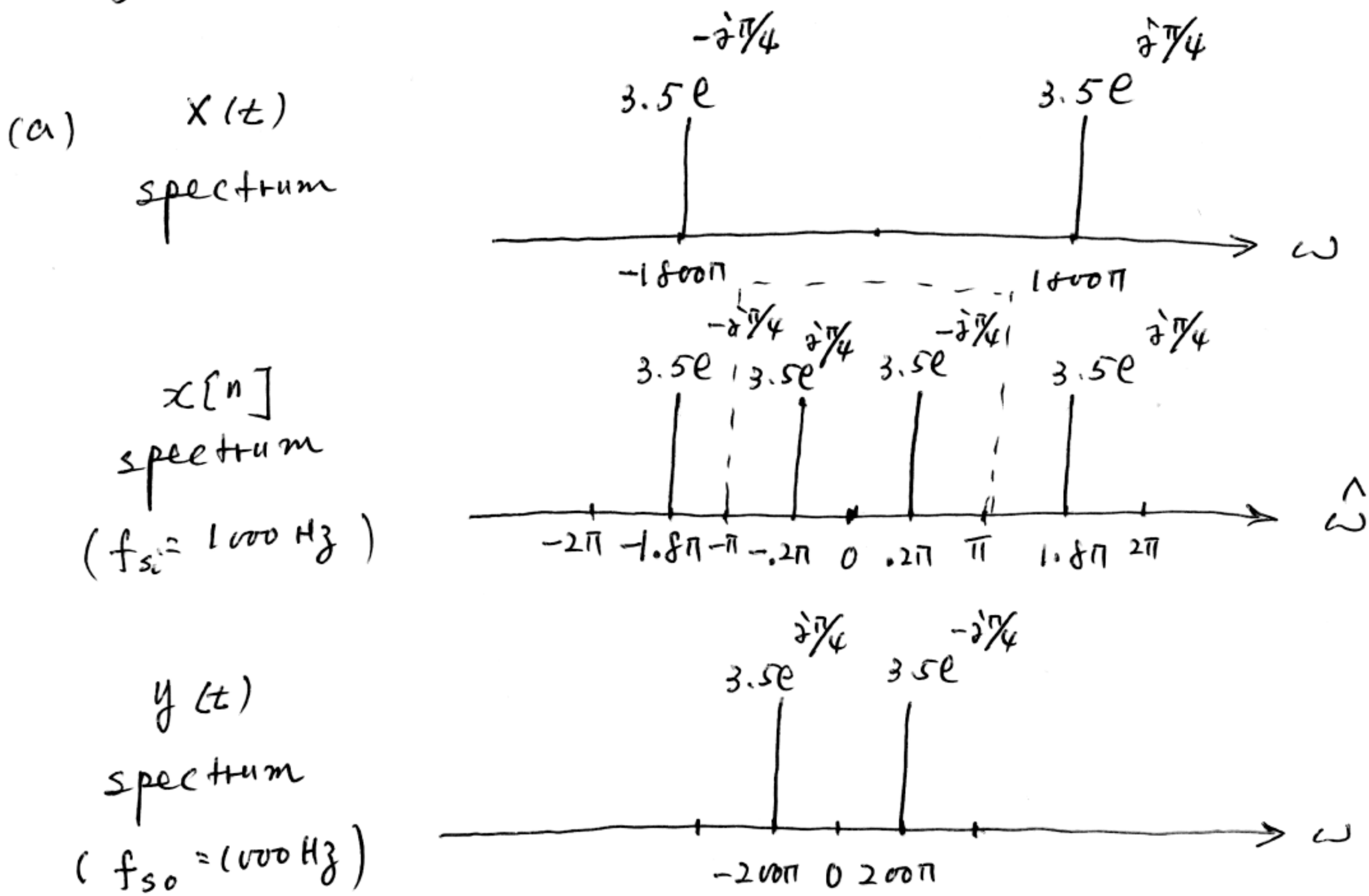


(c) $y(t)$ (continuous)



piece-wise linear due to shifted $p(t)$

5.3 sol:



