Solutions P.S. #6

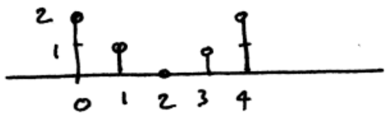
Problem 1:

a) $$y[n] = \sum_{k=0}^{4} |2-k|\, x[n-k]$$

$$= 2x[n] + x[n-1] + x[n-3] + 2x[n-4]$$

$$\Rightarrow \{b_k\} = \{2, 1, 0, 1, 2\}$$

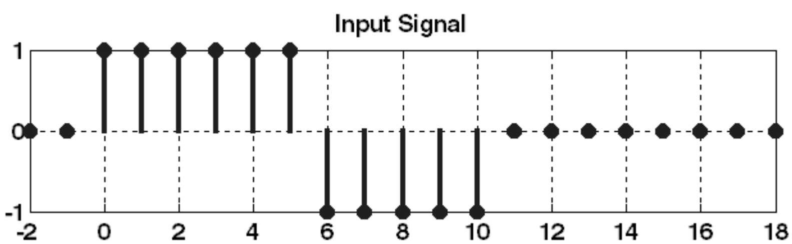
b)

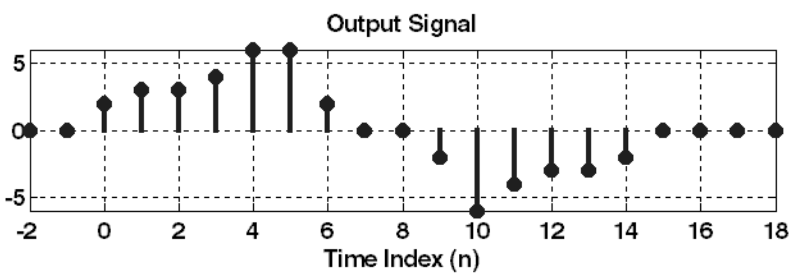


$$h[n] = 2\delta[n] + \delta[n-1] + \delta[n-3] + 2\delta[n-4]$$

c)

n	<0	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	>14
$x[n]$	0	1	1	1	1	1	1	-1	-1	-1	-1	-1	0				
$h[n]$	0	2	1	0	1	2	0										
$h[0]\,x[n-0]$	0	2	2	2	2	2	2	-2	-2	-2	-2	-2	0				
$h[1]\,x[n-1]$		0	1	1	1	1	1	1	-1	-1	-1	-1	-1	0			
$h[2]\,x[n-2]$			0												0		
$h[3]\,x[n-3]$				0	1	1	1	1	1	1	-1	-1	-1	-1	-1	0	
$h[4]\,x[n-4]$					0	2	2	2	2	2	2	-2	-2	-2	-2	-2	0
$y[n]$	0	2	3	3	4	6	6	2	0	0	-2	-6	-4	-3	-3	-2	0





Problem 2 :

a)

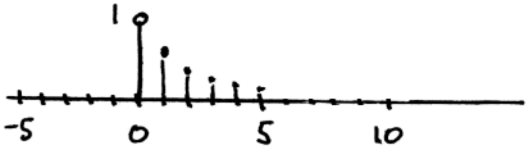
b) $x[n] = \left(\frac{2}{3}\right)^n \left(u[n] - u[n-6]\right)$

Since $u[n] - u[n-6] = 0$ if $n < 0$ or $n \geq 6$

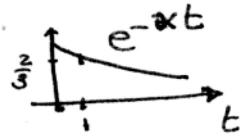
$= 1$ if $0 \leq n \leq 5$

It follows that $x[n] = \left(\frac{2}{3}\right)^n$, $0 \leq n \leq 5$

0 else



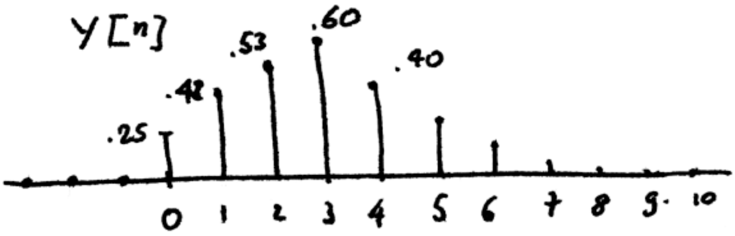
<u>Note</u>: the envelope is an exponential