so input $x(t) = 7\cos(1800\pi t + \pi/4)$

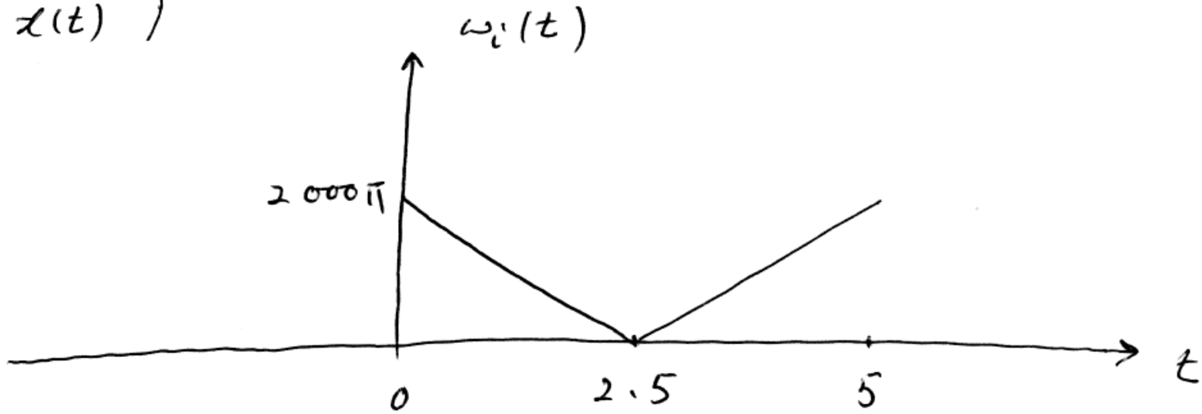
output $y(t) = 7\cos(200\pi t - \pi/4)$

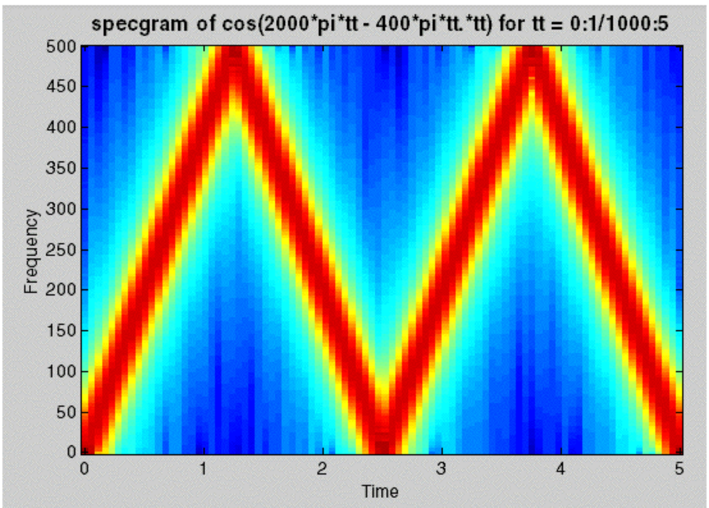
since $f_s < 2f$, alaising occurs (under-sampling which causes the loss of high freq.).

(b) Instenteneous freq. $\omega_i(t) = \left| \frac{d\varphi_i(t)}{dt} \right|$

$$\omega_i(t) = |2000\pi - 800\pi t| \qquad 0 \le t \le 5$$

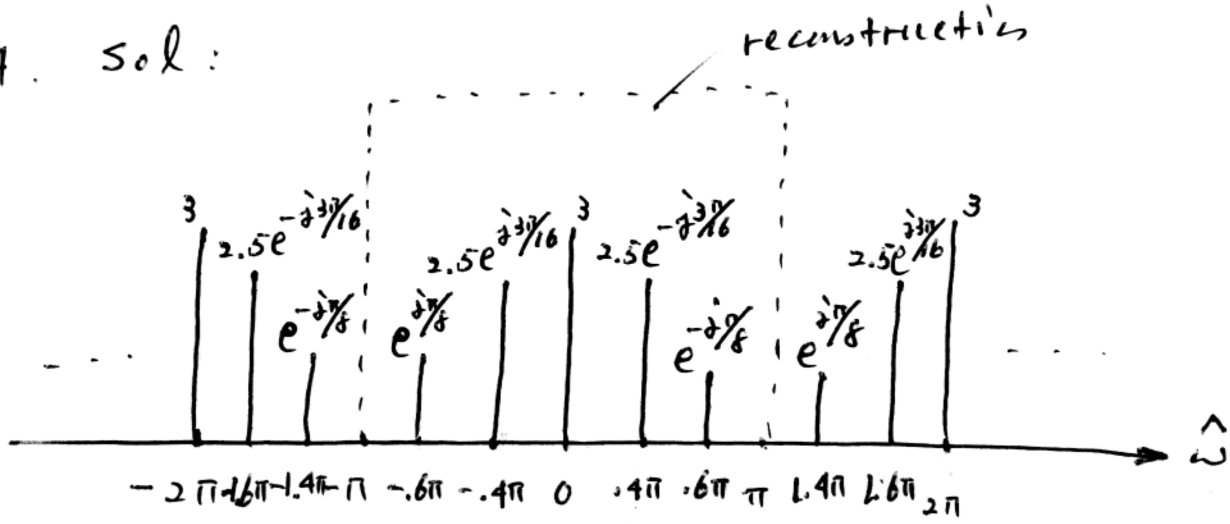
(for $x(t)$)





5.4. Sol:

(a)



(Three steps in plotting the spectrum:

- Locations of $\hat{\omega}$ from $x(t)$
- Amplititudes
- Those (locations and amplitudes) of alaises. Here $\pm.4\pi$, $\pm1.4\pi$, $\pm2\pi$ are alaises)

$\hat{\omega} = \omega / f_{si}$, $f_{si} = 10000$, $\omega_y = \hat{\omega} f_{so}$

ω (from $x(t)$)	0		$\pm2\pi\cdot3000$		$\pm2\pi\cdot8000$	
$\hat{\omega}$ (for $x[n]$)	0	$\pm.4\pi$	$\pm.6\pi$	$\pm1.4\pi$	$\pm1.6\pi$	[illegible]
(b) ω_y (for $y(t)$, $f_{so}=1000$)	0	$\pm2\pi\cdot2000$	$\pm2\pi\cdot3000$			
(c) ω_y (" , $f_{so}=2000$)	0			$\pm2\pi\cdot4000$	$\pm2\pi\cdot6000$	

Discussions:

① $f_{si} < 2f_{max}$: undersampling which causes alaising (the loss of high freq. component $(2\pi\cdot8000)$ in $x(t)$)

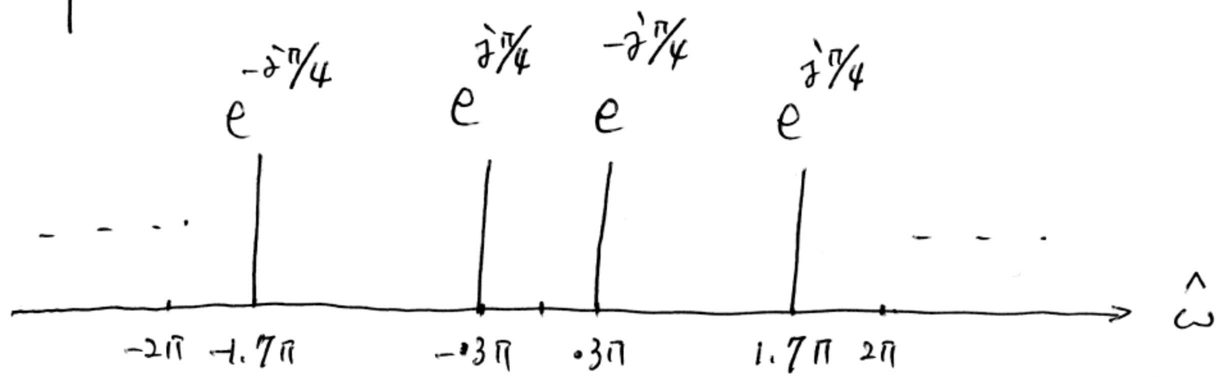
② The lost information can not be recovered through a larger f_{so}.

5.5. Sol:

(a) $x[n] = 2\cos(0.3\pi n - \pi/4)$

$\hat{\omega} = 0.3\pi$, $\omega = \hat{\omega} \cdot f_s$, $f_s = 10000$

$x[n]$ spectrum



$\hat{\omega}$	$x_1[n]$
0.3π	$2\cos(0.3\pi n - \pi/4)$
ω	$x_1(t)$
3000π	$2\cos(3000\pi t - \pi/4)$
$\hat{\omega}$	$x_2[n]$
1.7π	$2\cos(1.7\pi n + \pi/4)$
ω	$x_2(t)$
17000π	$2\cos(17000\pi t + \pi/4)$