from: $e^{-\alpha \cdot 1} = \frac{2}{3} \Rightarrow \alpha = -\ln\frac{2}{3} = 0.4055$

c)

$$
\begin{aligned}
y[n] &= 0 \qquad \text{for } n < 0 \\
y[0] &= \tfrac{1}{4}x[0] = \tfrac{1}{4} \\
y[1] &= \tfrac{1}{4}(x[1] + x[0]) = \tfrac{1}{4}\left(\tfrac{2}{3} + 1\right) = \tfrac{5}{12} = 0.4167 \\
y[2] &= \tfrac{1}{4}(x[2] + x[1] + x[0]) = \tfrac{1}{4}\left((\tfrac{2}{3})^2 + \tfrac{2}{3} + 1\right) = \tfrac{19}{36} = 0.5278 \\
y[3] &= \tfrac{1}{4}(x[3] + x[2] + x[1] + x[0]) = \tfrac{1}{4}\left((\tfrac{2}{3})^3 + (\tfrac{2}{3})^2 + \tfrac{2}{3} + 1\right) = 0.6019 \\
y[4] &= \tfrac{1}{4}(x[4] + x[3] + x[2] + x[1]) = \tfrac{1}{4}\left((\tfrac{2}{3})^4 + (\tfrac{2}{3})^3 + (\tfrac{2}{3})^2 + \tfrac{2}{3}\right) = 0.4012 \\
y[5] &= \tfrac{1}{4}(x[5] + x[4] + x[3] + x[2]) = \tfrac{1}{4}\left((\tfrac{2}{3})^5 + (\tfrac{2}{3})^4 + (\tfrac{2}{3})^3 + (\tfrac{2}{3})^2\right) = 0.2675 \\
y[6] &= \tfrac{1}{4}(x[5] + x[4] + x[3]) = \tfrac{1}{4}\left((\tfrac{2}{3})^5 + (\tfrac{2}{3})^4 + (\tfrac{2}{3})^3\right) = 0.1564 \\
y[7] &= \tfrac{1}{4}(x[5] + x[4]) = \tfrac{1}{4}\left((\tfrac{2}{3})^5 + (\tfrac{2}{3})^4\right) = 0.0823 \\
y[8] &= \tfrac{1}{4}(x[5]) = \tfrac{1}{4}(\tfrac{2}{3})^5 = 0.0329 \\
y[n] &= 0 \qquad \text{for } n \geq 9
\end{aligned}
$$



Problem 6.3 $\quad y[n] = \sum_{10}^{20} b_k \, x[n-k]$

$x[n]$ nonzero for $5 \le n \le 20$ only

The first nonzero sample of the input x launches a response $x[5]\,h[n-5]$. Since h is zero when its argument is less than 10 $\Rightarrow n-5 < 10 \Rightarrow n < 15$

greater than 20 $\Rightarrow n-5 > 20 \Rightarrow n > 25$

$\therefore \; x[5]\,h[n-5]$ is zero for $n \le 14$, $n \ge 25$

The last nonzero sample of the input launches a response $x[20]\,h[n-20]$

This is zero when $n-20 < 10 \Rightarrow n < 30$

$n-20 > 20 \Rightarrow n > 40$

Consequently, one is assured that the response of the given sequence $x[n]$ is zero if $n \le 14$ and $n \ge 41$

$\therefore$ y is nonzero at most in the range

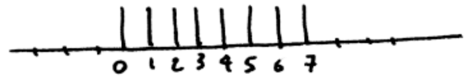
$$14 < n < 41$$

Problem 6.4

(a) $b_k = \begin{cases} 0 & k<0 \\ 0.125 & 0 \le k \le 7 \\ 0 & k>7 \end{cases}$ $\qquad y[n] = \frac{1}{8}\left(x[n] + x[n-1] + \cdots + x[n-7]\right)$

(8-term running average)

(b) If $x[n] = \delta[n] \Rightarrow v[n] = \begin{cases} \frac{1}{8} & 0 \le n \le 7 \\ 0 & \text{else} \end{cases}$



(c) $y[n] = -2v[n] + 2v[n-1]$ $\qquad h_2[n] = \begin{cases} -2 & n=0 \\ 2 & n=1 \\ 0 & \text{else} \end{cases}$