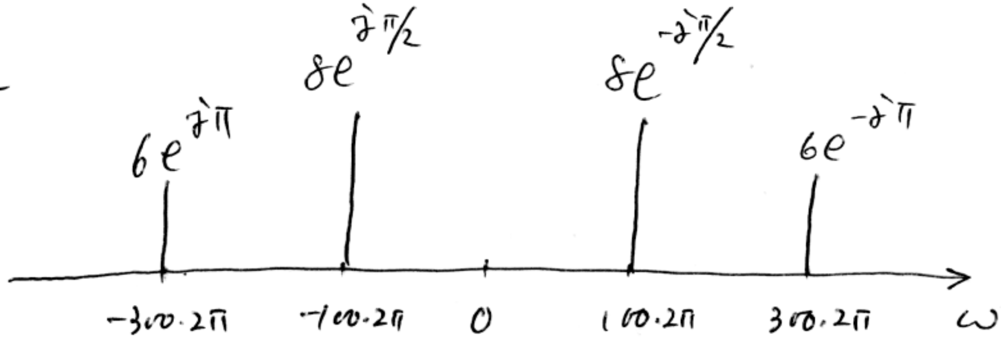
The same

different

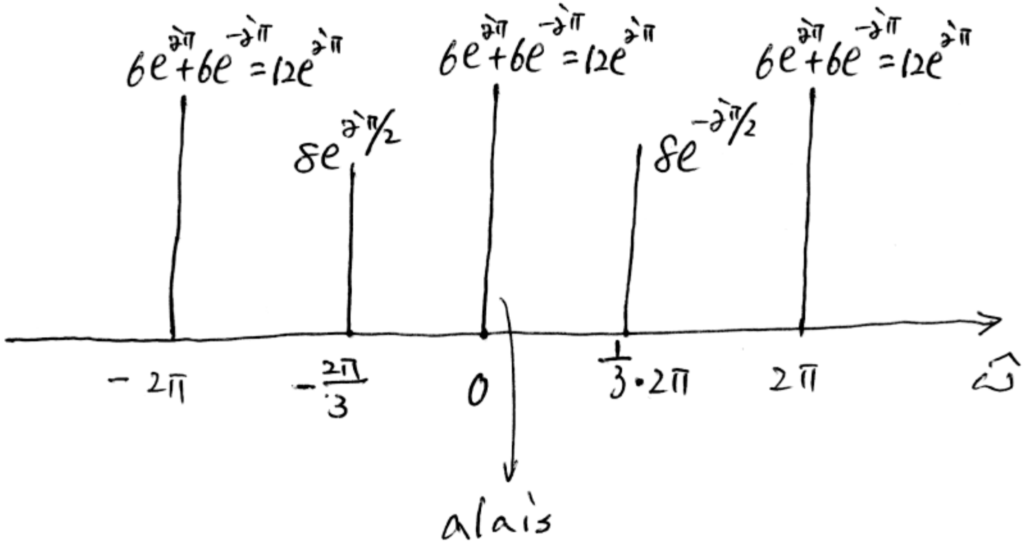
(b) $f_{max} = 300$ Hz

$f_s = 2 f_{max} = 600$ Hz

1(c) $x(t)$ spectrum



$x[n]$ spectrum ($f_s = 300$)



($f_s < 2 f_{max} \rightarrow$ aleising)

5.6 Sol:

(a) $\omega = 10 \cdot 2\pi$ ("+" sign ← counter clock wise rotation)

A rotating phaser:

$$p(t) = r e^{j\omega t + j\varphi}$$

$$= r e^{j\varphi} e^{j 2\pi \cdot 10 t}$$

φ: initial angle, r: radius.

(A ~~trodjectory~~ of a rotating phaser generates a continuous cosine function in time)

(b) n flash / sec : n samples of the cosine wave / sec.

Sampling frequency : $f_s = n$

$$\hat{\omega} = \frac{20\pi}{n}$$

For disk to be still, $\hat{\omega} = 2\pi l$, l: integer

Then $\frac{20\pi}{n} = 2\pi l$

$$n = \frac{10}{l}$$

n	10	5	2	1
l	1	2	5	10

⇒ possible flash rates: 10/sec, 1/sec, 2/sec, 5/sec.

(c) $f_s = \frac{1}{T_s}$

$= \frac{1}{0.06}$ flashes / sec

Rotating phasor

$$p[n] = re^{j\varphi} e^{j\frac{2\pi \cdot 10}{f_s} \cdot n + j2\pi \ell n}$$

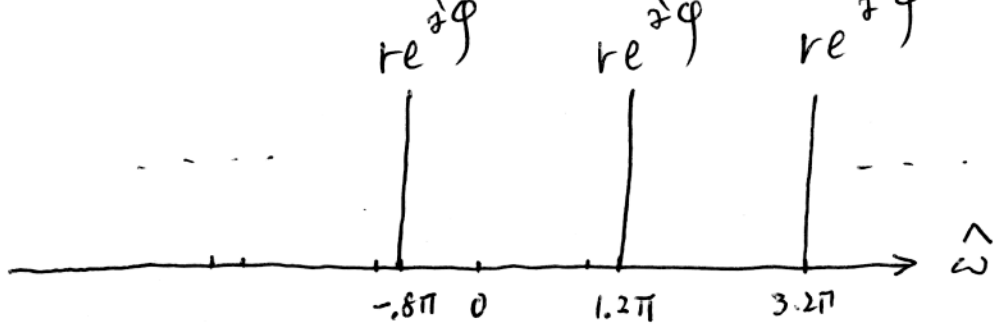
$$= re^{j\varphi} e^{j1.2\pi n + j2\pi \ell n}$$

$\ell = 0, \pm 1, \pm 2, \cdots$

For every flash, the phasor moves 1.2π $(216°)$ (or $-.8\pi$ $(-144°)$ equivalently).