(d) $h_{12}[n] = (h_1 * h_2)[n]$

n	0	1	2	3	4	5	6	7	8	...
$h_1[n]$	1/8	1/8	1/8	1/8	1/8	1/8	1/8	1/8	0	...
$h_2[n]$	-2	2								
$h_2[0]h_1[n]$	-1/4	-1/4	-1/4	-1/4	-1/4	-1/4	-1/4	-1/4	0	...
$h_2[1]h_1[n-1]$	0	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	0 ...
$h_{12}[n]$	-1/4	0	0	0	0	0	0	0	$\frac{1}{4}$	0 ...

(e) $y[n] = -\frac{1}{4}x[n] + \frac{1}{4}x[n-8]$

Problem 6.5:

(a) linear, time invariant, noncausal
($y[n]$ depends on $x[n+2]$)

(b) linear, causal, timevarying
$\because$ if $x[n] = \delta[n] \Rightarrow y[n] = n\delta[n] \equiv 0$ check!
but if $x[n] = \delta[n-1] \Rightarrow y[n] = n\delta[n-1] = \begin{cases} 1 & \text{if } n=1 \\ 0 & \text{else} \end{cases}$
not the shifted version!

(c) nonlinear (the input $2x[n]$ produces a 4 times larger output than $x[n]$.)
noncausal (e.g. at $n=-1$: $y[-1] = (x[1])^2$, depends on the future x)
timevarying (if $x[n] = \delta[n] \to y[n] = \delta[n]$
but if $x[n] = \delta[n-1] \to y[n] = \delta[n+1]$)

Problem 6.6 (a) Nonlinear (if input is scaled by α, output is scaled by α^3)

Time invariant (shifting input by k produces
$$(x[n+1-k])^3 = (x[n-k+1])^3 = y[n-k]$$
$\therefore$ corresponding output is also shifted by k.)

Noncausal: present of x depends on future of x

(b) $x_1[n] = -j e^{0.75\pi n} + j e^{-0.75\pi n}$

$$\Rightarrow y_1[n] = \left[-j e^{0.75\pi(n+1)} + j e^{-0.75\pi(n+1)}\right]^3$$

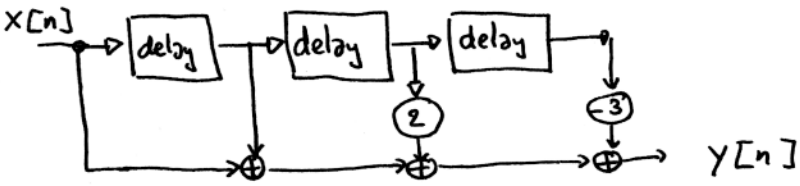
$$= \underbrace{-j^3}_{= j} e^{2.25\pi(n+1)} - 3j e^{0.75\pi(n+1)} + 3j e^{-0.75\pi(n+1)} + \underbrace{j^3}_{= -j} e^{-2.25\pi(n+1)}$$

$$= 6 \sin(0.75\pi(n+1)) - 2 \sin(2.25\pi(n+1))$$

Note that $\sin(2.25\pi(n+1)) = \sin(0.25\pi(n+1))$

$\downarrow$ frequency $\frac{0.25}{2}$ appears! aliasing.

Problem 6.7 (a)



(b)
$$h[n] = \begin{cases} 0 & n < 0 \\ 1 & n = 0, 1 \\ 2 & n = 2 \\ -3 & n = 3 \\ 0 & n > 3 \end{cases}$$

(c) input $\delta[n]$ produces output $h[n]$

$\delta[n-1]$ " " $h[n-1]$

$\delta[n-2]$ " " $h[n-2]$

$\delta[n-3]$ " " $h[n-3]$

$\Rightarrow x[n] = \delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3]$ " " $y[n] = h[n] + h[n-1] + h[n-2] + h[n-3]$

n	<0	0	1	2	3	4	5	6	7	>7
$h[n]$	0	1	1	2	−3	0				
$h[n-1]$		0	1	1	2	−3	0			
$h[n-2]$			0	1	1	2	−3	0		
$h[n-3]$				0	1	1	2	−3	0	0
$y[n]$	0	1	2	4	1	0	−1	−3	0	0