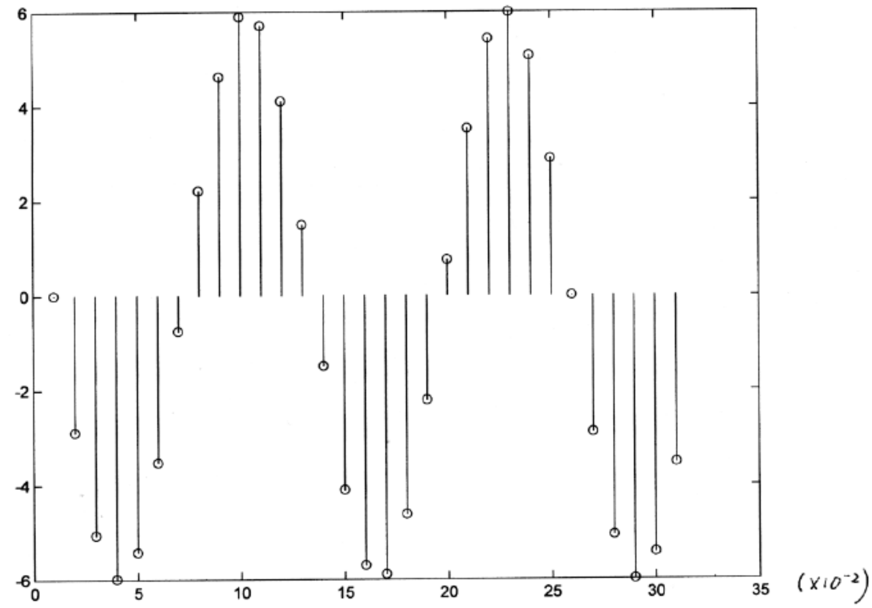
$\frac{-.8\pi}{2\pi} \cdot \frac{1}{0.06} = -\frac{20}{3}$ so the phasor makes $\frac{20}{3}$ revolutions / sec. clockwise

(d)



5.7 Sol.:

(a)

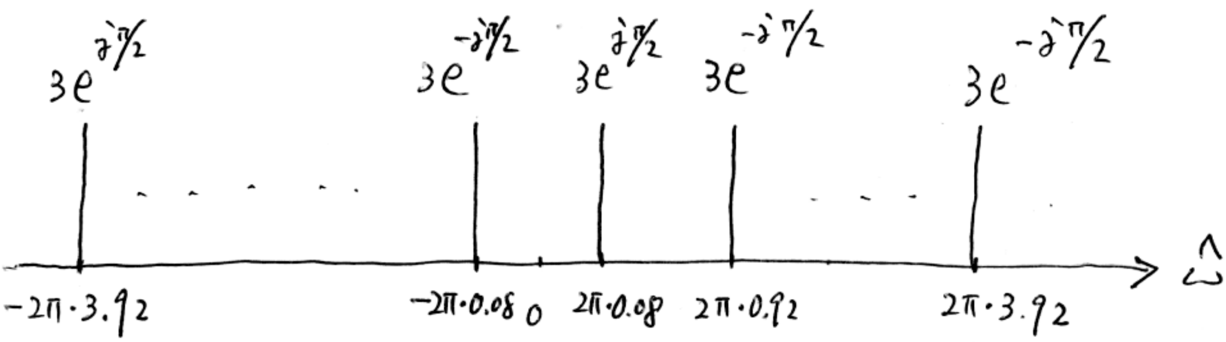


(b) $f_s = \frac{1}{T_s} = 100$

$x(t) = 6\cos(2\pi \cdot 392t - \pi/2)$, $\omega = 392$

$\hat{\omega} = \frac{2\pi \cdot 392}{f_s} = 2\pi \cdot 3.92$

$x[n]$ spectrum



(c) $x[n] = 6\cos(2\pi \cdot 0.08n + \pi/2)$

(d) $2\pi \cdot 0.08 \times f_s = 2\pi \cdot 8$

$y(t) = 6\cos(2\pi \cdot 8t + \pi/2)$ is what we see: a sinusoid with $f_0 = 8 \ll 392$.